مدرسة صقر الإمارات الدولية الخاصة
EMIRATES FALCON INT'L. PRIVATE SCHOOL

MATH REVISION SHEET FOR THE FINAL EXAM 2nd TERM 2019

grade10/___ ***Name: ___________________***

Exam will be in Algebra 2 lessons : 6.1+6.2+6.3+6.4+6.5+7.1+7.2

LESSON **6-1**

Adding and Subtracting Polynomials

Example $\left(-3x^4+2x-x^3-12\right)+\left(4+2x^4-x^2+9x\right)$

1. Write in standard form.		$-3x^4$	$-x^3$		$+2x$	-12
2. Align like terms.	$+$	$2x^4$		$-x^2$	$+9x$	$+4$
3. Add.		$-x^4$	$-x^3$	$-x^2$	$+11x$	-8

$$\left(-3x^4+2x-x^3-12\right)+\left(4+2x^4-x^2+9x\right)=-x^4-x^3-x^2+11x-8$$

Add the polynomials.

1. $\left(2x^2-7x+5x^4+4x^3-11\right)+\left(6x^3+x^4-3x^2+10x\right)$

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2. $\left(6x^2-9+x^3\right)+\left(3x^3-4-x\right)$

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3. $\left(4a^4-9a^2+3a^3-a\right)+\left(-5a^3+14-a\right)$

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4. $\left(y^2-5y+18\right)+\left(2y-y^2-11\right)$

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Example $\left(-x+5x^3+2x^4-10x\right)-\left(4x^2-2x-x^4+1\right)$

1. Write in standard form.		$2x^4$	$+5x^3$	$-x^2$	$-10x$	
2. Align like terms and add the opposite.	$+$	x^4		$-4x^2$	$+2x$	-1
3. Add.		$3x^4$	$+5x^3$	$-5x^2$	$-8x$	-1

$$\left(-x+5x^3+2x^4-10x\right)-\left(4x^2-2x-x^4+1\right)=3x^4+5x^3-5x^2-8x-1$$

Subtract the polynomials.

5. $\left(-4x^3+3x^2-x^4+8x\right)-\left(9+5x^4-3x^2+10x\right)$

6. $\left(x^3-7x+3x^4-5\right)-\left(3+2x^3-4x^2-2x\right)$

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7. $\left(c^4+7c-c^3-12\right)-\left(-c^4+4c^3-c^2+5\right)$

8. $\left(3r^3+r-8\right)-\left(-2r^2-8\right)$

LESSON 6-2 Multiplying Polynomials

You can multiply polynomials horizontally or vertically.

Example Find the product by multiplying horizontally. $(x-5)\left(3x+x^2-7\right)$

Multiply each term of the first polynomial by each term of the second polynomial, then simplify.

1. Write polynomials in standard form. $(x-5)\left(x^2+3x-7\right)$
2. Distribute x and –5. $x\left(x^2\right)+x(3x)+x(-7)+(-5)\left(x^2\right)+(-5)(3x)+(-5)(-7)$
3. Simplify. $x^3+3x^2-7x-5x^2-15x+35$
4. Combine like terms. $x^3-2x^2-22x+35$

Find the product by multiplying horizontally.

1. $(x+8)\left(6-2x^2+x\right)$

2. $(2x-3)\left(x^2+4-5x\right)$

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Example Find the product by multiplying vertically. $(x-5)\left(3x+x^2-7\right)$

1. Write each polynomial in standard form.
2. Multiply –5 and $\left(3x+x^2-7\right)$.

3. Multiply x and $(3x + x^2 - 7)$.

4. Combine like terms.

	x^2	$+3x$	-7
		x	-5
	$-5x^2$	$-15x$	$+35$
x^3	$+3x^2$	$-7x$	
x^3	$-2x^2$	$-22x$	$+35$

Find the product by multiplying vertically.

3. $(x-3)(5x-8+2x^2)$

4. $(5-3x)(4x^2-1+7x)$

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LESSON **6-3**

The Binomial Theorem

The coefficients of the expansion of $(x + y)^n$ are the numbers in Pascal's triangle.
The binomial coefficients are also combinations.

Pascal's Triangle	Combinations	Binomial Expansion
1	${}_0C_0$	$(x+y)^0 = 1$
1 1	${}_1C_0$ ${}_1C_1$	$(x+y)^1 = x + y$
1 2 1	${}_2C_0$ ${}_2C_1$ ${}_2C_2$	$(x+y)^2 = x^2 + 2xy + y^2$
1 3 3 1	${}_3C_0$ ${}_3C_1$ ${}_3C_2$ ${}_3C_3$	$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^2$

The **Binomial Theorem** summarizes the relationship.

$$(x+y)^n = {}_nC_0x^ny^0 + {}_nC_1x^{n-1}y^1 + {}_nC_2x^{n-2}y^2 + \ldots + {}_nC_{n-1}x^1y^{n-1} + {}_nC_nx^0y^n$$

The sum of the exponents for each term is n.

You can use the Binomial Theorem to expand any binomial.

Expand $(x + y)^5$.

Step 1 Use the Binomial Theorem with n = 5.

$$(x+y)^5 = {}_5C_0x^5y^0 + {}_5C_1x^4y^1 + {}_5C_2x^3y^2 + {}_5C_3x^2y^3 + {}_5C_4x^1y^4 + {}_5C_5x^0y^5$$

Step 2 Use the fifth row of Pascal's triangle to find the coefficients.
Continue the pattern and add pairs of terms from the fourth row to find the numbers in the fifth row.

1 3 3 1

3

The sum of the exponents for each term is 5.The exponents of x decrease as the exponents

1 4 6 4 1 ← n = 4

1 5 10 10 5 1 ← n = 5

Step 3 Write the coefficients.

$(x + y)^5 = 1x^5y^0 + 5x^4y^1 + 10x^3y^2 + 10x^2y^3 + 5x^1y^4 + 1x^0y^5$

Step 4 Simplify.

$(x + y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

Use the Binomial Theorem to expand each binomial.

1. $(2x + y)^3 = {}_3C_0(2x)^3y^0 + {}_3C_1(2x)^2y^1 + {}_3C_2(2x)^1y^2 + {}_3C_3(2x)^0y^3$

   $= {}_3C_0\ 8x^3 + {}_3C_1\ 4x^2y^1 + {}_3C_2\ 2xy^2 + {}_3C_3\ y^3$

   = ____

   = ____

2. $(x + 3y)^4$ = ____

   = ____

   = ____

LESSON 6-4

Factoring Polynomials

Factoring a sum of two cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

Example Factor $125a^3 + 8$.

$125x^3 + 8$

Recognize the sum of two cubes. $(5x)^3 + (2)^3$

Factor using factoring pattern. $(5x + 2)((5x)^2 - (5x)(2) + (2)^2)$

Simplify. $(5x + 2)(25x^2 - 10x + 4)$

Factor.

1. $27x^3 + 1$

2. $m^3 + \frac{1}{8}$

3. $p^3 + 216$

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Factoring a difference of two cubes: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Example Factor $27a^3 - 64$.

$27a^3 - 64$

Recognize the difference of two cubes. $(3a)^3 - (4)^3$

Factor using factoring pattern. $(3a - 4)((3a)^2 + (3a)(4) + (4)^2)$

Simplify. $(3a - 4)(9a^2 + 12a + 16)$

Factor.

4. $8x^3 - 1$

5. $b^3 - 1000$

6. $125t^3 - 343$

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LESSON 6-5

Dividing Polynomials

Example Divide $(x^3 - 2x^2 - 22x + 45)$ by $(x - 5)$ using synthetic division.

5 |				

→

5 |	1	–2	–22	45

→

5 |	1	–2	–22	45
	1			

→

5 |	1	–2	–22	45
		5		
	1			

→

5 |	1	–2	–22	45
		5		
	1	3		

→

5 |	1	–2	–22	45
		5	15	
	1	3		

→

5 |	1	–2	–22	45
		5	15	
	1	3	–7	

→

5 |	1	–2	–22	45
		5	15	–35
	1	3	–7	

→

5 |	1	–2	–22	45
		5	15	–35
	1	3	–7	|10

Quotient: $x^2 + 3x - 7$
Remainder: 10

Divide using synthetic division. Give the quotient and the remainder.

1. $(3x^3 - 4x^2 + x - 16) \div (x - 2)$

2. $(-2x^3 + 3x^2 + 5x + 12) \div (2x - 1)$

Quotient: ________________________ Quotient: ________________________

Remainder: ______________________ Remainder: ______________________

3. $\left(5x^3+6x^2-7x+8\right)\div\left(x+4\right)$

4. $\left(-2x^4+x^2+12x+1\right)\div\left(x-1\right)$

Quotient: ___________________________

Remainder: _________________________

Quotient: ___________________________

Remainder: _________________________

Use long division to divide.

5. $$\begin{array}{r} 4x \\ x+2\overline{)4x^2+7x+6} \\ -\left(4x^2+8x\right) \\ \hline -x+6 \end{array}$$

6. $x+4\overline{)2x^2+9x+9}$

Use synthetic division to divide.

7. $(4x^2+7x+10)\div(x+2)$

$a=-2$

$$\begin{array}{r|rrr} -2 & 4 & 7 & 10 \\ & & -8 & \\ \hline & 4 & & \end{array}$$

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8. $(2x^2-6x-12)\div(x-5)$

$a=$ _______________

$$\begin{array}{r|rrr} & 2 & -6 & -12 \\ & & & \\ \hline & & & \end{array}$$

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LESSON 7-1

Finding Rational Solutions of Polynomial Equations

Rational Root Theorem:	Possible rational roots are of the form $\frac{m}{n}$ where m = factor of the constant term n = factor of the leading coefficient

Example Find the rational zeros of $x^3 - 11x^2 + 23x + 35$, then write the function in factored form.

Step 1: List possible rational roots.

$x^3 - 11x^2 + 23x + 35$	Constant term: 35 Factors: $\pm1, \pm5, \pm7, \pm35$	Leading Coefficient: 1 Factors: ±1
Possible Rational Roots: $\frac{m}{n} = \frac{\pm1, \pm5, \pm7, \pm35}{\pm1} = \pm1, \pm5, \pm7, \pm35$		

Step 2: Use synthetic division to test for a zero remainder.

$\begin{array}{r\|rrrr} 1 & 1 & -11 & 23 & 35 \\ & & 1 & -10 & 13 \\ \hline & 1 & -10 & 13 & \|48 \end{array}$ Remainder is not 0, so 1 is not a root.	$\begin{array}{r\|rrrr} 5 & 1 & -11 & 23 & 35 \\ & & 5 & -30 & -35 \\ \hline & 1 & -6 & -7 & \|0 \end{array}$ Remainder is 0, so 5 is a root.

Step 3: Factor the remaining quadratic to find the zeros and write the polynomial in factored form.

$x^3 - 11x^2 + 23x + 35 = (x-5)(x^2 - 6x - 7)$ $= (x-5)(x-7)(x+1)$	Rational zeros are 5, 7, and –1, and $f(x) = (x-5)(x-7)(x+1)$.

Find the rational zeros of each polynomial function, then write each function in factored form.

1. $f(x) = 2x^3 + x^2 - 25x + 12$

2. $f(x) = x^3 - 11x^2 + 8x + 20$

3. $f(x) = x^4 - x^3 - 26x^2 - 24x$

4. $f(x) = x^3 + 2x^2 - 13x + 10$

LESSON 7-2 Finding Complex Solutions of Polynomial Equations

$a+bi$ and $a-bi$ are complex conjugates.

Example Give the complex conjugate of each number.

$-2-i$	$4+3i$	$5i$
Complex Conjugate: $-2+i$	Complex Conjugate: $4-3i$	Complex Conjugate: $-5i$

Give the complex conjugate of each number.

1. $1-i$
2. $-1+3i$
3. $-2i$

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Complex Conjugate Root Theorem:	If $a+bi$ is an imaginary root of a polynomial equation with real-number coefficients, then $a-bi$ is also a root.

Example Write the polynomial function with the least degree and a leading coefficient of 1 that has zeros $1-2i$, 5, and –1.

Complex roots come in conjugate pairs → zeros are $1-2i$, $1+2i$, 5, and –1.

Write the function in factored form. $p(x)=(x-(1-2i))(x-(1+2i))(x-5)(x+1)$

$=\left[x^2-(1+2i)x-(1-2i)x+(1-2i)(1+2i)\right](x-5)(x+1)$

Multiply the complex conjugate factors using FOIL, then simplify. $=\left[x^2+(-1-2i-1+2i)x+(1-4i^2)\right](x-5)(x+1)$

$=(x^2-2x+5)(x-5)(x+1)$

Multiply the binomials. $=(x^2-2x+5)(x^2-4x-5)$

Use the distributive property. $=x^2(x^2-4x-5)-2x(x^2-4x-5)+5(x^2-4x-5)$

$=x^4-4x^3-5x^2-2x^3+8x^2+10x+5x^2-20x-25$

Combine like terms. $=x^4-6x^3+8x^2-10x-25$

Write the polynomial function with the least degree and a leading coefficient of 1 that has the given zeros.

4. 1, –2, and $3+i$
5. –3, 5, and $2i$

Answer Key:

Reteach 6-1

1. $6x^4 + 10x^3 - x^2 + 3x - 11$
2. $4x^3 + 6x^2 - x - 13$
3. $4a^4 - 2a^3 - 9a^2 - 2a + 14$
4. $-3y + 7$
5. $-6x^4 - 4x^3 + 6x^2 - 2x - 9$
6. $3x^4 - x^3 + 4x^2 - 5x - 8$
7. $2c^4 - 5c^3 + c^2 + 7c - 17$
8. $3r^3 + 2r^2 + r$

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Reteach 6-2

1. $-2x^3 - 15x^2 + 14x + 48$
2. $2x^3 - 13x^2 + 23x - 12$
3. $2x^3 - x^2 - 23x + 24$
4. $-12x^3 - x^2 + 38x - 5$

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Reteach 6-3

1. $1 . 8x^3 + 3 . 4x^2y + 3 . 2xy^2 + 1 . y^3$;
   $8x^3 + 12x^2y + 6xy^2 + y^3$
2. ${}_4C_0x^4(3y)^0 + {}_4C_1x^3(3y)^1 + {}_4C_2x^2(3y)^2 + {}_4C_3x^1(3y)^3 + {}_4C_4x^0(3y)^4$;
   $1 . x^4 + 4 . (3x^3y) + 6 . (9x^2y^2) + 4 .$
   $(27xy^3) + 1 . (81y^4)$;
   $x^4 + 12x^3y + 54x^2y^2 + 108xy^3 + 81y^4$

Reteach 6-4

1. $(3x + 1)(9x^2 - 3x + 1)$
2. $\left(m + \frac{1}{2}\right)\left(m^2 - \frac{1}{2}m + \frac{1}{4}\right)$
3. $(p + 6)(p^2 - 6p + 36)$
4. $(2x - 1)(4x^2 + 2x + 1)$
5. $(b - 10)(b^2 + 100b + 100)$
6. $(5t - 7)(25t^2 + 35t + 49)$

Reteach 6-5

1. Quotient: $3x^2 + 2x + 5$; Remainder: -6
2. Quotient: $-2x^2 + 2x + 6$; Remainder: 15
3. Quotient: $5x^2 - 14x + 49$; Remainder: -188
4. Quotient: $-2x^3 - 2x^2 - x + 11$; Remainder: 12

5. $4x - 1 + \frac{8}{x+2}$ 6. $2x + 1 + \frac{5}{x+4}$

7. $4x - 1 + \frac{12}{x+2}$

8. $a = 5$

$2x + 4 + \frac{8}{x-5}$

Reteach 7-1

1. Zeros: –4, $\frac{1}{2}$, and 3; $f(x) = (x+4)(x-3)(2x-1)$
2. Zeros: –1, 2, and 10; $f(x) = (x+1)(x-2)(x-10)$
3. Zeros: 0, –4, –1, and 6; $f(x) = x(x+4)(x+1)(x-6)$
4. Zeros: –5, 1, and 2; $f(x) = (x+5)(x-1)(x-2)$

Reteach 7-2

1. $1 + i$
2. $-1 - 3i$
3. $2i$
4. $x^4 - 5x^3 + 2x^2 + 22x - 20$
5. $x^4 - 2x^3 - 11x^2 - 8x - 60$