Math 311
Spring 2018
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Day #20 Notes: Compactness

March 20, 2018

Contents

1 Finishing Up Closed Sets **2**

2 Compactness **3**

3 Conclusions **7**

1 Finishing Up Closed Sets

Definition: A point x in a set A is called *isolated* if it is not a limit point of A.

Example 1 $A = \{\frac{1}{n} : n \in \mathbf{N}\}$.

Definition: The *closures* of a set A is the union of A and its limit points.

Theorem 1 *A set C is closed if and only if every Cauchy sequence in C converges to a limit in C.*

Proof:

2 Compactness

Definition: A set $K \subset \mathbf{R}$ is *compact* if every sequence in K has a subsequence which converges to a limit in K.
How is this different from being closed?

Examples?

Theorem 2 (Heine-Borel) *A set $K \subset \mathbf{R}$ is compact if and only if it is both closed and bounded.*

Proof:

Theorem 3 *If* (K_i) *is a sequence of compact sets and* $K_i \supset K_{i+1}$ *for all* $i \in \mathbf{N}$, *then* $\cap_{i=1}^{\infty} K_i \neq \emptyset$.

Proof:

Theorem 4 *A set $K \subset \mathbf{R}$ is compact if and only if every open cover of K has a finite subcover.*

Proof:

3 Conclusions

Today we learned about:

1. More on Closed Sets
2. Compact Sets

Next Monday we will learn about:

1. Connectedness

Upcoming Deadlines:

- Next Wednesday: Homework #7
- Next Wednesday: Homework #5 Rewrites
- Next Wednesday: Chapter 2 project

Questions?