Day #9 Notes: More on Limits

February 7, 2018

Contents

1 Worksheet

Complete the worksheet and we will go over it together:

1. Fill in the holes in the proof below.

Proof. Let "_____", and choose $N \in \mathbb{N}$ such that _____. Suppose _____. Then

$$
\begin{aligned}
\left| \tfrac{n+1}{n} - 1 \right| &= |\text{_____}| && \text{(simplify this algebraically)} \\
&< \text{_____} && \text{(convert from } n \text{ to } N) \\
&< \epsilon. && \text{(use your choice of N to draw this conclusion)}
\end{aligned}
$$

Therefore, if $n > N$, we have $\left| \tfrac{n+1}{n} - 1 \right| < \epsilon$, as desired. $\square$

2. Prove that $\lim_{n\to\infty} \sin(n^2)/n^2 = 0$.

3. Complete the statement: To show $\left(\frac{n+1}{n}\right)$ does not converge to -37, we must show that ...

4. Prove that $\left(\frac{n+1}{n}\right)$ does not converge to -37.

5. If (a_n) converges to a real number a and also (a_n) converges to a real number b, then $a = b$.

2 Properties of Limits

Definition 1 *A sequence is* bounded *if* $\exists M > 0$ *so that* $\forall n \in \mathbb{N}$, $|a_n| < M$.

Proposition 1 *Every convergent sequence is bounded.*

Theorem 1 *Suppose that* (a_n) *and* (b_n) *are sequences and* $a, b, c \in \mathbb{R}$. *Suppose* $a_n \to a$ *and* $b_n \to b$. *Then:*

1. *$(ca_n) \to ca$*

2. *$(a_n + b_n) \to a + b$*

3. *$(a_n b_n) \to ab$*

4. *If* $b \neq 0$, $(\frac{a_n}{b_n} \to \frac{a}{b}$.

Proof:

(continued)

3 Conclusions

Today we learned about:

1. Limits and Their Properties

Friday we will learn about:

1. More Properties of Limits

Upcoming Deadlines:

- Wednesday, Feb 14, 2018: Homework #3