

# Day #13 Notes: Subsequences and Bolzano Weierstrass

February 16, 2018

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# 1 Series

**Example 1**  $\sum_{n=1}^{\infty} \frac{1}{n^2}$ .

**Example 2**  $\sum_{n=1}^{\infty} \frac{1}{n}$ .

**Theorem 1 (Cauchy Condensation Test)** *If  $b_n$  is a nonnegative, decreasing sequence of numbers, then  $\sum_{n=1}^{\infty} b_n$  converges if and only if  $\sum_{n=1}^{\infty} 2^n b_{2^n}$  converges.*

**Corollary 1**  $\sum_{n=1}^{\infty} \frac{1}{n^p}$  *converges if and only if  $p > 1$ .*

## 2 Subsequences

What is a subsequence?

How does subsequence convergence relate to sequence convergence?

### 3 How Subsequences Help Us

**Example 3** Consider a geometric sequence  $(b^n)$ , where  $0 < b < 1$ . To what does it converge?

**Proposition 1 Divergence Criterion** If  $(a_n)$  is a sequence and it has two subsequences which converge to different limits, then  $(a_n)$  diverges.

**Example 4**  $(-1)^n$

## 4 The Bolzano-Weierstrass Theorem

**Theorem 2 Bolzano-Weierstrass** *Every bounded sequence of real numbers has a convergent subsequence.*

**Proof:**

## 5 Conclusions

Today we learned about:

1. Cauchy Condensation Test
2. Subsequences
3. Bolzano Weierstrass Theorem

Monday we will learn about:

1. Cauchy Sequences

Upcoming Deadlines:

1. Next Wednesday Feb 21, 2018 H.W 4
2. Next Wednesday Feb 21, 2018 rewrites for H.W. 2