

## ULTIMATE PROPERTIES OF LOGARITHMS FORMULA SHEET

Property	Logarithm base b	Natural Log (base e)	Examples
1. Product Property	$\log_b(xy) = \log_b x + \log_b y$	$\ln(xy) = \ln x + \ln y$	$\log_3(10) = \log_3(5 \cdot 2)$ $= \log_3 5 + \log_3 2$
2. Quotient Property	$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$	$\ln\left(\frac{x}{y}\right) = \ln x - \ln y$	$\log_3\left(\frac{5}{7}\right) = \log_3 5 - \log_3 7$
3. Powers Property	$\log_b x^p = p \log_b x$	$\ln x^p = p \ln x$	$\ln 27 = \ln 3^3 = 3 \ln 3$
4. Root Property	$\log_b \sqrt[p]{x} = \frac{1}{p} \log_b x$	$\ln \sqrt[p]{x} = \frac{1}{p} \ln x$	$\log_2 \sqrt[3]{y} = \frac{1}{3} \log_2 y$
5. Inverse Property	$\log_b b^x = x$ or $b^{\log_b x} = x$	$\ln e^x = x$ or $e^{\ln x} = x$	$\log_3 3^4 = 4$
6. Identity Property	$\log_b b = 1$	$\ln e = 1$	$\log_{\sqrt{4}} \sqrt{4} = 1$
7. Zero Property	$\log_b 1 = 0$	$\ln 1 = 0$	$\log_4 1 = 0$
8. Change of base Property	$\log_b x = \frac{\log_a x}{\log_a b}$	$\ln x = \frac{\log_a x}{\log_a e}$	$\log_5 6 = \frac{\log 6}{\log 5}$
9. Equality Property	If $\log_b x = \log_b y$ then $x = y$	If $\ln x = \ln y$ then $x = y$	$\log_5 x = \log_5 6$ $x = 6$
10. Reciprocal Property	$\log_b \frac{1}{x} = -\log_b x$	$\ln \frac{1}{x} = -\ln x$	$\ln \frac{1}{5} = -\ln 5$

