Math 311
Spring 2018
Dr. Hussein Awala

Day #21 Notes: Fucntions

March 26, 2018

Contents

1 Connectedness

Proposition 1 *A set* $A \subset \mathbf{R}$ *is connected if and only if, whenever* $a, b \in A$*, then* $c \in A$ *for every* $c \in (a, b)$*.*

Proof:

2 Dirichlet's Function

Define $g : \mathbf{R} \to \mathbf{R}$ by

$$g(x) = \begin{cases} 1 & x \in \mathbf{Q} \\ 0 & x \notin \mathbf{Q} \end{cases}.$$

Is this function continuous? Why or why not?

Figure 4.1: Dirichlet's Function, $g(x)$.

Define $h : \mathbf{R} \to \mathbf{R}$ by

$$h(x) = \begin{cases} x & x \in \mathbf{Q} \\ 0 & x \notin \mathbf{Q} \end{cases}.$$

Is this function continuous? Why or why not?

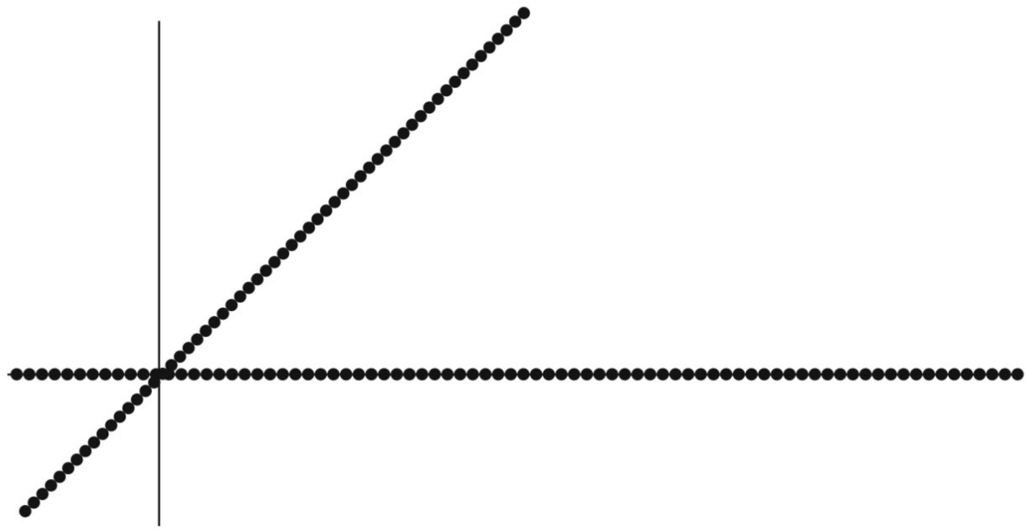

Figure 4.2: Modified Dirichlet Function, $h(x)$.

3 Thomae's Function

Define $t : \mathbf{R} \to \mathbf{R}$ by

$$t(x) = \begin{cases} \frac{1}{q} & x = \frac{p}{q} \in \mathbf{Q} \quad p, q \text{ coprime}, \\ 0 & x \notin \mathbf{Q} \end{cases}.$$

Is this function continuous? Why or why not?

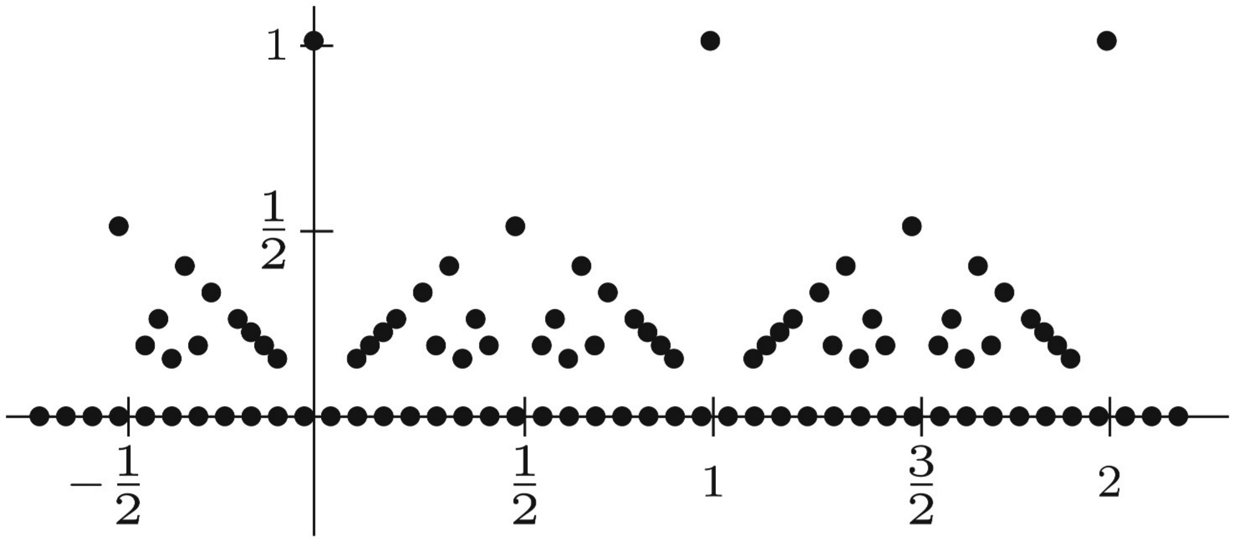

Figure 4.3: Thomae's Function, $t(x)$.

4 Discussion

What do we learn from these examples? Why are they interesting?

1. Is it possible for a function to be discontinuous at all points in the domain?

2. Is it possible for a function to be continuous at just one point in its domain?

3. Is it possible for a function to be discontinuous at just one point in its domain?

4. Is it possible for a function to be discontinuous just on $\mathbf{Q}$? What about $\mathbb{I}$?

5 Conclusions

Today we learned about:

1. Connected Sets
2. Functions

Friday we will learn about:

1. More on Continuity

Upcoming Deadlines:

- Tomorrow review session
- Wednesday: Test 2
- Wednesday April 4: Homework #7
- Wednesday April 4: Homework #5 Rewrites

Questions?