

**LESSON**  
**6-1**

# Algebraic Expressions

## Reteach

Algebraic expressions can be written from verbal descriptions. Likewise, verbal descriptions can be written from algebraic expressions. In both cases, it is important to look for word and number clues.

### Algebra from words

“One third of the participants increased by 25.”

#### Clues

Look for “number words,” like

- “One third.”
- “Of” means multiplied by.
- “Increased by” means add to.

Combine the clues to produce the expression.

- “One third of the participants.”  $\frac{1}{3}p$  or  $\frac{p}{3}$ .
- “Increased by 25.” +25

“One third of the participants increased by 25.”

$$\frac{1}{3}p + 25 \text{ or } \frac{p}{3} + 25$$

### Words from algebra

“Write  $0.75n - \frac{1}{2}m$  with words.”

#### Clues

Identify the number of parts of the problem.

- “ $0.75n$ ” means “three fourths of  $n$ ” or 75 hundredths of  $n$ . The exact meaning will depend on the problem.
- “ $-$ ” means “minus,” “decreased by,” “less than,” etc., depending on the context.

- “ $\frac{1}{2}m$ ” is “one half of  $m$ ” or “ $m$  over 2.”

Combine the clues to produce a description.  
“75 hundredths of the population minus half the men.”

**Write a verbal description for each algebraic expression.**

1.  $100 - 5n$

\_\_\_\_\_

\_\_\_\_\_

2.  $0.25r + 0.6s$

\_\_\_\_\_

\_\_\_\_\_

3.  $\frac{3m - 8n}{13}$

\_\_\_\_\_

\_\_\_\_\_

**Write an algebraic expression for each verbal description.**

4. Half of the seventh graders and one third of the eighth graders were divided into ten teams.

\_\_\_\_\_

5. Thirty percent of the green house flowers are added to 25 ferns for the school garden.

\_\_\_\_\_

6. Four less than three times the number of egg orders and six more than two times the number of waffle orders.

\_\_\_\_\_

**LESSON**  
**6-2****One-Step Equations with Rational Coefficients****Reteach****Using Addition to Undo Subtraction**

Addition “undoes” subtraction. Adding the same number to both sides of an equation keeps the equation balanced.

$$\begin{aligned}x - 5 &= -6.3 \\x - 5 + 5 &= -6.3 + 5 \\x &= -1.3\end{aligned}$$

**Using Subtraction to Undo Addition**

Subtraction “undoes” addition. Subtracting a number from both sides of an equation keeps the equation balanced.

$$\begin{aligned}n + \frac{3}{4} &= -15 \\n + \frac{3}{4} - \frac{3}{4} &= -15 - \frac{3}{4} \\n &= -15\frac{3}{4}\end{aligned}$$

Be careful to identify the correct number that is to be added or subtracted from both sides of an equation. The numbers and variables can move around, as the problems show.

**Solve using addition or subtraction.**

1.  $6 = m - \frac{7}{8}$

2.  $3.9 + t = 4.5$

3.  $10 = -3.1 + j$

**Multiplication Undoes Division**

To “undo” division, multiply both sides of an equation by the number in the denominator of a problem like this one.

$$\begin{aligned}\frac{m}{3} &= 6 \\3 \times \frac{m}{3} &= 3 \times 6 \\m &= 18\end{aligned}$$

**Division Undoes Multiplication**

To “undo” multiplication, divide both sides of an equation by the number that is multiplied by the variable as shown in this problem.

$$\begin{aligned}4.5p &= 18 \\ \frac{4.5p}{4.5} &= \frac{18}{4.5} = 4\end{aligned}$$

Notice that decimals and fractions can be handled this way, too.

**Solve using division or multiplication.**

4.  $\frac{y}{2.4} = 5$

5.  $0.35w = -7$

6.  $-\frac{a}{6} = 1$

**LESSON**  
**6-3**

## Writing Two-Step Equations

### Reteach

Many real-world problems look like this:

$$\text{one-time amount} + \text{number} \times \text{variable} = \text{total amount}$$

You can use this pattern to write an equation.

**Example:**

At the start of a month a customer spends \$3 for a reusable coffee cup. She pays \$2 each time she has the cup filled with coffee. At the end of the month she has paid \$53. How many cups of coffee did she get?

**one-time amount:** \$3

**number × variable:**  $2 \times c$  or  $2c$ , where  $c$  is the number of cups of coffee

**total amount:** \$53

The equation is:  $3 + 2c = 53$ .

**Write an equation to represent each situation.**

**Each problem can be represented using the form:**

$$\text{one-time amount} + \text{number} \times \text{variable} = \text{total amount}$$

1. The sum of twenty-one and five times a number  $f$  is 61.

\_\_\_\_\_

$$\text{one-time amount} + \text{number} \times \text{variable} = \text{total amount}$$

2. Seventeen more than seven times a number  $j$  is 87.

\_\_\_\_\_

3. A customer's total cell phone bill this month is \$50.50. The company charges a monthly fee of \$18 plus five cents for each call. Use  $n$  to represent the number of calls.

\_\_\_\_\_

4. A tutor works with a group of students. The tutor charges \$40 plus \$30 for each student in the group. Today the tutor has  $s$  students and charges a total of \$220.

\_\_\_\_\_

**LESSON**  
**6-4****Solving Two-Step Equations****Reteach**

Here is a key to solving an equation.

**Example:** Solve  $3x - 7 = 8$ .

**Step 1:**

- Describe how to form the expression  $3x - 7$  from the variable  $x$ :
- Multiply by 3. Then subtract 7.

**Step 2:**

- Write the parts of Step 1 in the reverse order and use inverse operations:
- Add 7. Then divide by 3.

**Step 3:**

- Apply Step 2 to *both sides* of the original equation.
- Start with the original equation.  $3x - 7 = 8$
- Add 7 to both sides.  $3x = 15$
- Divide both sides by 3.  $x = 5$

**Describe the steps to solve each equation. Then solve the equation.**

1.  $4x + 11 = 19$

\_\_\_\_\_

2.  $-3y + 10 = -14$

\_\_\_\_\_

3.  $\frac{r - 11}{3} = -7$

\_\_\_\_\_

4.  $5 - 2p = 11$

\_\_\_\_\_

5.  $\frac{2}{3}z + 1 = 13$

\_\_\_\_\_

6.  $\frac{w - 17}{9} = 2$

\_\_\_\_\_

**LESSON**  
**7-1**

# Writing and Solving One-Step Inequalities

## Reteach

When solving an inequality, solve it as if it is an equation. Then decide on the correct inequality sign to put in the answer.

When adding or subtracting a number from each side of an inequality, the sign stays the same. When multiplying or dividing by a positive number, the sign stays the same. When multiplying or dividing by a negative number, the sign changes.

$x + 5 > -5$ $x + 5 - 5 > -5 - 5$ $x > -10$ <p>Check: Think: 0 is a solution because <math>0 &gt; -10</math>. Substitute 0 for <math>x</math> to see if your answer checks.</p> $x + 5 > -5$ $0 + 5 ? -5$ $5 > -5 \checkmark$	$x - 3 \leq 8$ $x - 3 + 3 \leq 8 + 3$ $x \leq 11$ <p>Check: Think: 0 is a solution because <math>0 \leq 11</math>. Substitute 0 for <math>x</math> to see if your answer checks.</p> $x - 3 \leq 8$ $0 - 3 ? 8$ $-3 \leq 8 \checkmark$	$-2x \geq 8$ $\frac{-2x}{-2} \leq \frac{8}{-2}$ $x \leq -4$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin: 5px;">                     Dividing by a negative, so reverse the inequality sign.                 </div> <p>Check: Think: -6 is a solution because <math>-6 \leq -4</math>. Substitute -6 for <math>x</math> to see if your answer checks.</p> $-2x \geq 8$ $-2 \cdot -6 ? 8$ $12 \geq 8 \checkmark$	$\frac{x}{3} < -6$ $\frac{x}{3}(3) < (-6)(3)$ $x < -18$ <p>Check: Think: -21 is a solution because <math>-21 &lt; -18</math>. Substitute -21 for <math>x</math> to see if your answer checks.</p> $\frac{x}{3} < -6$ $\frac{-21}{3} ? -6$ $-7 < -6 \checkmark$
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**Solve each inequality. Check your work.**

1.  $n + 6 \geq -3$

\_\_\_\_\_

2.  $-2n < -12$

\_\_\_\_\_

3.  $\frac{n}{3} \leq -21$

\_\_\_\_\_

4.  $n - (-3) \geq 7$

\_\_\_\_\_

5.  $-15 + n < -8$

\_\_\_\_\_

6.  $6n > -12$

\_\_\_\_\_

7.  $-6 + n < -9$

\_\_\_\_\_

8.  $\frac{n}{-6} > -2$

\_\_\_\_\_

**LESSON**  
**7-2**

# Writing Two-Step Inequalities

## Reteach

Two-step inequalities involve

- a division or multiplication
- an addition or subtraction.

### Step 1

The description indicates whether division or multiplication is involved:

“One half of a number ”

$\frac{1}{2}n$  or  $\frac{n}{2}$

### Step 2

The description indicates whether addition or subtraction is involved:

“...less 25...” or  
“...decreased by

“ -25”

### Step 3

Combine the two to give two steps:

“One half of a number less

$\frac{1}{2}n - 25$

### Step 4

Use an inequality symbol:

“...is more than 15.” means “>.”

$\frac{1}{2}n - 25 > 15$

**Fill in the steps as shown above.**

1. Five less than 3 times a number is greater than the opposite of 8.

**Step 1:** \_\_\_\_\_

**Step 2:** \_\_\_\_\_

**Step 3:** \_\_\_\_\_

**Step 4:** \_\_\_\_\_

2. Thirteen plus 5 times a number is no more than 30.

**Step 1:** \_\_\_\_\_

**Step 2:** \_\_\_\_\_

**Step 3:** \_\_\_\_\_

**Step 4:** \_\_\_\_\_

**LESSON**  
**7-3****Solving Two-Step Inequalities****Reteach**

When you solve a real-world two-step inequality, you have to

- be sure to solve the inequality correctly, and
- interpret the answer correctly in the context of the problem.

**Example**

The catfish pond contains 2,500 gallons of water. The pond can hold no more than 3,000 gallons. It is being filled at a rate of 110 gallons per hour. How many whole hours will it take to fill but not overflow the pond?

**Step 1:** Solve the inequality.

- The pond already contains 2,500 gallons.
- The pond can be filled at a rate of 110 gallons per hour, or  $110h$  for the number of gallons added in  $h$  hours.
- The pond can hold no more than 3,000 gallons, so  $2,500 + 110h \leq 3,000$ .
- Solve the inequality:  
 $2,500 + 110h \leq 3,000$   
 $2,500 - 2,500 + 110h \leq 3,000 - 2,500$   
 $110h \leq 500$ , or  $h \leq 4.5$  hours.

**Step 2:** Interpret the results.

The problem asks for how many *whole* hours would be needed to fill the pond with not more than 3,000 gallons.  
 Since  $h \leq 4.5$  hours, 5 hours would fill the pool to overflowing. So, the nearest number of *whole* hours to fill it but not to overflow it would be 4 hours.

1. A cross-country racer travels 20 kilometers before she realizes that she has to cover at least 75 kilometers in order to qualify for the next race. If the racer travels at a rate of 10 kilometers per hour, how many whole hours will it take her to reach the 75-kilometer mark?

With inequality problems, many solutions are possible. In real-world problems, these have to be interpreted in the context of the problem and its information.

**Example**

An animal shelter has \$2,500 in its reserve fund. The shelter charges \$40 per animal placement and would like to have at least \$4,000 in its reserve fund. If the shelter places 30 cats and 10 dogs, will that be enough to meet its goal?

**Step 1**

Write and solve the inequality:  
 $2,500 + 40a \geq 4,000$ , or  $40a \geq 1,500$   
 $a \geq 37.5$

**Step 2**

If the shelter places 30 cats and 10 dogs, or 40 animals, that will be enough to meet its goal, because  $a = 40$  is a solution to the inequality  $a \geq 37.5$ .

2. Alissa has \$75 worth of bird seed, which she will put into small bags. She will sell each bag for \$7. What is the greatest number of bags she must sell in order to have no less than \$10 worth of bird seed left over?

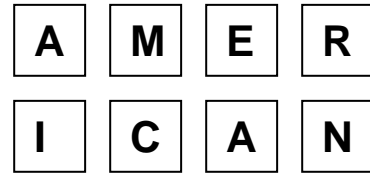
**LESSON**  
**12-1**

# Probability

## Reteach

Picturing a thermometer can help you rate probability.

At right are 8 letter tiles that spell AMERICAN.



If something will always happen, its probability is **certain**.

If you draw a tile, the letter will be in the word "American."

$$P(\text{A, M, E, R, I, C, or N}) = 1$$

If something will never happen, its probability is **impossible**.

If you draw a tile, you cannot draw a "Q."

$$P(\text{Q}) = 0$$

The probability of picking a vowel is **as likely as not** because there are 4 vowels and 4 consonants.

$$P(\text{a vowel}) = \frac{4 \text{ vowels}}{8 \text{ letters}} = \frac{1}{2}$$

Picking the letter "C" is **unlikely** because there is only one "C."

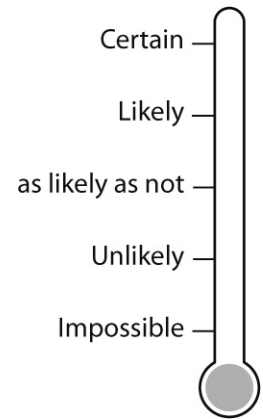
$$P(\text{C}) = \frac{1 \text{ "c" }}{8 \text{ letters}} = \frac{1}{8}$$

Picking a letter besides "A" is **likely** because there are 6 letters that are not "A".

$$P(\text{not A}) = \frac{6 \text{ letters}}{8 \text{ letters}} = \frac{3}{4}$$

Another way to find  $P(\text{not A})$  is to subtract  $P(\text{A})$  from 1.

$$P(\text{not A}) = 1 - P(\text{A}) = 1 - \frac{1}{4} = \frac{3}{4}$$



**Tell whether each outcome is *impossible, unlikely, as likely as not, likely, or certain*. Then write the probability in simplest form.**

- choosing a red crayon from a box of 24 different colored crayons, including red crayons

\_\_\_\_\_

- rolling an odd number on a number cube containing numbers 1 through 6

\_\_\_\_\_

- randomly picking a white card from a bag containing all red cards

\_\_\_\_\_



4. Ramon plays outfield. In the last game, 15 balls were hit in his direction. He caught 12 of them. What is the experimental probability that he will catch the next ball hit in his direction?

- a. What is the number of favorable events? \_\_\_\_\_
- b. What is the total number of trials? \_\_\_\_\_
- c. What is the experimental probability that Ramon will catch the next ball hit in his direction?

\_\_\_\_\_

5. In one inning Tori pitched 9 strikes and 5 balls. What is the experimental probability that the next pitch she throws will be a strike?

- a. What is the number of favorable events? \_\_\_\_\_
- b. What is the total number of trials? \_\_\_\_\_
- c. What is the experimental probability that the next pitch Tori throws will be a strike?

\_\_\_\_\_

6. Tori threw 5 pitches for one batter. Kevin, the catcher, caught 4 of those pitches. What is the experimental probability that Kevin will **not** catch the next pitch? Show your work.

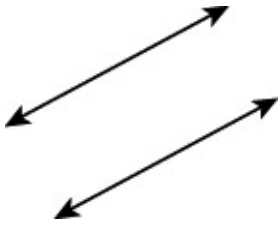
\_\_\_\_\_

**LESSON**  
**21-1**

# Parallel Lines Cut by a Transversal

## Reteach

### Parallel Lines



Parallel lines never meet.

### Parallel Lines Cut by a Transversal

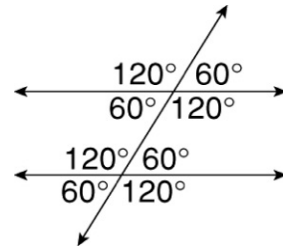
A line that crosses parallel lines is a **transversal**.

Eight angles are formed. If the transversal is not perpendicular to the parallel lines, then four angles are acute and four are obtuse.

The acute angles are all congruent.

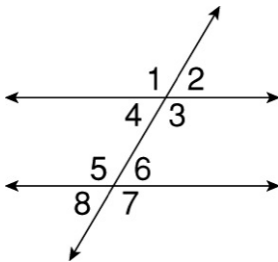
The obtuse angles are all congruent.

Any acute angle is supplementary to any obtuse angle.



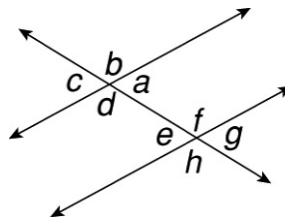
In each diagram, parallel lines are cut by a transversal. Name the angles that are congruent to the indicated angle.

1.



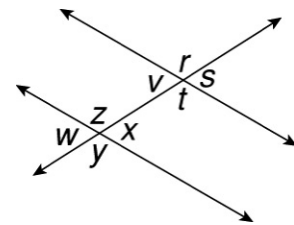
The angles congruent to  $\angle 1$  are:

\_\_\_\_\_



2. The angles congruent to  $\angle a$  are:

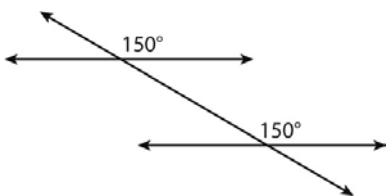
\_\_\_\_\_



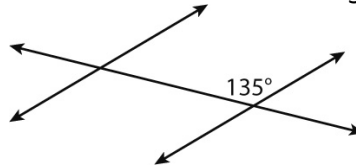
3. The angles congruent to  $\angle z$  are:

\_\_\_\_\_

In each diagram, parallel lines are cut by a transversal and the measure of one angle is given. Write the measures of the remaining angles on the diagram.



4.



5.



6.

**LESSON**  
**21-2**

**Angle Theorems for Triangles**

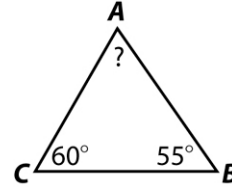
**Reteach**

If you know the measure of two angles in a triangle, you can subtract their sum from 180°. The difference is the measure of the third angle.

The two known angles are 60° and 55°.

$$60^\circ + 55^\circ = 115^\circ$$

$$180^\circ - 115^\circ = 65^\circ$$



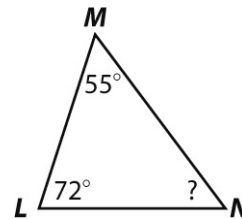
**Solve.**

1. Find the measure of the unknown angle.

Add the two known angles: \_\_\_\_ + \_\_\_\_ = \_\_\_\_

Subtract the sum from 180°: 180 - \_\_\_\_ = \_\_\_\_

The measure of the unknown angle is: \_\_\_\_

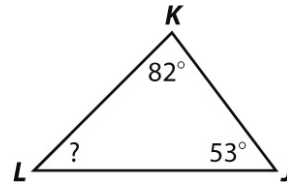


2. Find the measure of the unknown angle.

Add the two known angles: \_\_\_\_ + \_\_\_\_ = \_\_\_\_

Subtract the sum from 180°: 180 - \_\_\_\_ = \_\_\_\_

The measure of the unknown angle is: \_\_\_\_



$\angle DEG$  is an **exterior angle**.

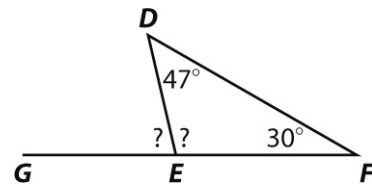
The measure of  $\angle DEG$  is equal to the sum of  $\angle D$  and  $\angle F$ .

$$47^\circ + 30^\circ = 77^\circ$$

You can find the measure of  $\angle DEF$  by subtracting 77° from 180°.

$$180^\circ - 77^\circ = 103^\circ$$

The measure of  $\angle DEF$  is 103°.



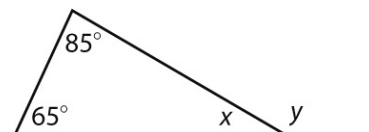
**Solve.**

3. Find the measure of angle  $y$ .

$$85^\circ + 65^\circ = \underline{\hspace{2cm}}$$

4. Find the measure of angle  $x$ .

$$180^\circ - \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$



**Answer key****6.1**

1. Answers will vary. Sample answer: one hundred minus five times the number of cars.
2. Answers will vary. Sample answer: twenty-five hundredths of the apartments and six tenths of the condos.
3. Answers will vary. Sample answer: one thirteenth of the difference between three times the number of hammers and eight times the number of pliers.
4.  $\frac{1}{10}\left(\frac{1}{2}s + \frac{1}{3}e\right)$
5.  $0.3f + 25$
6.  $(3e - 4) + (6 + 2w)$

**6.2**

1.  $m = 6\frac{7}{8}$
2.  $t = -0.6$
3.  $j = 13.1$
4.  $y = 12$
5.  $w = -20$
6.  $a = -6$

**6.3**

1.  $21 + 5f = 61$
2.  $7j + 17 = 87$
3.  $18 + 0.05n = 50.50$

**6.4**

1. Subtract 11 from both sides. Then divide both sides by 4.  $x = 2$
2. Subtract 10 from both sides. Then divide both sides by  $-3$ .  $y = 8$
3. Multiply both sides by 3. Then add 11 to each side.  $r = -10$
4. Subtract 5 from each side. Then divide both sides by  $-2$ .  $p = -3$
5. Subtract 1 from each side. Then multiply both sides by  $\frac{3}{2}$   
 $\left(\text{or divide both sides by } \frac{2}{3}\right)$ .  $z = 18$
6. Multiply both sides by 9. Then add 17 to each side.  $w = 35$

**7.1**

1.  $n \geq -9$
2.  $n > 6$
3.  $n \leq -63$
4.  $n \geq 4$
5.  $n < 7$
6.  $n > -2$
7.  $n < -3$
8.  $n < 12$

**7.2**

1.  $3n; 5 -; 3n - 5; 3n - 5 > -8$
2.  $5n; + 13; 5n + 13; 5n + 13 \leq 30$

**7.3**

1.  $h \geq 5.5$ , or 6 whole hours; 5 hours would not be enough to reach the 75-kilometer goal.
2.  $b \leq 9.29$  bags, so 9 bags would be the greatest number that could be sold and still leave \$10 worth of bird seed left over.

**12.1**

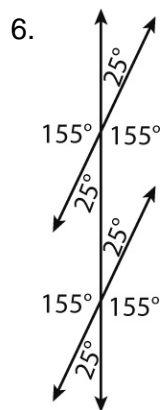
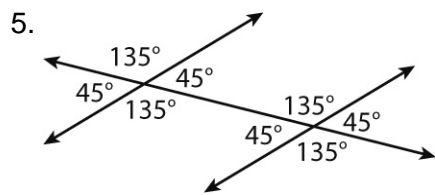
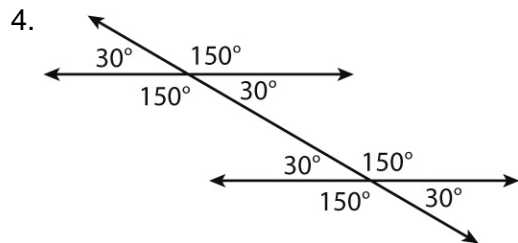
1. unlikely;  $\frac{1}{24}$
2. as likely as not;  $\frac{1}{2}$
3. impossible; 0
4. a. 12  
b. 15  
c.  $\frac{12}{15} = \frac{4}{5}$
5. a. 9  
b. 14  
c.  $\frac{9}{14}$
6.  $P(\text{catch}) = \frac{4}{5}$ ;  $P(\text{no catch}) = 1 - \frac{4}{5} = \frac{1}{5}$

### 21.1

1.  $\angle 3, \angle 5, \angle 7$

2.  $\angle c, \angle e, \angle g$

3.  $\angle y, \angle t, \angle r$



### 21.2

1.  $55^\circ + 72^\circ = 127^\circ$ ;  $180^\circ - 127^\circ = 53^\circ$ ;  $53^\circ$

2.  $82^\circ + 53^\circ = 135^\circ$ ;  $180^\circ - 135^\circ = 45^\circ$ ;  $45^\circ$

3.  $y = 150^\circ$

4.  $150^\circ$ ;  $30^\circ$