Math 311
Spring 2018
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Day #8 Notes: Sequences and Limits

February 5, 2018

Contents

1 Infinite Series

What does $\sum_{i=1}^{\infty} \frac{(-1)^n}{2^n}$ equal?

What does $\sum_{i=1}^{\infty} \frac{(-1)^i}{i}$ equal?

Are you sure?

How can you add up the following?

$$\begin{bmatrix} -1 & \frac{1}{2} & \frac{1}{4} & \frac{1}{8} & \dots \\ 0 & -1 & \frac{1}{2} & \frac{1}{4} & \dots \\ 0 & 0 & -1 & \frac{1}{2} & \dots \\ 0 & 0 & 0 & -1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

What about $1 + (-1) + 1 + (-1) + 1 + (-1) + \dots$?

2 Sequences

What is a sequence?

Examples...

1. $(1, 2, 3, \ldots)$
2. $\left(\frac{1}{n}\right)_{n=1}^{\infty}$.
3. (a_n) where $a_n = \frac{1}{2^n} \quad \forall n \in \mathbb{N}$.
4. (x_n) where $x_1 = 1$ and $x_n = 3x_n - 1 \quad \forall n \in \mathbb{N}$.

What is the difference between a sequence and a set?

3 Limits

What does it mean to say that $\lim_{n\to\infty} \frac{1}{n} = 0$?

Definition 1 *Let* (a_n) *be a sequence of real numbers. We say that* (a_n) converges *to a real number* a *if, for every* $\epsilon > 0$*, there is an* $N \in \mathbb{N}$ *so that,* $\forall n > N$, $|a_n - a| < \epsilon$.

What's so great about this definition?

4 Worksheet

Complete the worksheet and we will go over it together:

1. Fill in the holes in the proof below.

Proof. Let "________", and choose $N \in \mathbb{N}$ such that ________. Suppose ________. Then

$$\begin{aligned} \left|\frac{n+1}{n} - 1\right| &= |\_\_\_\_\_\_\_\_| && \text{(simplify this algebraically)} \\ &< \_\_\_\_\_\_\_\_ && \text{(convert from } n \text{ to } N\text{)} \\ &< \epsilon. && \text{(use your choice of N to draw this conclusion)} \end{aligned}$$

Therefore, if $n > N$, we have $\left|\frac{n+1}{n} - 1\right| < \epsilon$, as desired. □

2. Prove that $\lim_{n\to\infty} \sin(n^2)/n^2 = 0$.

3. Complete the statement: To show $\left(\frac{n+1}{n}\right)$ does not converge to -37, we must show that ...

4. Prove that $\left(\frac{n+1}{n}\right)$ does not converge to -37.

5. If (a_n) converges to a real number a and also (a_n) converges to a real number b, then $a = b$.

5 Conclusions

Today we learned about:

1. Sequences

2. Limits

Wednesday we will learn about:

1. More on Sequence Proofs

2. Properties of Limits

Upcoming Deadlines:

- Wednesday, Feb 7, 2018 : Homework #2