Math 311
Spring 2018
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Day #13 Notes: Subsequences and Bolzano Weierstrass

February 19, 2018

Contents

1 How Subsequences Help Us

Example 1 *Consider a geometric sequence* (b^n), *where* $0 < b < 1$. *To what does it converge?*

Proposition 1 Divergence Criterion *If* (a_n) *is a sequence and it has two subsequences which converge to different limits, then* (a_n) *diverges.*

Example 2 $(-1)^n$

2 The Bolzano-Weierstrass Theorem

Theorem 1 Bolzano-Weierstrass *Every bounded sequence of real numbers has a convergent subsequence.*

Proof:

3 Cauchy sequences–worksheet

Definition: A sequence (a_n) is called a *Cauchy* sequence if, for every $\epsilon > 0$, there exists an $N \in \mathbf{N}$ such that whenever $m, n \geq N$, it follows that $|a_m - a_n| < \epsilon$.

1. How is the definition for a Cauchy sequence different from the definition for a convergent sequence?

2. [T/F] If a sequence $(a_n) \subset \mathbf{N}$ converges to an element of $\mathbf{N}$, then (a_n) is Cauchy.

3. [T/F] If a sequence $(a_n) \subset \mathbf{N}$ is Cauchy, then (a_n) converges to an element of $\mathbf{N}$.

4. [T/F] If a sequence $(a_n) \subset \mathbf{Q}$ converges to an element of $\mathbf{Q}$, then (a_n) is Cauchy.

5. [T/F] If a sequence $(a_n) \subset \mathbf{Q}$ is Cauchy, then (a_n) converges to an element of $\mathbf{Q}$.

6. [T/F] Every Cauchy sequence in $\mathbf{R}$ is bounded.

7. [T/F] Every bounded sequence in $\mathbf{R}$ is Cauchy.

8. [T/F] Every Cauchy sequence in $\mathbf{R}$ is monotone.

9. [T/F] Every monotone sequence in $\mathbf{R}$ is Cauchy.

10. [T/F] If a sequence $(a_n) \subset \mathbf{R}$ converges to an element of $\mathbf{R}$, then (a_n) is Cauchy.

11. [T/F] If a sequence $(a_n) \subset \mathbf{R}$ is Cauchy, then (a_n) has a convergent subsequence.

12. [T/F] If a sequence $(a_n) \subset \mathbf{R}$ is Cauchy, then (a_n) converges to an element of $\mathbf{R}$.

4 Conclusions

Today we learned about:

1. Bolzano Weierstrass Theorem
2. Cauchy Sequences

Wednesday we will learn about:

1. Series

Upcoming Deadlines:

1. Next Wednesday Feb 21, 2018 H.W 4
2. Next Wednesday Feb 21, 2018 rewrites for H.W. 2