Math 311
Spring 2018
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Day #21 Notes: Compactness and Connectedness

March 23, 2018

Contents

1 Compactness

Theorem 1 *A set $K \subset \mathbf{R}$ is compact if and only if every open cover of K has a finite subcover.*

Relevant Definitions:

Proof:

2 Connectedness

Definition: Two nonempty sets $A, B \subset \mathbf{R}$ are *separate* if $\overline{A} \cap B = a \cap \overline{B} = \emptyset$. A set S is *disconnected* if it can be written as the union of two separated sets. A set S is *connected* if it is not disconnected.

Examples:

Proposition 1 *A set* $A \subset \mathbf{R}$ *is connected if and only if, whenever* $a, b \in A$*, then* $c \in A$ *for every* $c \in (a, b)$*.*

Proof:

1. Classify the following sets

$$\begin{pmatrix} & \text{closed} & \text{bounded} & \text{compact} & \text{connected} \\ \{5\} & & & & \\ \\ (2,4) & & & & \\ \\ \{x \in \mathbf{R} : 2 \le |x| \le 4\} & & & & \\ \\ \mathbf{Q} & & & & \\ \\ \mathbf{R} & & & & \\ \\ \{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\} & & & & \\ \\ \{0\} \cup \{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\} & & & & \\ \\ C^* & & & & \end{pmatrix}$$

* C here is the Cantor set.

2. [T/F] $\overline{\overline{B}} = \overline{B}$.

3. [T/F] If $B \subset \mathbf{R}$, then $\overline{B}$ is closed.

4. [T/F] If a set $F \subset \mathbf{R}$ is closed, then every Cauchy sequence contained in F has a limit that is also an element of F.

5. [T/F] If every Cauchy sequence contained in $F \subset \mathbf{R}$ has a limit that is also an element of F, then F is closed.

6. [T/F] A set $A \subset \mathbf{R}$ is compact if it is closed and bounded.

7. [T/F] A set $A \subset \mathbf{R}$ is compact only if it is closed and bounded.

8. [T/F] A set $A \subset \mathbf{R}$ is connected if, whenever $a, b \in A$, then $c \in A$ for every $c \in (a, b)$.

9. [T/F] A set $A \subset \mathbf{R}$ is connected only if, whenever $a, b \in A$, then $c \in A$ for every $c \in (a, b)$.

3 Conclusions

Today we learned about:

1. More on Closed Sets
2. Compact Sets

Next Monday we will learn about:

1. Functions

Upcoming Deadlines:

- Next Wednesday: Test 2
- Wednesday April 4: Homework #7
- Wednesday April 4: Homework #5 Rewrites

Questions?