Final Revision sheet Term2 Grade 11 Mathematics

LESSON 19-1

Probability and Set Theory

Reteach

Basic Set Vocabulary:

- **element** an object in a set (often a number or event)
- **empty set** a set with no elements; the symbol is $\varnothing$
- **set** a collection of distinct objects called elements
- **subset** a smaller set of elements within a universal set
- **universal set** a complete collection of elements

The **theoretical probability** of an event is the likelihood that an event will happen, where all possible outcomes are equally likely.

Theoretical Probability of *A*: $P(A) = \dfrac{\text{number of outcomes for event } A}{\text{total possible outcomes in universal set}} = \dfrac{n(A)}{n(U)}$

1. Consider the universal set of all odd numbers between 1 and 15: $U = \{1, 3, 5, 7, 9, 11, 15\}$. Complete the chart for each subset of the universal set.

Description of Subset	Set Notation	Number of Elements in Subset	Number of Elements in Universal Set
Multiples of 3	$A = \{$ ________________ $\}$	$n(A) =$ ______	$n(U) = 7$
Multiples of 5	$B = \{$ ________________ $\}$	$n(B) =$ ______	$n(U) = 7$

Suppose each element in *U* is written on a card. Calculate the theoretical probability of randomly choosing a number of each type from the set of cards.

2. $P(A) = \dfrac{n(A)}{n(U)} =$ ___________

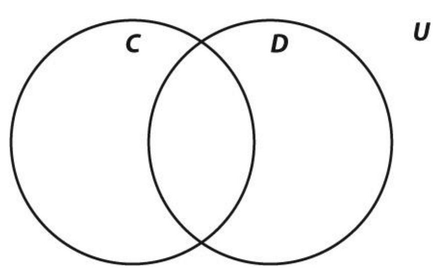
3. $P(B) = \dfrac{n(B)}{n(U)} =$ ___________

The **intersection** of sets *A* and B, $A \cap B$, is the set of all elements in **both** *A* **and** *B*.

The **union** of sets *A* and *B*, $A \cup B$, is the set of all elements in *A* **and/or** in *B*.

The **complement** of set *A* is the set of all elements in the universal set that are **not** in *A*.

4. Use the Venn diagram at right to organize the elements in $U = \{0, 4, 6, 9, 12, 16, 20\}$, when $C = \{0, 6, 12\}$ and $D = \{0, 4, 16\}$.



LESSON 19-2

Permutations and Probability

Reteach

To find the number of permutations of 3 objects chosen out of 10 total objects, use the "blank method."

A Draw the number of blanks in which the objects will be placed: $\boxed{}$ $\boxed{}$ $\boxed{}$.

B Record the number of options you have for the first blank: $\boxed{10}$ $\boxed{}$ $\boxed{}$.

C Record the number of options you have for the second blank, after the first object has been placed: $\boxed{10}$ $\boxed{9}$ $\boxed{}$

D Record the number of options you have for the third blank, after the first two objects have been placed: $\boxed{10}$ $\boxed{9}$ $\boxed{8}$

E Continue until all blanks are filled with a number. Multiply the numbers in the blanks to find the total permutations of 3 objects chosen out of 7 objects: $10 \cdot 9 \cdot 8 = 720$.

You can use this method to find the total number of permutations in the sample space *n*(*S*) as well as the total number of permutations of a specific type *n*(*A*). Calculate the probability of *A* as $\frac{n(A)}{n(S)}$.

1. A jewelry store clerk will choose 4 pieces of jewelry to display on a shelf. The clerk will choose the 4 pieces randomly from a group of 8 pieces. Solve for $n(S)$, the total number of different possible ways to choose the jewelry to display on the shelf.
   a. Use the space above to draw the appropriate number of blanks.
   b. How many options are there for the first blank, given that the clerk is choosing from 8 pieces? Record that number in the first blank.
   c. After the first piece is placed, how many options are there for the second blank? Record that number in the second blank.
   d. Fill in each remaining blank with the number of options that are left for each one.
   e. Write and solve a multiplication sentence to find how many possible ways there are to choose the jewelry to display on the shelf.

      $n(S) =$ ___________

2. Suppose that 5 of the pieces of jewelry the clerk can select from are made of gold. Let event *A* be "All 4 pieces on display are gold." What is the probability of *A*?
   a. Use the blank method to find $n(A)$, the number of different possible ways there are to choose 4 pieces of jewelry that are gold.

      $n(A) =$ ___________

   b. Find the probability of *A* by substituting the values you found into the equation below.

$$P(\text{all gold}) = P(A) = \frac{\text{number of permutations of 4 gold pieces}}{\text{total possible permutations of 4 pieces}} = \frac{n(A)}{n(S)} = \_\_\_\_\_\_\_\_\_\_\_\_$$

LESSON 19-3 Combinations and Probability

Reteach

Permutations vs. Combinations

- Remember to use a **permutation** when **order matters** and a **combination** when **order does not matter**.
- There are fewer combinations of a group of objects than there are permutations. You must divide the number of permutations by the number of permutations of the “blanks” in order to find the smaller number of combinations.

To find the number of combinations of 4 objects chosen out of 9 total objects, use the following steps.

A Draw the number of blanks in which the objects will be placed: $\boxed{}\ \boxed{}\ \boxed{}\ \boxed{}$.

B Use the “blank method” to find the number of permutations: $\boxed{9} \cdot \boxed{8} \cdot \boxed{7} \cdot \boxed{6} = 3024$.

C Find the factorial of the number of blanks: $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

D Divide the number from **B** by the number from **C**: $\frac{3024}{24} = 126$

You can use this method to find the total number of combinations in the sample space *n*(*S*) as well as the total number of combinations of a specific type *n*(*A*). Calculate the probability of A as $\frac{n(A)}{n(S)}$.

For 1–2, determine whether each is an example of a combination or a permutation.

1. Picking a group of 3 winners to come in first place, second place, and third place. ______
2. Picking a group of 3 winners from a group of 8 finalists. ______
3. Mike is choosing 3 books to borrow from a friend. He selects the books from the shelf at random. The shelf contains 11 books.
   a. Use the blank method to find how many permutations there are of 3 books chosen from the 11 on the shelf.

      ____

   b. Find the factorial of the number of blanks.

      ____

   c. Write and solve a division sentence to find the total number of combinations $n(S)$ of 3 books.

      $n(S) =$ __________________________
4. Suppose that the shelf contains 5 fiction books. Let event A be “All books that Mike chooses are fiction.” What is the probability of A?
   a. Write and solve a division sentence to find the total number of combinations $n(A)$ of 3 fiction books.

      $n(A) =$ __________________________

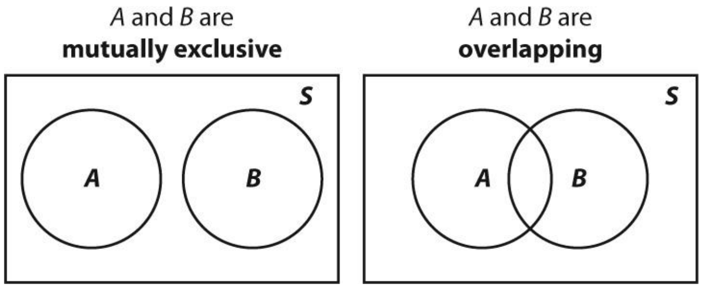
   b. Find the probability of A by substituting the values you found into the equation below.

$$P(\text{all fiction}) = P(A) = \frac{\text{number of combinations of 3 fiction books}}{\text{total possible combinations of 3 books}} = \frac{n(A)}{n(S)} = \underline{\qquad\qquad}$$

Mutually Exclusive and Overlapping Events

Reteach

If two events are **mutually exclusive**, then they cannot occur at the same time.
If two events are **overlapping**, then they can occur at the same time.



The **Addition Rule** tells you how to find the probability that *A* **or** *B* occurs.

- Addition Rule for mutually exclusive *A* and *B*: $P(A \text{ or } B) = P(A) + P(B)$
- Addition Rule for overlapping *A* and *B*: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Examples of the Addition Rule:

- Find the probability that a multiple of 5 or an even number is rolled on a number cube. The events are mutually exclusive because they cannot both occur at once:
  $P(A \text{ or } B) = P(A) + P(B) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$
- Find the probability that a multiple of 3 or an even number is rolled on a number cube. The events are overlapping because both occur when 6 is rolled:
  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = \frac{2}{6} + \frac{3}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$

For 1–4, determine whether the two events are mutually exclusive or overlapping.

1. rolling an even number on a die; rolling a 6 on a die ___________
2. rolling an odd number on a die; rolling a 2 on a die ___________
3. drawing a face card from a deck of cards; drawing a 5 from a deck of cards ___________
4. drawing a spade from a deck of cards; drawing a 7 from a deck of cards ___________

Use the appropriate version of the Addition Rule to find each probability.

5. *P*(rolling an even number on a die or rolling a 6 on a die) = ________
6. *P*(rolling an odd number on a die or rolling a 2 on a die) = ________
7. *P*(drawing a spade from a deck of cards or drawing a 7 from a deck of cards) = ________

LESSON 20-2

Independent Events

Reteach

If $P(A) = P(A|B)$ and $P(A \text{ and } B) = P(A) \cdot P(B)$, then A and B are independent.

	Prefers Rock	Prefers Classical	TOTAL
Male	12	3	15
Female	24	6	30
TOTAL	36	9	45

The table shows the genders and music preference in a group of people. Let *A* be that a person prefers rock and let *B* be that a person is male. Are *A* and *B* independent?

Method 1: Test whether $P(A) = P(A|B)$.

- $P(A) = \frac{36}{45} = \frac{4}{5}$ and $P(A|B) = \frac{12}{15} = \frac{4}{5}$. Therefore, $P(A) = P(A|B) = \frac{4}{5}$ and whether a person likes rock is independent of whether the person is male.

Method 2: Test whether $P(A \text{ and } B) = P(A) \cdot P(B)$.

- $P(A) = \frac{36}{45} = \frac{4}{5}$, $P(B) = \frac{15}{45} = \frac{1}{3}$, and $P(A \text{ and } B) = \frac{4}{15}$. Therefore, $P(A \text{ and } B) = P(A) \cdot P(B) = \frac{4}{5} \cdot \frac{1}{3} = \frac{4}{15}$ and whether a person likes rock is independent of whether the person is male.

If events A and B are independent, then $P(A \text{ and } B) = P(A) \cdot P(B)$.

Let *C* be that a person is female. Let *D* be that a person prefers rock. Use the table above to answer the questions.

1. Use **Method 1** to determine whether *C* is independent of *D*.

   a. $P(C) =$ _______________ b. $P(C|D) =$ _______________

   c. Is *C* independent of *D*? _______________

2. Use **Method 2** to determine whether *C* is independent of *D*.

   a. $P(C) =$ __________ b. $P(D) =$ __________ c. $P(C \text{ and } D) =$ __________

   d. Is *C* independent of *D*? ___________

3. Events *E* and *F* are independent. $P(E) = \frac{1}{2}$ and $P(F) = \frac{2}{5}$. What is the probability that both *E* and *F* will occur?

   $P(E \text{ and } F) =$ ________________________

LESSON 20-3

Dependent Events

Reteach

If A and B are **independent events**, then whether A happens or not has no effect on whether B happens. In cases of **selection with replacement**, the events are independent.

If A and B are **dependent events**, then the outcome of A has an effect on whether B happens. In cases of **selection without replacement**, the events are dependent.

Suppose a card is randomly selected from a deck, the deck is shuffled, and then a second card is drawn from the deck. Let A be selecting a spade on the first draw. Let B be selecting a spade on the second draw.

- If the first card is replaced, you start with the same set of cards for each draw. In this case, A and B are **independent** events.
- If the first card is **not** replaced before the second draw, the deck on the second draw is not exactly the same as the deck on the first draw. In this case, A and B are **dependent** events.

For each pair of events, determine whether *A* and *B* are independent or dependent.

1. Event A is rolling a 3 on the first roll of a number cube.
   Event B is rolling a 2 on the second roll of the number cube. ________________________

2. Event A is pulling a blue marble from a bag of colored marbles.
   Event B is pulling a blue marble on the second draw, given that the first marble pulled is **not** put back in the bag. ________________________

If events A and B are dependent, then $P(A \text{ and } B) = P(A) \cdot P(B|A)$.

A card is randomly selected from a deck and not replaced, the deck is shuffled, and then a second card is selected from the deck. Let A be selecting a spade on the first draw. Let B be selecting a spade on the second draw. What is the probability of getting a spade on both draws?

- There are 13 spades in a complete deck of 52 cards: $P(A) = \frac{13}{52}$
- If a spade is drawn first, there are 12 spades and 51 cards at the start of the second draw: $P(B|A) = \frac{12}{51}$
- $P(A \text{ and } B) = P(A) \cdot P(B|A) = \frac{13}{52} \cdot \frac{12}{51} = \frac{1}{17}$

3. A card is randomly selected from a deck and not replaced. The deck is shuffled, and then a second card is drawn. Let A be selecting a 2 on the first draw. Let B be selecting a 2 on the second draw. What is the probability that a 2 will be drawn both times?

   a. $P(A) =$ _________ b. $P(B|A) =$ _________ c. $P(A \text{ and } B) =$ _________

Data-Gathering Techniques

Reteach

Data gathered as a sample can be used to make predictions about the entire population.

<u>Types of samples</u> (simple random, Convenience, self-selected, systematic, stratified, cluster)

Example

A local high school drama department is gathering data about the audience members at its annual spring musical. A total of 624 people attended the show over three nights, and a random sample of 20 audience members per night was surveyed about their connections to the show and the number of tickets they purchased.

Type of Audience Member	Number of Audience Members	Number of Tickets Purchased
Family of Cast	22	96
School Faculty	8	24
Student	19	21
Other Community Member	11	45

Calculate the mean number of tickets purchased by a single audience member.

Step 1	Step 2	Step 3
Sum the total tickets purchased by members of this 60-person sample. $96+24+21+45=186$	Divide the number of tickets by the number of people. $\frac{186}{60}=3.1$	Interpret the result. Each audience member purchased ≈3.1 tickets.

Predict the total number of tickets purchased by non-students.

Step 1	Step 2	Step 3
Calculate the proportion of tickets purchased by non-students. $\frac{96+24+45}{96+24+21+45}=\frac{165}{186}$ $\approx 0.89=89\%$	Multiply this percentage by the total population to make your prediction. $(624)(0.89)\approx 555$	Interpret the result. ≈555 tickets were purchased by non-students.

Use the data table from the example above to answer the following questions.

1. Predict the total number of tickets purchased by faculty and students. ________________
2. Calculate the average number of tickets purchased by family of cast members. ________________
3. The drama department would like for 15% of its audience to be other community members not connected to the school. Did the drama department reach its goal with this musical? Explain.

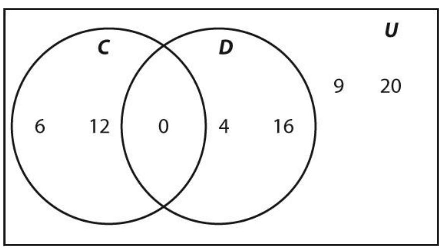
Reteach 19-1

1.

Description of Subset	Set Notation	Number of Elements in Subset	Number of Elements in Universal Set
Multiples of 3	$A = \{3, 9, 15\}$	$n(A) = 3$	$n(U) = 7$
Multiples of 5	$B = \{5, 15\}$	$n(B) = 2$	$n(U) = 7$

2. $\frac{3}{7}$

3. $\frac{2}{7}$

4. 

Reteach 19-2

1. a–d. $\boxed{8}$ $\boxed{7}$ $\boxed{6}$ $\boxed{5}$

   e. $8 \bullet 7 \bullet 6 \bullet 5 = 1680$

2. a. 120

   b. $\frac{1}{14}$

Reteach 19-3

1. permutation
2. combination
3. a. 990

   b. 6

   c. $\frac{990}{6} = 165$

4. a. $\frac{60}{6} = 10$

   b. $\frac{2}{33}$

Reteach 19-4

1. overlapping
2. mutually exclusive
3. mutually exclusive
4. overlapping
5. $\frac{1}{2}$
6. $\frac{2}{3}$
7. $\frac{4}{13}$

Reteach 20-2

1. a. $\frac{2}{3}$

   b. $\frac{2}{3}$

   c. yes

Reteach 20-3

1. independent
2. dependent
3. a. $\frac{4}{52}$ or $\frac{1}{13}$

   b. $\frac{3}{51}$ or $\frac{1}{17}$

   c. $\frac{1}{221}$

Reteach 22-1

1. 151 tickets
2. 4.4 tickets
3. $\frac{11}{60} \approx 18\%$ of the audience were other community members, so the drama department did reach the goal.