MST 311 - Exam 1 - Spring 2018

Real Analysis

March 2, 2018

Instruction:

- You have 50 minutes to finish this exam.

- No Calculators, phones or laptops are allowed during this exam.

- It is expected that each student during this exam will conduct himself, herself or themselves within the guidelines of the WFU Honor Code. All academic work should be done with the high level of honesty and integrity that the university demands.

Justify All Claims You Make

Name: _____

(I) (28pts) State and prove the Monotone Convergence Theorem.

(II) (29pts) Consider the sequence $(a_n)_{n\in\mathbb{N}}$, given by

$$a_n = \frac{2}{n^2 - n}$$

(a) Use induction to prove that $n \leq \frac{1}{2}n^2$, for $n \geq 2$.

(b) Use part (a) to prove that $\lim_{n\to\infty} a_n = 0$.

(III) (28pts) Consider to countable sets A and B such that $A \cap B = \emptyset$. Prove that the union of the two sets will be countable as well. (i.e. prove that $A \cup B$ is countable). Hint: Use the bijections between A, B and the set $\mathbb{N}$ to find a bijection from $A \cup B$ to $\mathbb{Z}$.

(IV) (15pts) Give an example of a set of real numbers that is uncountable but contains no intervals. Make sure you provide a complete proof as to why this example satisfies this property.