Math 311
Spring 2018
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Day #2 Notes: More Preliminaries

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Contents

1 The rationals

What are the properties of the rational numbers?

$\mathbb{Q}$ is an Abelian group under $+$
- Associative
- Commutative
- 0 = identity
- inverse

The multiplication operation has the following properties.
- Associative
- Commutative
- 1 = identity
- inverse (for all elements except for 0)

- Distribution: addition and multiplication. $a\cdot(b+c) = a\cdot b + a\cdot c$
- Order:
  - if $a \leq b$ and $b \leq a \Rightarrow a = b$
  - for all $a, b \in \mathbb{Q}$ then $a \leq b$ or $b \leq a$
  - for a, b and $c \in \mathbb{Q}$ if $a \leq b$ and $b \leq c \Rightarrow a \leq c$
  - for $a, b, c \in \mathbb{Q}$ $a \geq 0$ and $b \leq c \Rightarrow a.b \leq a.c$

How can we construct $\mathbb{Q}$?

- Consider the set: $X = \left\{ \frac{m}{n} \,/\, m \in \mathbb{Z} \text{ and } n \in \mathbb{N} \right\}$

consider the relation: $\sim$: $(m,n) \sim (p,q)$ iff $m.q = n.p$

then $\mathbb{Q} = X/\sim$.

2 Preliminaries

* Set is a collection of objects.
* These objects are called elements.

2.1 Set Operations

Intersection and Union: for two sets A and B:

Intersection : $A \cap B = \{x$, such that $x \in A$ and $x \in B\}$

Union : $A \cup B = \{x$, such that $x \in A$ or $x \in B\}$

Complement: for a set $A \subset \Omega$

The complement set $A^c := \{x \in \Omega$, such that $x \notin A\}$
(complement in Ω)

Infinite Operations: for sets $A_1, A_2, A_3, \ldots$

$\bigcup_{i=1}^{n} A_i := \{x \in \Omega$, such that $\exists\, i \in \mathbb{N}$ and $x \in A_i\}$

$\bigcap_{i=1}^{n} A_i := \{x \in \Omega$, such that $\forall\, i \in \mathbb{N}, x \in A_i\}$

Example 1 $A_n = \{k \in \mathbb{N} : k \geq n\}$. *Describe* A_n *and compute* $\cup_{n=1}^{\infty} A_n$ *and* $\cap_{n=1}^{\infty} A_n$.

$A_n = \{n, n+1, n+2, \ldots\}$

$\bigcup_{n=1}^{\infty} A_n = \mathbb{N}$ and $\bigcap_{n=1}^{\infty} A_n = \emptyset$

$\emptyset$ is the empty set.

3 Functions

What is a function?

A function is an operation that maps one set to another.

To define a function you need
- domain
- codomain

} they matter (they define the function).

- rule of assignment

eg: $f(x) = x^2$

$\mathbb{R} \to \mathbb{R}$

not injective or surjective.

$f(x) = x^2$

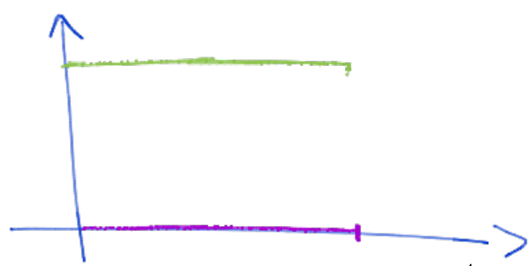
$\mathbb{R}^+ \longrightarrow \mathbb{R}^+$

is injective and surjective (bijective)

Example 2 *Define* $g : [0, 1] \to \mathbb{R}$ *by*

$$g(x) = \begin{cases} 1 & x \in \mathbb{Q} \ \ast \\ 0 & x \notin \mathbb{Q} \ \ast \end{cases}.$$

This is known as Dirichlet's function. *What does it look like?*



4 The Triangle Inequality

Complete today's worksheet in groups and we will go over the answers.

1. [T/F] For every pair of real numbers a and b, $|a+b| \leq |a| + |b|$. T

   Take cases for the signs of a and b.

2. [T/F] For every pair of real numbers a and b, $|a+b| \leq |a| - |b|$. F

   eg: $a = 6$ $b = 1$

   $|6+1| = 7$

   $|6| - |1| = 5$

3. [T/F] For every pair of real numbers a and b, $|a-b| \leq |a| + |b|$. T

   Apply 1 to a and -b

4. [T/F] For every pair of real numbers a and b, $|a-b| \leq |a| - |b|$. F

   $a = 3$ $b = -1$

   $|a-b| = 4$

   $|a| - |b| = 2$.

5. [T/F] Given $a, b \in \mathbb{R}$, we have $a = b$ if for every $\epsilon > 0$, it follows that $|a - b| < \epsilon$.

6. [T/F] Given $a, b \in \mathbb{R}$, we have $a = b$ only if for every $\epsilon > 0$, it follows that $|a - b| < \epsilon$.

7. [T/F] Given $a, b \in \mathbb{R}$, we have $a = b$ if for some $\epsilon > 0$, it follows that $|a - b| < \epsilon$.

8. [T/F] Given $a, b \in \mathbb{R}$, we have $a = b$ only if for some $\epsilon > 0$, it follows that $|a - b| < \epsilon$.

5 Conclusions

Today we learned about:

1. Functions
2. The triangle inequality
3. Induction
4. A start on the Axiom of COmpleteness

Monday we will learn about:

1. MOre about Completeness

Upcoming Deadlines:

- September 6: Homework 1
- September 8: Project Preferences

Questions?