Math 311
Spring 2018
Dr. Hussein Awala

Day #15 Notes: Series

February 21, 2018

Contents

1 Cauchy sequences–worksheet

Definition 1 *A sequence* (a_n) *is called a Cauchy sequence if, for every* $\epsilon > 0$*, there exists an* $N \in \mathbf{N}$ *such that whenever* $m, n \geq N$*, it follows that* $|a_m - a_n| < \epsilon$*.*

1. *How is the definition for a Cauchy sequence different from the definition for a convergent sequence?*

2. *[T/F] If a sequence* $(a_n) \subset \mathbf{N}$ *converges to an element of* $\mathbf{N}$*, then* (a_n) *is Cauchy.*

3. *[T/F] If a sequence* $(a_n) \subset \mathbf{N}$ *is Cauchy, then* (a_n) *converges to an element of* $\mathbf{N}$*.*

4. *[T/F] If a sequence* $(a_n) \subset \mathbf{Q}$ *converges to an element of* $\mathbf{Q}$*, then* (a_n) *is Cauchy.*

5. *[T/F] If a sequence* $(a_n) \subset \mathbf{Q}$ *is Cauchy, then* (a_n) *converges to an element of* $\mathbf{Q}$.

6. *[T/F] Every Cauchy sequence in* $\mathbf{R}$ *is bounded.*

7. *[T/F] Every bounded sequence in* $\mathbf{R}$ *is Cauchy.*

8. *[T/F] Every Cauchy sequence in* $\mathbf{R}$ *is monotone.*

9. *[T/F] Every monotone sequence in* $\mathbf{R}$ *is Cauchy.*

10. *[T/F] If a sequence* $(a_n) \subset \mathbf{R}$ *converges to an element of* $\mathbf{R}$*, then* (a_n) *is Cauchy.*

11. *[T/F] If a sequence* $(a_n) \subset \mathbf{R}$ *is Cauchy, then* (a_n) *has a convergent subsequence.*

12. *[T/F] If a sequence* $(a_n) \subset \mathbf{R}$ *is Cauchy, then* (a_n) *converges to an element of* $\mathbf{R}$*.*

2 Series

Theorem 1 *If $\sum_{n=1}^{\infty} a_n$ converges to A and $\sum_{n=1}^{\infty} b_n$ converges to B, and $c \in \mathbf{R}$, then*

1. $\sum_{n=1}^{\infty}(ca_n) = cA$, *and*
2. $\sum_{n=1}^{\infty}(a_n + b_n) = A + B$.

Proof:

Cauchy Convergence Theorem for Series:

Theorem 2 *We have that* $\sum_{n=1}^{\infty} a_n$ *converges if and only if* $\forall \epsilon > 0,\ \exists N \in \mathbf{N}$ *such that* $\forall n > m \geq N$,

$$|\sum_{k=m+1}^{n}| < \epsilon.$$

Proof:

Divergence Test

Corollary 1 *If* $\sum_{n=1}^{\infty} a_n$ *converges, then* $a_n \to 0$.

Comparison Test

Theorem 3 *If $0 \leq a_k \leq b_k$ for all $k \in \mathbf{N}$, then*

1. *If $\sum_{n=1}^{i} nftyb_n$ converges, so does $\sum_{n=1}^{\infty} a_n$.*
2. *If $\sum_{n=1}^{\infty} a_n$ diverges, so does $\sum_{n=1}^{\infty} b_n$.*

Proof:

Example 1 *A geometric series has the form* $\sum_{n=0}^{\infty} ar^n$*. It converges if and only if* $|r| < 1$ *and, if so, it converges to* $\frac{a}{1-r}$*.*

Proof:

2.1 Absolute and Conditional Convergence

Theorem 4 *If* $\sum_{n=1}^{\infty} |a_n|$ *converges, then so does* $\sum_{n=1}^{\infty} a_n$*.*

Proof:

Definition 2 *A series* $\sum_{n=1}^{\infty} a_n$ *converges absolutely if* $\sum_{n=1}^{\infty} |a_n|$ *also converges. If* $\sum_{n=1}^{\infty} a_n$ *converges but* $\sum_{n=1}^{\infty} |a_n|$ *does not, we say that the series converges conditionally.*

Examples:

Alternating Series Test:

Theorem 5 *Suppose that a_n is a nonnegative, decreasing sequence. Then $\sum_{n=1}^{\infty}(-1)^n a_n$ converges if $a_n \to 0$.*

3 Conclusions

Today we learned about:

1. *Cantor Diagonalization*
2. *Convergence of Series*
3. *Absolute vs. Conditional Convergence*

Friday we will learn about:

1. *The Cantor Set*
2. *Open and Closed sets*

Upcoming Deadlines:

- *Wednesday: Homework #5 and Rewrites #3.*

Questions?