Math 311
Spring 2018

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Day #16 Notes: More on Series

February 23, 2018

Contents

1 Series

Theorem 1 *If* $\sum_{n=1}^{\infty} a_n$ *converges to* A *and* $\sum_{n=1}^{\infty} b_n$ *converges to* B, *and* $c \in \mathbf{R}$, *then*

1. $\sum_{n=1}^{\infty}(ca_n) = cA$, *and*
2. $\sum_{n=1}^{\infty}(a_n + b_n) = A + B$.

Proof:

Cauchy Convergence Theorem for Series:

Theorem 2 *We have that* $\sum_{n=1}^{\infty} a_n$ *converges if and only if* $\forall \epsilon > 0,\ \exists N \in \mathbf{N}$ *such that* $\forall n > m \geq N$,

$$|\sum_{k=m+1}^{n} a_n| < \epsilon.$$

Proof:

Divergence Test

Corollary 1 *If* $\sum_{n=1}^{\infty} a_n$ *converges, then* $a_n \to 0$.

Comparison Test

Theorem 3 *If $0 \leq a_k \leq b_k$ for all $k \in \mathbf{N}$, then*

1. *If $\displaystyle\sum_{n=1}^{\infty} b_n$ converges, so does $\displaystyle\sum_{n=1}^{\infty} a_n$.*

2. *If $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges, so does $\displaystyle\sum_{n=1}^{\infty} b_n$.*

Proof:

Example 1 *A geometric series has the form* $\sum_{n=0}^{\infty} ar^n$. *It converges if and only if* $|r| < 1$ *and, if so, it converges to* $\frac{a}{1-r}$.

Proof:

1.1 Absolute and Conditional Convergence

Theorem 4 *If* $\sum_{n=1}^{\infty} |a_n|$ *converges, then so does* $\sum_{n=1}^{\infty} a_n$.

Proof:

Definition 1 *A series* $\sum_{n=1}^{\infty} a_n$ *converges absolutely if* $\sum_{n=1}^{\infty} |a_n|$ *also converges. If* $\sum_{n=1}^{\infty} a_n$ *converges but* $\sum_{n=1}^{\infty} |a_n|$ *does not, we say that the series converges conditionally.*

Examples:

Alternating Series Test:

Theorem 5 *Suppose that a_n is a nonnegative, decreasing sequence. Then* $\sum_{n=1}^{\infty}(-1)^n a_n$ *converges if* $a_n \to 0$.

2 Conclusions

Today we learned about:

1. Series
2. Absolute vs. Conditional Convergence

Monday we will learn about:

1. The Cantor Set
2. Open and Closed sets

Upcoming Deadlines:

- Next Wednesday: Homework #4
- Next Wednesday: Homework #3 Rewrites