Math 311
Spring 2018
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Day #7 Notes: Cardinality and More

February 2, 2018

Contents

1 Cardinality

Complete the worksheet in class and we will go over it together:

Definition 1 *Two sets A and B have the same cardinality if there exists a function $f : A \to B$ that is one-to-one and onto. In this case, we write $A \sim B$.*

Definition 2 *We say a set has cardinality n if $A \sim \{1, \ldots, n\}$. If $\exists n \in \mathbb{N}$ so that A has cardinality n, we say that A is finite. We say a set A is countable if $A \sim \mathbb{N}$. If A is neither finite nor countable, then we say A is uncountable.*

1. [T/F] If $A = \{1, 2, 3\}$ and $B = \{e, \pi, \sqrt{2}\}$ then $A \sim B$.

2. [T/F] The even integers $2\mathbb{Z}$ have the same cardinality as the integers; that is, $2\mathbb{Z} \sim \mathbb{Z}$.

3. [T/F] $\mathbb{Z} \sim \mathbb{N}$; that is, the integers are countable.

4. [T/F] $\mathbb{Q}$ is countable.

5. [T/F] $\mathbb{R}$ is countable.

2 And More: Infinite Series

What does $\sum_{i=1}^{\infty} \frac{(-1)^n}{2^n}$ equal?

What does $\sum_{i=1}^{\infty} \frac{(-1)^i}{i}$ equal?

Are you sure?

How can you add up the following?

$$\begin{bmatrix} -1 & \frac{1}{2} & \frac{1}{4} & \frac{1}{8} & \dots \\ 0 & -1 & \frac{1}{2} & \frac{1}{4} & \dots \\ 0 & 0 & -1 & \frac{1}{2} & \dots \\ 0 & 0 & 0 & -1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

What about $1 + (-1) + 1 + (-1) + 1 + (-1) + \dots$?

3 Sequences

What is a sequence?

Examples...

1. $(1, 2, 3, \ldots)$
2. $\left(\frac{1}{n}\right)_{n=1}^{\infty}$.
3. (a_n) where $a_n = \frac{1}{2^n} \forall n \in \mathbb{N}$.
4. (x_n) where $x_1 = 1$ and $x_n = 3x_n - 1 \forall n \in \mathbb{N}$.

What is the difference between a sequence and a set?

4 Limits

What does it mean to say that $\lim_{n \to \infty} \frac{1}{n} = 0$?

Definition 3 *Let (a_n) be a sequence of real numbers. We say that (a_n)* converges *to a real number a if, for every $\epsilon > 0$, there is an $N \in \mathbb{N}$ so that, $\forall n > N$, $|a_n - a| < \epsilon$.*

What's so great about this definition?

5 Conclusions

Today we learned about:

1. The Cardinality of $\mathbb{Q}$ and $\mathbb{R}$.
2. A first look at the vagaries of infinite series.

Monday we will learn about:

1. More on Sequences.

Upcoming Deadlines:

- Wednesday Feb 7, 2018: Homework #2.