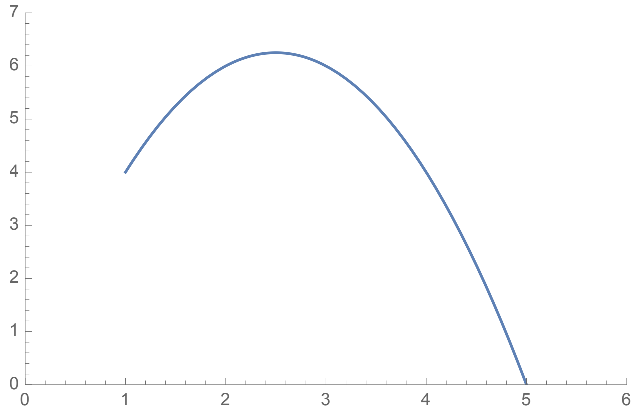
Test-1 Sample

I) The graph of the function $f(x) = 5x - x^2$ on the interval [1,5] is shown below.

(a) You will need to estimate the area under the graph $f(x) = 5x - x^2$ from $x = 1$ to $x = 5$ using M_2, i.e., using to rectangles and midpoint.

(i) Sketch and shade the approximating rectangles.

(ii) Find M_2.



(b) Find the exact area under the graph of the function $f(x) = 5x - x^2$, $1 \leq x \leq 5$, using the Fundamental Theorem of Calculus.

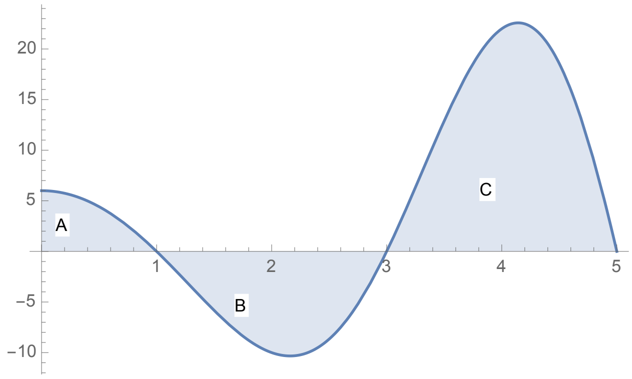
II) Sketch the graph of the function $f(x) = 2 + \sqrt{4 - x^2}, -2 \leq x \leq$. Then evaluate the integral $\int_{-2}^{2} (2 + \sqrt{4 - x^2})\, dx$ by interpreting it in terms of areas.

III) Let $F(x) = \int_0^{\sqrt{\sin(x)}} \frac{1}{\sqrt{1 - t^2}}\, dt,\ 0 \leq x \leq \frac{\pi}{2}$.

Use Part 1 of the Fundamental Theorem of Calculus to find the derivative $F'(x)$. Simplify your answer.

IV) Let $g(x) = \int_0^x f(t)\,dt$, where f is the function whose graph is shown below on the interval [0,5]. Assume that the area of the regions A, B, and C are $4, 7,$ and 11 respectively.

(a) Find $g(3)$ and $g(5)$. Show your work.



(b) Find $g'(1)$.

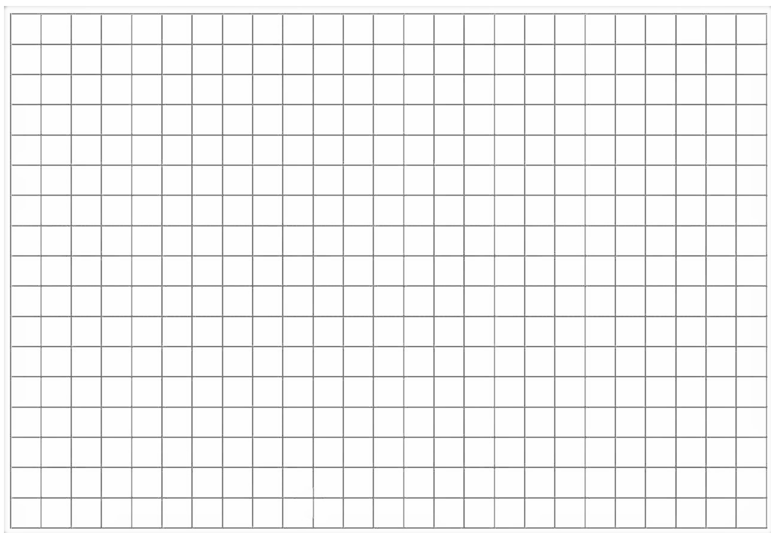
(c) On what interval(s) is $g(x)$ increasing?

(d) At what value on [0,5] does $g(x)$ attain its absolute minimum? What is the minimum value?

(e) At what value on [0,5] does $g(x)$ attain its absolute maximum? What is the maximum value?

(f) On What interval(s) is $g(x)$ concave down?

(g) Sketch the graph of $g(x)$ on the interval [0,5].



V) Evaluate the integrals. Show your work.

(a) $\displaystyle\int t\sqrt{t+7}\,dt$

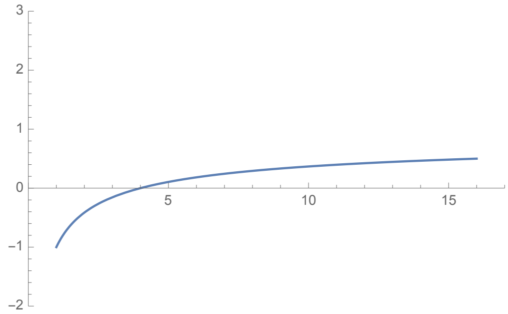
(b) $\displaystyle\int_1^e \frac{\ln x}{x^2}\,dx$

(c) $\displaystyle\int (5-2x)e^{2x}\,dx$

(d) $\displaystyle\int_0^{\frac{1}{2}} \frac{(\arcsin x)^2}{\sqrt{1-x^2}}\,dx$

VI) Let $v(t) = \dfrac{t - 2\sqrt{t}}{t}$, $t > 0$ be the velocity (in centimeters per second) of a particle moving along a line (see graph at the right).

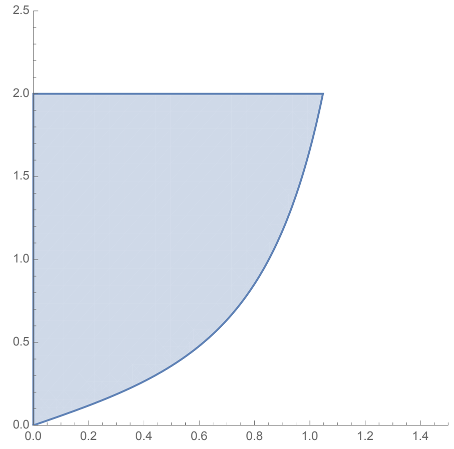
(a) Find the displacement Δs of the particle over the interval [1,16].



(b) Find the distance traveled by the particle during the time interval [1,16].

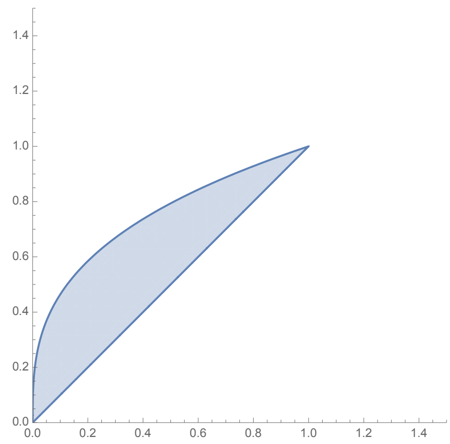
VII) The shaded region below is bounded be $y = \dfrac{1}{\sqrt{3}} \sec x \tan x$, $y = 2$, and $y-axis$, for $0 \leq x \leq \frac{\pi}{3}$.

(a) Find the area of the shaded region.



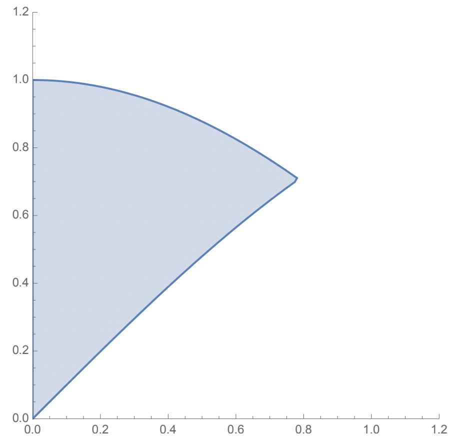
(b) Find the volume of the solid obtained by rotating the shaded region about $x - axis$.

VIII) The Shaded region below is bounded by the curves $y = x^{1/3}$, $y = x$, and $0 \leq x \leq 1$. Set up the integral for the volume of the solid obtained by rotating the shaded region about $x = 1$. (Don't evaluate the integral.)



IX) The base of a solid is the shaded region enclosed by $y = \sin x$, $y = \cos x$, for $0 \leq x \leq \frac{\pi}{4}$ and its cross-sections, perpendicular to the *x-axis*, are squares.

(i) Express $A(x)$, the area of the cross-section as a function of x.



(ii) Find the volume of the solid.