Math 311
Spring 2018
Dr. Hussein Awala

Day #23 Notes: Continuity

April 2, 2018

Contents

1 Definition of a Functional Limit

Definition: Let $f : \mathbf{R} \to \mathbf{R}$ and $x_0 \in \mathbf{R}$. We say that $\lim\limits_{x \to x_0} f(x) = L$ if, $\forall \epsilon > 0$, $\exists \delta > 0$ so that whenever $0 < |x - x_0| < \delta$, it follows that $|f(x) - L| < \epsilon$.

What does this mean?

Definition: Let $f : \mathbf{R} \to \mathbf{R}$ and $x_0 \in \mathbf{R}$. We say that f is continuous at x_0 if, $\forall \epsilon > 0$, $\exists \delta > 0$ so that whenever $0 < |x - x_0| < \delta$, it follows that $|f(x) - f(x_0)| < \epsilon$.

2 Worksheet

1. Draw a picture of this definition. In Figure 1, demonstrate that f is continuous at $x = 1$. That is, let $c = 1$ and let $\epsilon = 0.25$. Draw dashed lines at $y = f(1) \pm \epsilon$, and then draw dashed lines for $x = 1 \pm \delta$, for an appropriate value of δ.

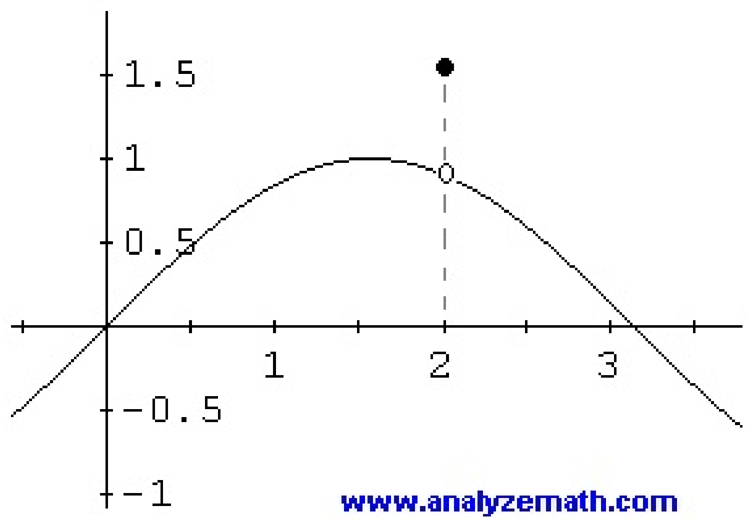

Figure 1: A function that is continuous at $x = 1$.

2. Draw another picture of this definition. In Figure 2, demonstrate that f is not continuous at $x = 2$. That is, let $c = 2$ and let $\epsilon = 0.25$. Draw dashed lines at $y = f(2) \pm \epsilon$, and then conclude that there is no corresponding appropriate value of δ.

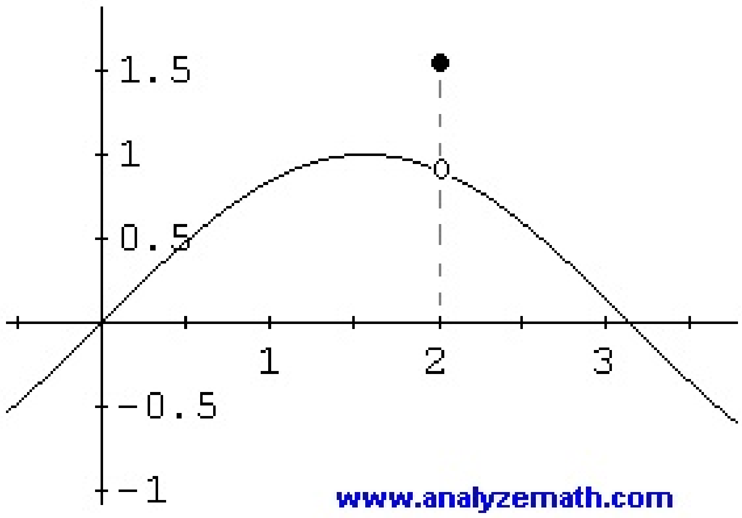

Figure 2: A function that is not continuous at $x = 2$.

3. [T/F] If $f(x) = 4x + 8$, then $\lim_{x \to 3} f(x) = 20$.

4. [T/F] If $f(x) = 3x - 5$, then $\lim_{x \to 2} f(x) = 20$.

5. [T/F] If $f(x) = 3x - 5$, then f is continuous at $x = 2$.

6. [T/F] $\lim_{x \to 2}(x^2 + x - 1) = 5$.

7. [T/F] The modified Dirichlet function h given by

$$h(x) = \begin{cases} x & x \in \mathbf{Q} \\ 0 & x \notin \mathbf{Q} \end{cases}$$

is continuous at $x = 1$.

8. [T/F] The modified Dirichlet function h is continuous at $x = 0$.

Proposition 1 *Given* $f : \mathbf{R} \to \mathbf{R}$ *and let* $c \in \mathbf{R}$*. Then* $\lim_{x \to c} f(x) = L$ *iff* $\lim_{n \to \infty} f(x_n) = L$ *for every sequence* $(x_n) \in \mathbf{R} - \{c\}$ *with* $x_n \to c$*.*

How is this useful?

1. Algebraic Limit Theorem

2. Divergence Criterion

3 Conclusions

Today we learned about:

1. Functional Limits
2. Continuity

Wednesday we will learn about:

1. More on Limits and Continuity

Upcoming Deadlines:

- Wednesday: Homework #7
- Wednesday: Homework #5 Rewrites

Questions?