Math 311
Spring 2018
Dr. Hussein Awala

Day #12 Notes: Monotone Convergence and Series

February 14, 2018

Contents

Definition: Let x_n be a sequence. An *infinite series* is a formal expression of the form

$$\sum_{n=1}^{\infty} x_n = x_1 + x_2 + x_3 + \cdots .$$

The corresponding *sequence of partial sums* (s_m) is given by $s_m = x_1 + x_2 + \cdots + x_m$, and we say the series $\sum_{n=1}^{\infty} x_n$ *converges to* S if the sequence (s_m) converges to S. In this case, we write $\sum_{n=1}^{\infty} x_n = S$.

6. [T/F] If (x_n) is a sequence of positive real numbers, then the partial sums for the series $\sum_{n=1}^{\infty} x_n$ form a bounded sequence.

7. [T/F] If (x_n) is a sequence of positive real numbers, then the partial sums for the series $\sum_{n=1}^{\infty} x_n$ form a monotone sequence.

8. [T/F] If $\sum_{n=1}^{\infty} x_n$ converges, then $(x_n) \to 0$.

9. [T/F] If $(x_n) \to 0$, then $\sum_{n=1}^{\infty} x_n$ converges.

1 Wrap Up From the Worksheet

Theorem 1 (Monotone Convergence Theorem (MCT)) *If a sequence is monotone and bounded, then it is convergent.*

Proof:

Proposition 1 *If* $\sum_{n=1}^{\infty} x_n$ *converges, then* $(x_n) \to 0$.

Proof:

2 Series

Example 1 $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

Example 2 $\sum_{n=1}^{\infty} \frac{1}{n}$.

Theorem 2 (Cauchy Condensation Test) *If b_n is a nonnegative, decreasing sequence of numbers, then $\sum_{n=1}^{\infty} b_n$ converges if and only if $\sum_{n=1}^{\infty} 2^n b_{2^n}$ converges.*

Corollary 1 $\sum_{n=1}^{\infty} \frac{1}{n^p}$ *converges if and only if $p > 1$.*

3 Conclusions

Today we learned about:

1. Monotone Convergence
2. A start on Series

Friday we will learn about:

1. Subsequences
2. How subsequences help us
3. Bolzano Weierstrass Theorem

Upcoming Deadlines:

- Next Wednesday: Homework #4
- Next Wednesday: Homework #2 Rewrites

Questions?