Revision sheet Grade 9 Mathematics

LESSON 19-1

Understanding Quadratic Functions

To analyze a function of the form $y = ax^2$, where a is not 0, you can take notes about the equation.

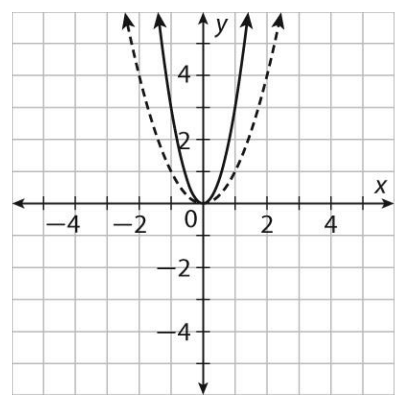
Look.	Think.	Look.	Think.
$y = 3x^2$	variable squared quadratic function U shaped graph	$y = 3x^2$	number times variables squared Point (0, 0) is the highest or lowest point on the

Remember:

> The line $x = 0$ divides the U into left and right parts that are identical.
> The U is symmetric with $x = 0$ as the line of symmetry.

Now look at
the number 3, the coefficient of x^2.

Look.	Think.
3 is greater than 0.	The U opens **upward.** (0, 0) is the **lowest point** on the
3 is greater than 1.	The U is **narrower** than the U that represents the graph of $y = x^2$.



vertical stretch

In this example, the coefficient of x^2 is positive. Use similar thinking when the coefficient is negative. The U will flip over the x-axis of the one shown here.

Answer each question about $y = -3x^2$.

1. Does the graph open up or down? ______________________
2. Is (0, 0) the highest (maximum) or lowest (minimum) point on the graph? _______________

Answer each question about $y = 0.1x^2$.

3. Is the graph wider or more narrow than the graph of $y = x^2$? ______________________
4. What is an equation of the axis of symmetry of the graph? ______________________

Answer each question about $y = -0.1x^2$.

5. Does the graph open up or down? ______________________
6. What are the coordinates of the highest (maximum) or lowest (minimum) point on the graph? ______________________

LESSON 19-2 Transforming Quadratic Functions
Reteach

A parabola has the equation $f(x) = a(x - h)^2 + k$. Identify:

a. a, a stretch if $a > 1$ or compression if $0 < a < 1$
b. h, the horizontal translation
c. k, the vertical translation

The vertex is (h, k) and the parabola opens up if $a > 0$ and opens down if $a < 0$.

In parabola $f(x) = 4(x - 3)^2 + 5$, the stretch is 4, the horizontal translation is 3 to the right, and the vertical translation is up 5. The vertex is (3, 5), and the parabola opens up.

Complete 1–4 for parabola $f(x) = 2(x + 7)^2 + 9$.

1. Stretch or shrink? ________________
2. Open up or down? _______________
3. Horizontal translation? _____________
4. Vertical translation? _______________

Complete 5–8 for parabola $f(x) = \frac{1}{2}(x - 4)^2 - 8$.

5. Stretch or shrink? ________________
6. Open up or down? _______________
7. Horizontal translation? _____________
8. Vertical translation? _______________

For a parabola that opens up, the vertex represents the minimum point. For a parabola that opens down, the vertex represents the maximum point.

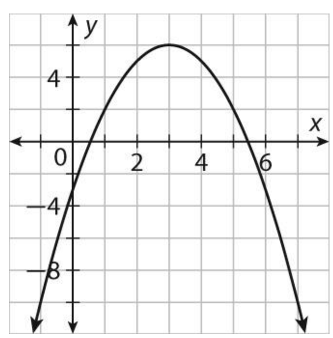
The following graph is a translation of $y = x^2$.

9. The vertex is (_____, ______).
10. Is the vertex a maximum or a minimum?

____

11. The quadratic equation for the graph is

___________________________________.



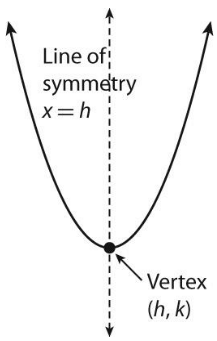
LESSON 19-3

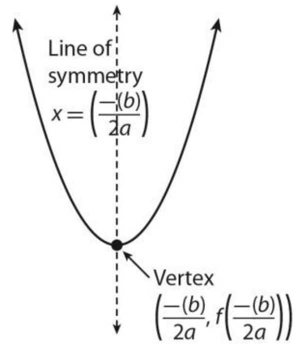
Interpreting Vertex Form and Standard Form

Reteach

The equation of a parabola can be written in either **vertex** or **standard** form.

Vertex Form	**Standard Form**
$y = a(x-h)^2 + k$	$y = ax^2 + bx + c$





Find the vertex of the quadratic equation $y = 2(x-1)^2 - 4$.

The **vertex** is the lowest point of a parabola when the parabola opens up.
When the equation is written in vertex form, the coordinates of the vertex are (*h, k*).

$y = a(x-h)^2 - k$

$y = 2(x-1)^2 - 4$ $\quad h = 1,\ k = -4$

The vertex is the ordered pair $(1, -4)$ and the line of symmetry is $x = 1$.

To change the equation from vertex form to standard form, do the following:

$y = 2(x-1)^2 - 4$	
$y = 2(x-1)(x-1) - 4$	Show the factors.
$y = 2(x^2 - 2x + 1) - 4$	Expand.
$y = 2x^2 - 4x + 2 - 4$	Multiply.
$y = 2x^2 - 4x - 2$	Simplify.

When written in standard form, the coordinates of the vertex are $\left(\frac{-(b)}{2a}, f\left(\frac{-(b)}{2a}\right)\right)$.

$y = ax^2 + bx + c$

$y = 2x^2 - 4x - 2$ $\quad a = 2,\ b = -4$, so $x = \frac{-(b)}{2a} = \frac{-(-4)}{2(2)} = 1$, and $y = 2(1)^2 - 4(1) - 2 = -4$

The vertex is the ordered pair (1, –4) and the line of symmetry is $x = 1$.

Find the vertex and axis of symmetry of each quadratic equation.

1. $y = (x-5)^2 + 2$ ____________________
2. $y = x^2 + 6x + 8$ ____________________
3. $y = 2(x+4)^2 + 1$ ____________________
4. $y = 2x^2 - 12x + 24$ ____________________
5. $y = 8(x-9)^2 + 5$ ____________________
6. $y = 4x^2 + 16x + 1$ ____________________

LESSON
20-1

Connecting Intercepts and Zeros

Use your calculator to graph the function $f(x) = x^2 - 4x - 5$.

If you place the cursor on a point on the graph, the *x*- and *y*-values of the point will be displayed. Use the graph to answer the following questions.

1. Complete the table.

x	0	1	2	3	4
y					

2. What are the zeros of the function? Hint: the zeros are the *x*-values at which the graph intercepts the *x*-axis.

____

3. What is the value of *y* for zeros of a function? ________________________________

To complete a table like the one above without using a calculator, substitute each *x*-value into the expression and solve for $f(x)$. For the function $f(x) = x^2 + 2x - 3$, start with $x = -3$.

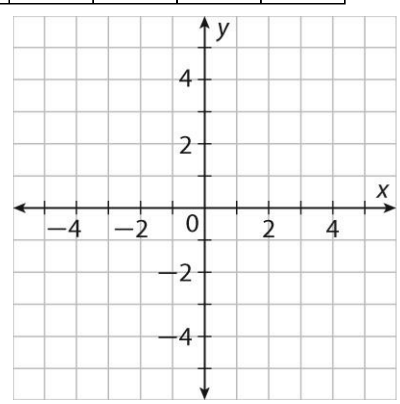
$$f(-3) = (-3)^2 + 2(-3) - 3$$
$$f(-3) = 9 + -6 - 3$$
$$f(-3) = 3 - 3$$
$$f(-3) = 0$$

x	–3	–2	–1	0	1
y	0				

4. Complete the table for $f(x) = x^2 + 2x - 3$.

x	–3	–2	–1	0	1
y	0				

5. Graph the function on the axes provided. Identify the zeros of the function.



LESSON **20-2**

Connecting Intercepts and Linear Factors

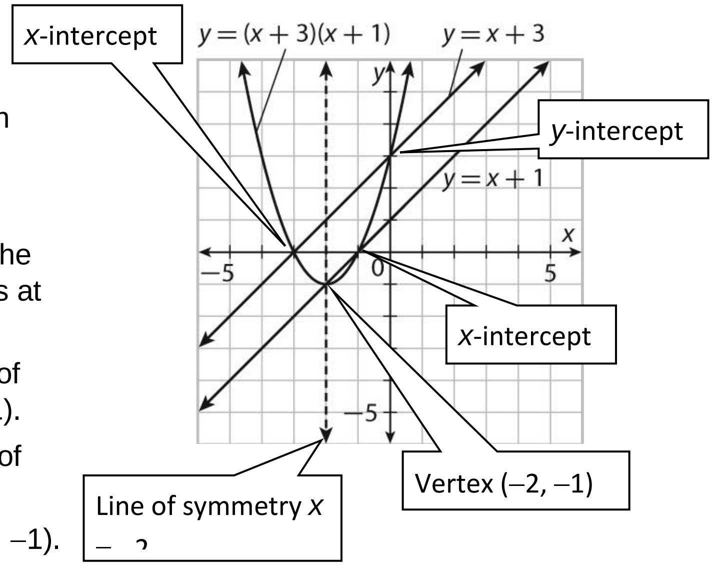
The x-intercepts of a quadratic function and the x-intercepts of its linear factors are the same.

Graph the lines $y = x + 3$ and $y = x + 1$.

- The x-intercept of a line is where the line crosses the x-axis and y is equal to 0.
- Substitute 0 for y in each equation.

  $0 = x + 3$, $-3 = x$

  $0 = x + 1$, $-1 = x$
- Find the value that is halfway between the x-intercepts. $\dfrac{-3+(-1)}{2} = -2$
- This is the line of symmetry, $x = -2$. The vertex of the quadratic function occurs at this x-value.
- The quadratic function is the product of the two linear factors, $y = (x + 3)(x + 1)$.
- Substitute -2 for x to find the y-value of the vertex. $y = (-2 + 3)(-2 + 1)$
- The coordinates of the vertex are $(-2, -1)$.



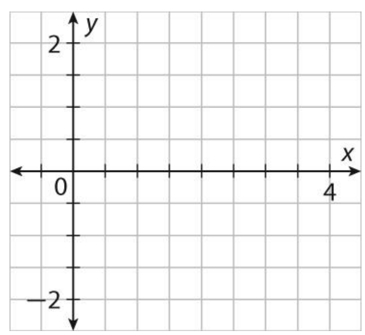
- Multiply the linear factors to convert to standard form $y = x^2 + 4x + 3$.
- Substitute 0 for x to find the y-intercept.
- Graph the quadratic equation.

For the quadratic function $y = (x - 2)(x - 1)$, follow the instructions.

1. Graph the lines.
2. Plot points at the x-intercepts.
3. Draw the line of symmetry.
4. Find the coordinates of the vertex and plot the vertex point.
5. Write the quadratic equation in standard form.

   ____

6. Plot a point at the y-intercept.
7. Graph the quadratic function.



LESSON **20-3**

Applying the Zero Product Property to Solve Equations

Quadratic equations in factored form can be solved by using the Zero Product Property.

If the product of two quantities equals zero, at least one of the quantities must equal zero.

If $(x)\ (y) = 0$, then → $x = 0$ or $y = 0$

If $(x + 3)\ (x - 2) = 0$, then → $x + 3 = 0$ or $x - 2 = 0$

You can use the Zero Product Property to solve any quadratic equation written in factored form, such as $(a + b)(a - b) = 0$.

Examples

Find the zeros of $(x + 5)(x - 1) = 0$.

$x + 5 = 0$ or $x - 1 = 0$ *Set each factor equal to 0.*

$x = -5$ or $x = 1$ *Solve each equation for x.*

Solve $(x - 7)(x + 2) = 0$.

$x - 7 = 0$ or $x + 2 = 0$ *Set each factor equal to 0.*

$x = 7$ or $x = -2$ *Solve each equation for x.*

Use the Zero Product Property to solve each equation by filling in the blanks below. Then find the solutions. Check your answer.

1. $(x - 6)\ (x - 3) = 0$

   $x =$ ________ or $x =$ ________

   ____

2. $(x + 8)\ (x - 5) = 0$

   $x =$ ________ or $x =$ ________

   ____

Use the Zero Product Property to solve each equation.

3. $(y - 7)(y - 3) = 0$ ______________________
4. $0 = (x + 6)(x - 3)$ ______________________
5. $(x + 4)(x + 3) = 0$ ______________________
6. $(t + 9)(t - 3) = 0$ ______________________
7. $(n - 5)(n + 3) = 0$ ______________________
8. $(a - 10)(a + 3) = 0$ ______________________
9. $(z - 6)(z + 4) = 0$ ______________________
10. $0 = (x + 4)(x - 2)$ ______________________
11. $0 = (g + 3)(g - 3)$ ______________________

LESSON 21-1

Solving Equations by Factoring $x^2 + bx + c$

Reteach

To find the factors for a trinomial in the form $x^2 + bx + c$, answer these 2 questions.

1. What numbers have a product equal to c?
2. What numbers have a sum equal to b?

Find numbers for which the answer to both is yes.

Factor $x^2 + 5x + 6$.

What numbers have a product equal to c, 6?

1 and 6   –1 and –6   2 and 3   –2 and –3

What numbers have a sum equal to b, 5?

1 and 6   –1 and –6   (2 and 3)   –2 and –3

The factors of $x^2 + 5x + 6$ are $(x + 2)$ and $(x + 3)$.

Solve the trinomial by setting it equal to 0. Factor and use the Zero Product Property to solve.

Example

Solve $x^2 + 5x + 6 = 0$.

$x^2 + 5x + 6 = 0$

$(x + 2)(x + 3) = 0$ *Factor $x^2 + 5x + 6$.*

$x + 2 = 0$ or $x + 3 = 0$ *Set each factor equal to 0.*

$x = -2$ or $x = -3$ *Solve each equation for x.*

Complete the factoring.

1. $x^2 + x - 2$

   What numbers have a product equal to c, _____? ________________

   What numbers have a sum equal to b, _____? ________________

   Factors: ____

Factor.

2. $x^2 + 4x + 4$ ____________________
3. $x^2 - 4x + 3$ ____________________
4. $x^2 + 3x - 10$ ____________________

Solve.

5. $x^2 + 12x + 35 = 0$ ____________________
6. $x^2 - 9x + 18 = 0$ ____________________
7. $x^2 - x - 20 = 0$ ____________________

LESSON 21-3

Using Special Factors to Solve Equations

Reteach

Use the difference of squares method or the perfect-square method to solve many projectile-motion word problems. The height of a projectile is often represented by one of these equations (where h is height in feet and t is time in seconds, and $-16t^2$ represents the force of gravity for all projectiles on Earth).

$h = -16t^2 + h_0$	$h = -16t^2 + v_0t + h_0$
Use when the initial velocity = 0 (the projectile is *dropped* from a height, h_0).	Use when the initial velocity ≠ 0 (the projectile is launched from a height, h_0, with an initial upward velocity of v_0).

Problem 1: $h = -16t^2 + 64$	**Problem 2:** $h = -16t^2 + 24t - 9$
✓ 64 represents the initial height of the projectile. ✓ To find when the projectile hits the ground set $h = 0$ and use the difference of squares to solve for t.	✓ 24 represents the initial velocity of the launched projectile. ✓ The −9 represents the initial height, in this case 9 feet under ground. ✓ Set $h = 0$ and use perfect-squares to solve for t.
1. Set $h = 0$. $h = -16t^2 + 64 = 0$ 2. Factor. $h = -16(t^2 - 4) = 0$ 3. $a = \sqrt{t^2} = t$ and $b = \sqrt{4} = 2$ 4. Use difference of squares to solve for t. $h = -16(t + 2)(t - 2) = 0$ $t = -2$ or 2 Pick positive t, so $t = 2$.	1. Set $h = 0$. $h = -16t^2 + 24t - 9 = 0$ 2. Factor. $h = -1(16t^2 - 24t + 9) = 0$ 3. $a = \sqrt{16t^2} = 4t$ and $b = \sqrt{9} = 3$ 4. Check middle term. $2ab = 24t$ 5. Use perfect-squares to solve for t. $h = -1(4t - 3)^2 = 0$, so $t = \frac{3}{4}$ seconds.

Find when each projectile below hits the ground.

1. $h = -16t^2 + 128$
   a. 128 represents ___________.
   b. Set $h = 0$ and solve for t.

   $t =$ ________

2. $h = -16t^2 + 40t - 25$
   a. 40 represents ___________.
   b. −25 represents ___________.
   c. Set $h = 0$ and solve for t.

   $t =$ ________

Answer key

LESSON 19-1 Reteach

1. down
2. highest
3. wider
4. $x = 0$
5. down
6. (0, 0)

LESSON 19-2 Reteach

1. stretch
2. up
3. left 7
4. up 9
5. shrink
6. up
7. right 4
8. down 8
9. (3, 6)
10. maximum
11. $y = -(x - 3)^2 + 6$

LESSON 19-3 Reteach

1. Vertex $(5,\ 2)$, axis of symmetry $x = 5$
2. Vertex $(-3,\ -1)$, axis of symmetry $x = -3$
3. Vertex $(-4,\ 1)$, axis of symmetry $x = -4$
4. Vertex $(3,\ 6)$, axis of symmetry $x = 3$
5. Vertex $(9,\ 5)$, axis of symmetry $x = 9$
6. Vertex $(-2,\ -15)$, axis of symmetry $x = -2$

LESSON 20-1 Reteach

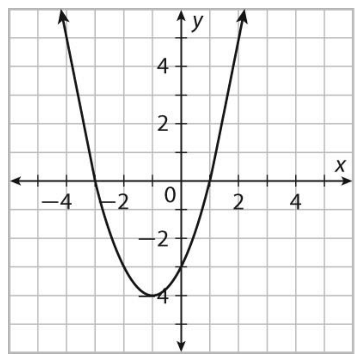
1.

x	0	1	2	3	4
y	–5	–8	–9	–8	–5

2. $x = -1$ and $x = 5$
3. 0
4.

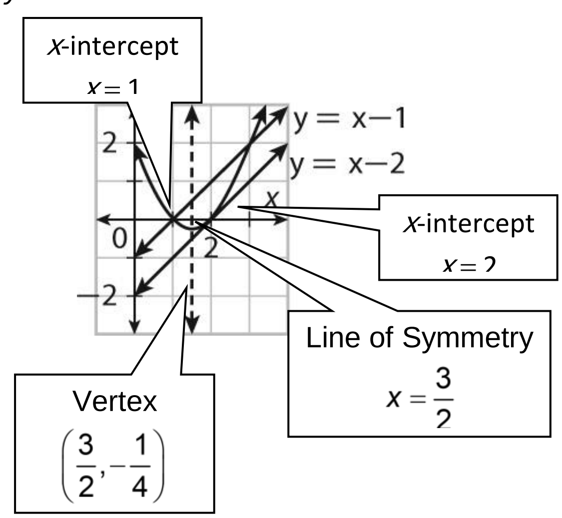
x	–3	–2	–1	0	1
y	0	–3	–4	–3	0

5. $x = -3$ and $x = 1$



LESSON 20-2 Reteach

$y = x^2 - 3x + 2$



LESSON 20-3 Reteach

1. $x = 6$, $x = 3$
2. $x = -8$, $x = 5$
3. $y = 7$, $y = 3$
4. $x = -6$, $x = 3$
5. $x = -4$, $x = -3$
6. $t = -9$, $t = 3$
7. $n = 5$, $n = -3$
8. $a = 10$, $a = -3$
9. $z = 6$, $z = -4$
10. $x = -4$, $x = 2$
11. $g = -3$, $g = 3$

LESSON 21-1 Reteach

1. –2; –1, 2; 1, –2; $(x - 1)(x + 2)$
2. $(x + 2)(x + 2)$
3. $(x - 3)(x - 1)$
4. $(x - 2)(x + 5)$
5. –7, –5
6. 6, 3
7. 5, –4

LESSON 21-3 Reteach

1. a) initial height, b) $2\sqrt{2}$
2. a) initial velocity, b) initial height, c) $\frac{5}{4}$