LIMIT LAWS

1. $\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1$ 2. $\lim\limits_{x\to\infty}\dfrac{\sin x}{x}=0$ 3. $\lim\limits_{x\to 0}\dfrac{\sin^2 x}{x}=0$
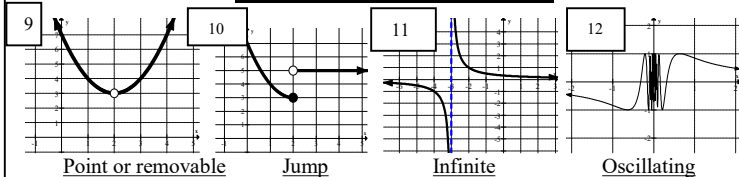
4. $\lim\limits_{x\to 0}\dfrac{\cos x-1}{x}=0$ 5. $\lim\limits_{x\to 0}\dfrac{\sin(ax)}{(bx)}=\lim\limits_{x\to 0}\dfrac{\sin(ax)}{\sin(bx)}=\lim\limits_{x\to 0}\dfrac{(bx)}{\sin(ax)}=\dfrac{a}{b}$

6. $\lim\limits_{x\to a}f(x)=L$ (exists) If and only if $\lim\limits_{x\to a^+}f(x)=\lim\limits_{x\to a^-}f(x)=L$

7. $f(x)$ is cont a if $\lim\limits_{x\to a}f(x)=f(a)$

8. **Continuity at** a **if** $\lim\limits_{x\to a^-}f(x)=f(a)=\lim\limits_{x\to a^-}f(x)$

TYPES OF DISCONTINUITY

Point or removable   Jump   Infinite   Oscillating

ARITHMETIC OF INFINITY

$(+)$
1. $\infty+\infty=\infty$ 2. $n+\infty=\infty$
3. $\infty+0=\infty$ 4. $0+\infty=\infty$

$(\times)$
1. $\infty\times\infty=\infty$ 2. $n\times\infty=\infty$
3. $0\times\infty=$ Ind.

$(-)$
1. $\infty-\infty=$ Ind.
2. $n-\infty=-\infty$
3. $\infty-n=\infty$
4. $n-n^-=0^-=-\dfrac{1}{\infty}$
5. $n^+-n=0^+=\dfrac{1}{\infty}$

$(\div)$
1. $\infty/\infty=$ Ind.
2. $n/\pm\infty=0$
3. $\infty/n=\infty$
4. $n/0=\pm\infty$
5. $n/0^+=\infty$
6. $n/0^-=-\infty$
7. $0/0=$ Ind.

power
1. $\infty^\infty=\infty$ 2. $\infty^n=\infty$
3. $\infty^0=$ Ind 4. $0^\infty=0$
5. $n^\infty=\infty$ 6. $1^\infty=$ Ind
7. $0^0=$ Ind.

Trig
1. $-1\le\sin(\pm\infty)\le 1$
2. $-1\le\cos(\pm\infty)\le 1$

Basic Rules

1. $\dfrac{d}{dx}c=0$ (Constant) 2. $\dfrac{d}{dx}c[f(x)]=c\dfrac{d}{dx}f(x)\begin{pmatrix}\text{constant}\\\text{multiple}\end{pmatrix}$

3. $\dfrac{d}{dx}x^n=nx^{n-1}$ (power) 4. $\dfrac{d}{dx}(u\pm v)=\dfrac{d}{dx}u\pm\dfrac{d}{dx}v\begin{pmatrix}\text{Sum \&}\\\text{Difference}\end{pmatrix}$

5. $\dfrac{d}{dx}(uv)=v\dfrac{d}{dx}u+u\dfrac{d}{dx}v$ (Product) 6. $\dfrac{d}{dx}\left(\dfrac{u}{v}\right)=\dfrac{v\frac{d}{dx}u-u\frac{d}{dx}v}{v^2}$ (Quotient)

7. $\left(f^{-1}\right)'(x)=\dfrac{1}{f'\left(f^{-1}(x)\right)}$ (Inverse)

8. $\dfrac{d}{dx}f(g(x))=\dfrac{d}{dx}f(u)\square\dfrac{d}{dx}g(x)$ where $u=g(x)$ (Quotient)
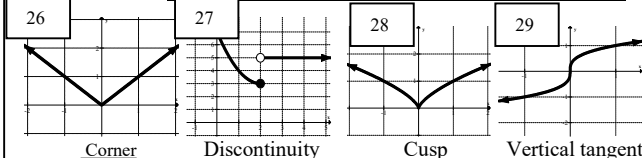
Exponential and Logarithmic Functions

20. $\dfrac{d}{dx}\ln u=\dfrac{1}{u}\dfrac{du}{dx}$ 21. $\dfrac{d}{dx}\log_a u=\dfrac{1}{u\ln a}\dfrac{du}{dx}$

22. $\dfrac{d}{dx}e^u=e^u\dfrac{du}{dx}$ 23. $\dfrac{d}{dx}a^u=a^u\ln a\dfrac{du}{dx}$

24. $\dfrac{d}{dx}\sqrt{u}=\dfrac{1}{2\sqrt{u}}\dfrac{du}{dx}$ 25. $\dfrac{d}{dx}|u|=\dfrac{u}{|u|}\dfrac{du}{dx}$

WHERE THE DERIVATIVE DOES NOT EXIST

Corner   Discontinuity   Cusp   Vertical tangent

Linearization: $L(x)=f(a)+f'(a)(x-a)$

Trigonometric Functions

8. $\dfrac{d}{dx}\sin u=\cos u\dfrac{du}{dx}$ 9. $\dfrac{d}{dx}\cos u=-\sin u\dfrac{du}{dx}$

10. $\dfrac{d}{dx}\tan u=\sec^2 u\dfrac{du}{dx}$ 11. $\dfrac{d}{dx}\cot u=-\csc^2 u\dfrac{du}{dx}$

12. $\dfrac{d}{dx}\sec u=\sec u\tan u\dfrac{du}{dx}$ 13. $\dfrac{d}{dx}\csc u=-\csc u\cot u\dfrac{du}{dx}$

Inverse Trigonometric Functions

14. $\dfrac{d}{dx}\sin^{-1}u=\dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx}$ 15. $\dfrac{d}{dx}\cos^{-1}u=\dfrac{-1}{\sqrt{1-u^2}}\dfrac{du}{dx}$

16. $\dfrac{d}{dx}\tan^{-1}u=\dfrac{1}{1+u^2}\dfrac{du}{dx}$ 17. $\dfrac{d}{dx}\cot^{-1}u=\dfrac{-1}{1+u^2}\dfrac{du}{dx}$

18. $\dfrac{d}{dx}\sec^{-1}u=\dfrac{1}{|u|\sqrt{u^2-1}}\dfrac{du}{dx}$ 19. $\dfrac{d}{dx}\csc^{-1}u=\dfrac{-1}{|u|\sqrt{u^2-1}}\dfrac{du}{dx}$

30. Mean Value Theorem: If f is cont on $[a,b]$ on and diff on (a,b)

$\Rightarrow$ exist a $c\in(a,b)$ s.t. $f'(c)=\dfrac{f(b)-f(a)}{b-a}$

31. Rolle's Theorem : MVT where $f'(c)=\dfrac{f(b)-f(a)}{b-a}=0$

32. Intermediate Value Theorem. : If f is cont on $[a,b]$ and $d\in[f(a),f(b)]$ then there is a $c\in[a,b]$ st $f(c)=d$

Definition of Derivative

33. $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

34. $f'(a)=\lim\limits_{x\to a}\dfrac{f(x)-f(a)}{x-a}$

35. $f'(x)\approx$ **Average rate of change**
   = Slope of Secant line
   $=\dfrac{f(b)-f(a)}{b-a}$

APPROXIMATING AREA

11. $\mathbf{LRAM}_n=w(f(x_1)+f(x_2)+..+f(x_{n-1}))$ or $w_1f(x_1)+w_2f(x_2)+..+w_{n-1}f(x_{n-1})$

12. $\mathbf{RRAM}_n=w(f(x_2)+f(x_3)+\cdots+f(x_n))$ or $w_1f(x_2)+w_2f(x_2)+..+w_{n-1}f(x_n)$

13. $\mathbf{MRAM}_n=w\left(f\left(\dfrac{x_1+x_2}{2}\right)+f\left(\dfrac{x_2+x_3}{2}\right)+\cdots+f\left(\dfrac{x_{n-1}+x_n}{2}\right)\right)$ or

$w_1f\left(\dfrac{x_1+x_2}{2}\right)+w_2f\left(\dfrac{x_2+x_3}{2}\right)+\cdots+w_{n-1}f\left(\dfrac{x_{n-1}+x_n}{2}\right)$

Note: $w=\dfrac{b-a}{n}$ **and applies only for equal sub intervals**

14. $\mathbf{T}_n=\dfrac{w}{2}(y_1+2y_2+\cdots+2y_{n-1}+y_n)$ **or** $\dfrac{1}{2}(w_1(y_1+y_2)+w_2(y_2+y_3)+...)$

20. Area $=\displaystyle\int_{x_{left}}^{x_{right}}\left[f(x)_{top}-f(x)_{down}\right]dx$ or Area $=\displaystyle\int_{y_{down}}^{y_{top}}\left[f(y)_{left}-f(y)_{right}\right]dy$

21. Vol of rev $=\pi\displaystyle\int_{x_{left}}^{x_{right}}\left\{\left[f(x)_{top}-a\right]^2-\left[f(x)_{down}-a\right]^2\right\}dx$ Vol Cross sect.$=\displaystyle\int_a^b A(x)dx$

ANTIDIFFERENTION (INTEGRATION) RULES

1. $\displaystyle\int x^n dx=\dfrac{x^{n+1}}{n+1}+C$ 2. $\displaystyle\int\dfrac{1}{x}dx=\ln|x|+C$ 3. $\displaystyle\int\dfrac{1}{ax+b}dx=\dfrac{\ln|ax+b|}{a}+C$ 4. $\displaystyle\int e^{kx}dx=\dfrac{e^{kx}}{k}+C$

5. $\displaystyle\int a^x dx=\dfrac{a^u}{\ln a}+C$ 6. $\displaystyle\int\sin kx\,dx=-\dfrac{\cos kx}{k}+C$ 7. $\displaystyle\int\cos kx\,dx=\dfrac{\sin kx}{k}+C$

8. $\displaystyle\int\sec x\tan x\,dx=\sec x+C$ 9. $\displaystyle\int\sec^2 x\,dx=\tan x+C$ 10. $\displaystyle\int\csc x\cot x\,dx=-\csc x+C$
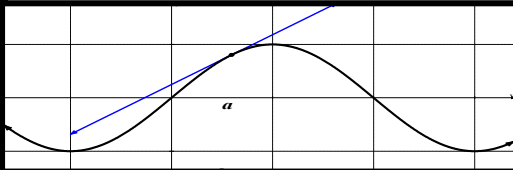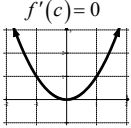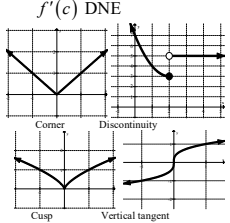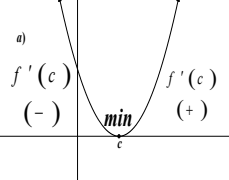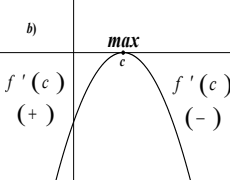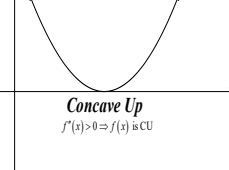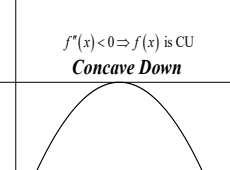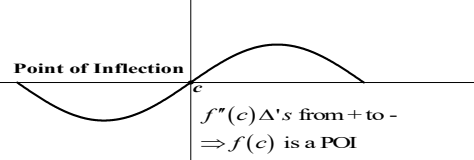
15. **Average Value of f** $\operatorname{av}(f)=f_{ave}=\dfrac{1}{b-a}\displaystyle\int_a^b f(x)dx$

16. FTC I: $\displaystyle\int_a^b f'(x)dx=f(b)-f(a)$ 17 FTC II i) $\displaystyle\int_a^x f(t)dt=F(x)$

ii) $\dfrac{d}{dx}\displaystyle\int_a^x f(t)dt=f(x)$ iii) $\dfrac{d}{dx}\displaystyle\int_{h(x)}^{g(x)}f(t)dt=f(g(x))\cdot g'(x)-h(g(x))\cdot h'(x)$

18. Integration by parts: $\displaystyle\int v\,du=uv-\int v\,du$ use LIPET to select u

19. Integration by substitution: $\displaystyle\int f(g(x))g'(x)=\int f(u)du$

Term	Verbal Description	Symbolic	Graphical
1. Derivative of f at a:	The instantaneous rate of change of the function at a or the slope of the tangent line at a	$f'(a) = \dfrac{df}{dx}\Big|_{x=a}$ $= \lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$	
2. Critical Number c	A number c in an open (a, b) interval where the derivative is zero or does not exist	$c \in (a,b)$ where $f'(c) = 0$ or $f'(c)$ DNE	$f'(c)=0$ $f'(c)$ DNE Corner Discontinuity Cusp Vertical tangent
3. First Derivative Test	a) If the first derivative changes from negative to positive at c then the function has a relative minimum at c b) If the first derivative changes from positive to negative at c then the function has a relative maximum at c	a) If $f'(c)$ Δ's from $-$ to $+$ $\Rightarrow f'(c)$ is a min b) If $f'(c)$ Δ's from $+$ to $-$ $\Rightarrow f'(c)$ is a max	a) $f'(c)$ $(-)$ $f'(c)$ $(+)$ min b) max c $f'(c)$ $(+)$ $f'(c)$ $(-)$
4. Concavity Test	a) If the second derivative is positive on an interval I then the function is Concave Up on I b) If the second derivative is negative on an interval I the function is Councave down on I	a) If $f''(c) > 0$ on I $\Rightarrow f(x)$ is CU on I b) If $f''(c) < 0$ on I $\Rightarrow f(x)$ is CD on I	Concave Up $f'(x)>0 \Rightarrow f(x)$ is CU $f''(x)<0 \Rightarrow f(x)$ is CU Concave Down
5. Point of Inflection at c	**f:** Is a point where the concavity of f changes **f':** Is a point where f' changes from increasing to decreasing or decreasing to increasing **f'':** Is a point where f'' changes from positive to negative or negative to positive	f Δ's from CU to CD or CD to CU $f'\Delta's$ from □ to □ or □ to □ $f''(x)$ Δ's from $+$ to $-$ or $-$ to	**Point of Inflection** c $f''(c)\Delta's$ from $+$ to $-$ $\Rightarrow f(c)$ is a POI

VOCABULARY (4)

LIMITS (1)

Motion definitions and Equations

6. Displacement: A Vector quantity that represents the net change in position
$$s(t) = x(b) - x(a) = \int_a^b v(t)$$

7. Distance: A scalar quantity that represents total movement regardless of sign
$$d(t) = |x(b) - x(a)| = \int_a^b |v(t)|\,dt$$

8. Velocity: A Vector quantity that represents the rate of change of position
$$v(t) = s'(t)$$

9. Speed: A scalar quantity that represents the rate of covering distance
$$\text{Speed} = |v(t)|$$

10. Acceleration: A vector quantity that represents the rate of change of velocity
$$a(t) = v'(t) = s''(t)$$

11. Given initial position $s(a) = C$ the final position is given by $s(b) = s(a) + \int_a^b s'(t)\,dt$

Reciprocal	**Quotient**	**Pythogorean**
$\sin x = \dfrac{1}{\csc x}$ $\csc x = \dfrac{1}{\sin x}$ $\cos x = \dfrac{1}{\sec x}$ $\sec x = \dfrac{1}{\cos x}$ $\tan x = \dfrac{1}{\cot x}$ $\cot x = \dfrac{1}{\tan x}$	$\tan x = \dfrac{\sin x}{\cos x}$ $\cot x = \dfrac{\cos x}{\sin x}$	$\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $\cot^2 x + 1 = \csc^2 x$
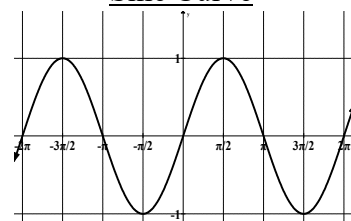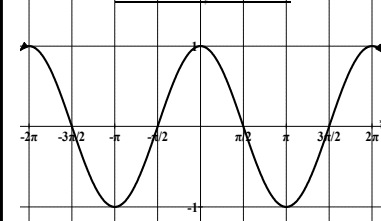
Sine Curve

Cosine Curve

	0	$\pi/6\ (30°)$	$\pi/4\ (45°)$	$\pi/3\ (60°)$	$\pi/2\ (90°)$
$\sin x$	0	$\dfrac{1}{2}$	$\dfrac{\sqrt{2}}{2}$	$\dfrac{\sqrt{3}}{2}$	1
$\cos x$	1	$\dfrac{\sqrt{3}}{2}$	$\dfrac{\sqrt{2}}{2}$	$\dfrac{1}{2}$	0
$\tan x$	0	$\dfrac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Und.
$\csc x$	Und.	2	$\dfrac{2}{\sqrt{2}}$	$\dfrac{2}{\sqrt{3}}$	1
$\sec x$	1	$\dfrac{2}{\sqrt{3}}$	$\dfrac{2}{\sqrt{2}}$	2	Und.
$\cot x$	Und.	$\dfrac{2}{\sqrt{3}}$	1	$\dfrac{1}{\sqrt{3}}$	0

Linear $y = x$ **quadratic** $y = x^2$ **Cubic** $y = x^3$ **Radical** $y = \sqrt{x}$

Logarithmic $y = \ln x$ **Exponential** $y = e^x$ **Absolute value** $y = |x|$ **circular** $y = \sqrt{9 - x^2}$

AP Calculus Survival Kit