

Day #10 Notes: Properties of Limits

February 9, 2018

Contents

1	Properties of Limits	2
2	Conclusions	8

1 Properties of Limits

Definition 1 *A sequence is bounded if $\exists M > 0$ so that $\forall n \in \mathbb{N}$, $|a_n| < M$.*

Proposition 1 *Every convergent sequence is bounded.*

Theorem 1 (Algebraic Limit Theorem) *Suppose that (a_n) and (b_n) are sequences and $a, b, c \in \mathbb{R}$. Suppose $a_n \rightarrow a$ and $b_n \rightarrow b$. Then:*

1. $(ca_n) \rightarrow ca$

2. $(a_n + b_n) \rightarrow a + b$

3. $(a_nb_n) \rightarrow ab$

4. If $b \neq 0$, $(\frac{a_n}{b_n}) \rightarrow \frac{a}{b}$.

Proof:

(continued)

(continued)

Theorem 2 (Order Limit Theorem) *Suppose $a, b, c \in \mathbb{R}$ and $(a_n), (b_n)$ are sequences of real numbers so that $a_n \rightarrow a$ and $b_n \rightarrow b$. Then*

1. *If $a_n \geq 0 \forall n \in \mathbb{N}$, $a \geq 0$.*

2. *If $a_n \leq b_n \forall n \in \mathbb{N}$, $a \leq b$.*

3. *If $c \leq b_n \forall n \in \mathbb{N}$, $c \leq b$ and similarly, if $a_n \leq c \forall n \in \mathbb{N}$, then $a \leq c$.*

Proof:

(continued)

2 Conclusions

Today we learned about:

1. Properties of limits

Monday we will learn about:

1. The Monotone Convergence Theorem
2. Infinite series

Upcoming Deadlines:

- Wednesday February 14, 2018: Homework #3.
- Wednesday February 14, 2018: Homework #1 rewrite.