Math 311
Spring 2018
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Day #18 The Cantor Set - Open Closed Sets

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Contents

1 The Cantor Set

Construction:

We construct the Cantor set in the following fashion:

- Start with an interval of the real line.
- Remove the middle third of the interval, not including the endpoints.
- Remove the middle third of each remaining interval.
- Repeat forever.

When all middle thirds have been removed, the remaining set is the Cantor set, or Cantor dust.

1.1 How Big is the Cantor Set?

How many elements does the Cantor set have?

- Is the Cantor set empty?
- Is the Cantor set finite?
- Is the Cantor set countable?

Tercimal Representations It turns out that a number is in the Cantor set if and only if it can be represented as a base-three decimal (tercimal) that doesn't have any ones.

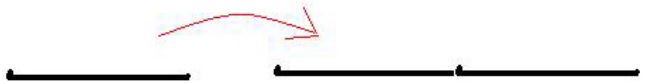
For example, the endpoint $\frac{1}{3} = .0\overline{2}$ in base 3. But not all elements of the Cantor set are endpoints: The element $\frac{1}{4} = .\overline{02}$ in tercimals is also an element of the Cantor set.

In fact, the Cantor set is **not** countable! There are as many elements in the Cantor set as in the set of all real numbers.

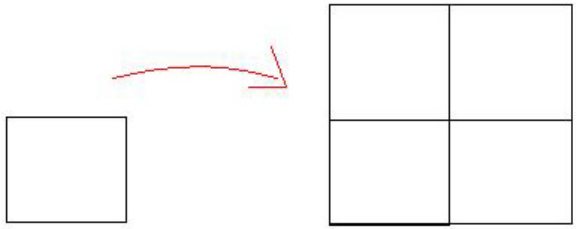
What is the Length of the Cantor Set?

So, How big is the Cantor Set Really?

One way to distinguish the "size" of different sets is by the concept of dimension. There are many different definitions of dimension. The simplest is called *Similarity Dimension*.



How many copies of a line do you get if you double its size?



How many copies of a rectangle do you get if you double its size?

How many copies of a cube would you get if you double its size? What is the pattern?

What is the similarity dimension of the Cantor set?

2 Worksheet 1: Open Sets

Definition 1 *An ϵ-ball around the point $a \in \mathbf{R}$ is the set $B_\epsilon(a) = \{x \in \mathbf{R} : |x - a| < \epsilon\}$. Actually, your book uses the symbol "V_ϵ" (which I haven't seen other people use), and uses the word neighborhood. Both "ball" and "neighborhood" are common words for this concept.*

Definition 2 *A set $O \subset \mathbf{R}$ is open if for every point $a \in O$, there is some $\epsilon > 0$ so that $B_\epsilon(a) \subset O$.*

1. *Is the empty set open?*

2. *Is $\mathbf{R}$ open?*

3. *Is $\mathbf{R} - \{5\}$ open? (That's the real line with the point "5" removed).*

4. Is $\{5\}$ *open?*

5. Is $\mathbf{Q}$ *open?*

6. Is the interval $(2, 4)$ *open?*

7. Is the interval $[2, 4]$ *open?*

8. *Is the interval* $[2, 4)$ *open?*

9. *Name a non-empty, bounded, open set that is not of the form* (a, b)*.*

10. *[T/F] The union of two open sets is an open set.*

11. *[T/F] The union of finitely many open sets is an open set.*

12. [T/F] The union of infinitely many open sets is an open set.

13. [T/F] The intersection of two open sets is an open set.

14. [T/F] The intersection of finitely many open sets is an open set.

15. [T/F] The intersection of infinitely many open sets is an open set.

3 Conclusions

Today we learned about:

1. *The Cantor Set*
2. *Open and Closed sets*

Friday we will learn about:

1. *More on Open and Closed sets*

Upcoming Deadlines:

- *Next Friday: Homework #6*

Questions?