Principles of Lasers

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The course

❑ 4 credits, 64 credit hours

❑ 70% exame, 20% homework, 10% Lab

❑ Textbook:

O. Svelto, *Principles of Lasers*, Springer, 2010 (main)

❑ Reference：

B. Saleh and M. Teich, *Fundamentals of Photonics*, Wiley, 2007

William T. Silfvast, Laser Fundamentals, Cambridge, 2004

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Electromagnetic spectrum

Electromagnetic spectrum			
Name	Wavelength	Frequency (Hz)	Photon energy (eV)
Gamma ray	< 0.02 nm	> 15 EHz	> 62.1 keV
X-ray	0.01 nm – 10 nm	30 EHz – 30 PHz	124 keV – 124 eV
Ultraviolet	10 nm – 400 nm	30 PHz – 750 THz	124 eV – 3 eV
Visible light	390 nm – 750 nm	770 THz – 400 THz	3.2 eV – 1.7 eV
Infrared	750 nm – 1 mm	400 THz – 300 GHz	1.7 eV – 1.24 meV
Microwave	1 mm – 1 m	300 GHz – 300 MHz	1.24 meV – 1.24 µeV
Radio	1 mm – 100,000 km	300 GHz – 3 Hz	1.24 meV – 12.4 feV

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Electromagnetic spectrum
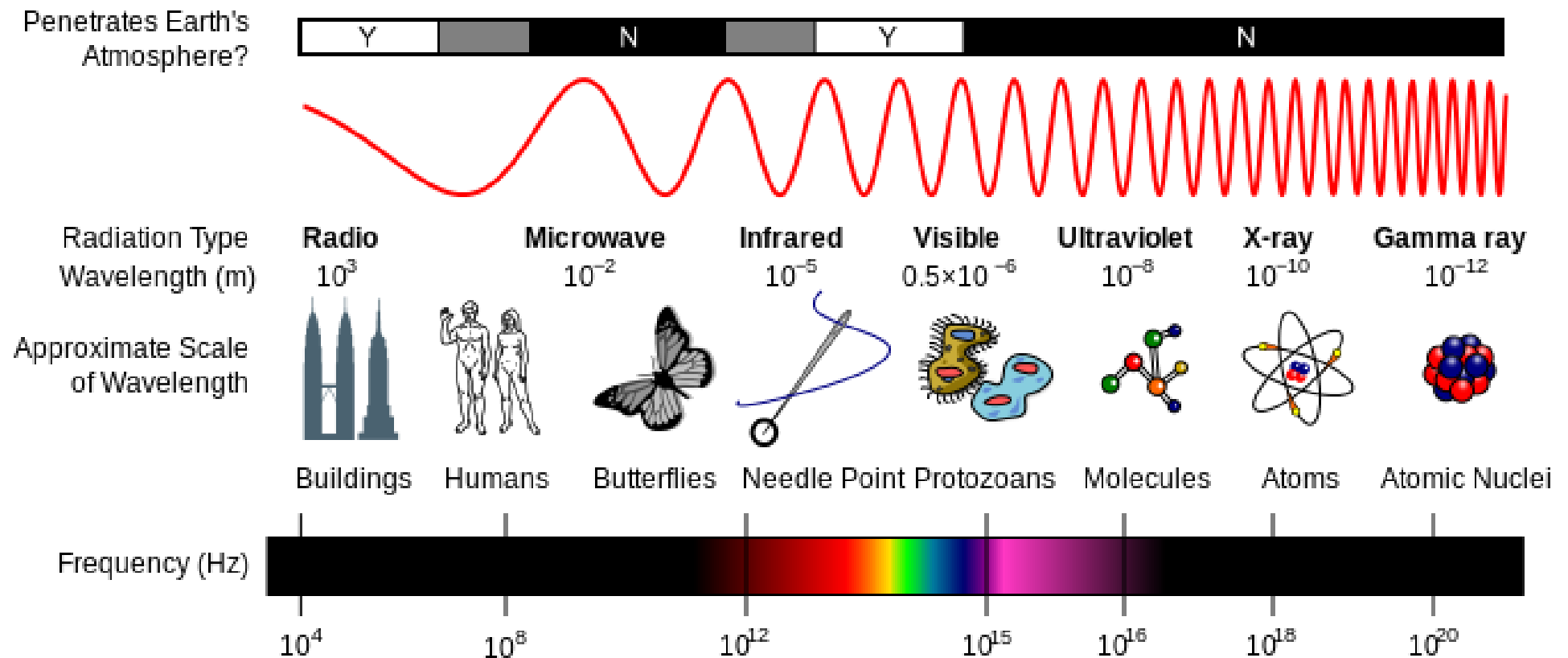
Ref: Wikipedia

Photons

❑ A photon is an elementary particle, the quantum form of light, it has energy, momentum, and mass.

❑ Photon exhibits wave-particle duality, both waves and particles.
  -The nature of waves shows frequency, phase, polarization, etc.
  -The nature of particles shows energy, momentum, mass, etc.

❑ Photon energy is related to the frequency

$$E = h\nu$$

❑ Photon has moving mass, while the rest mass is zero

$$m = E / c^2$$

❑ Photon momentum is related to the wave vector

$$\vec{P} = mc\vec{e} = \frac{E}{c}\vec{e} = \frac{h\nu}{c}\vec{e} = \frac{h}{2\pi}\frac{2\pi}{\lambda}\vec{e} = \hbar\vec{k}$$
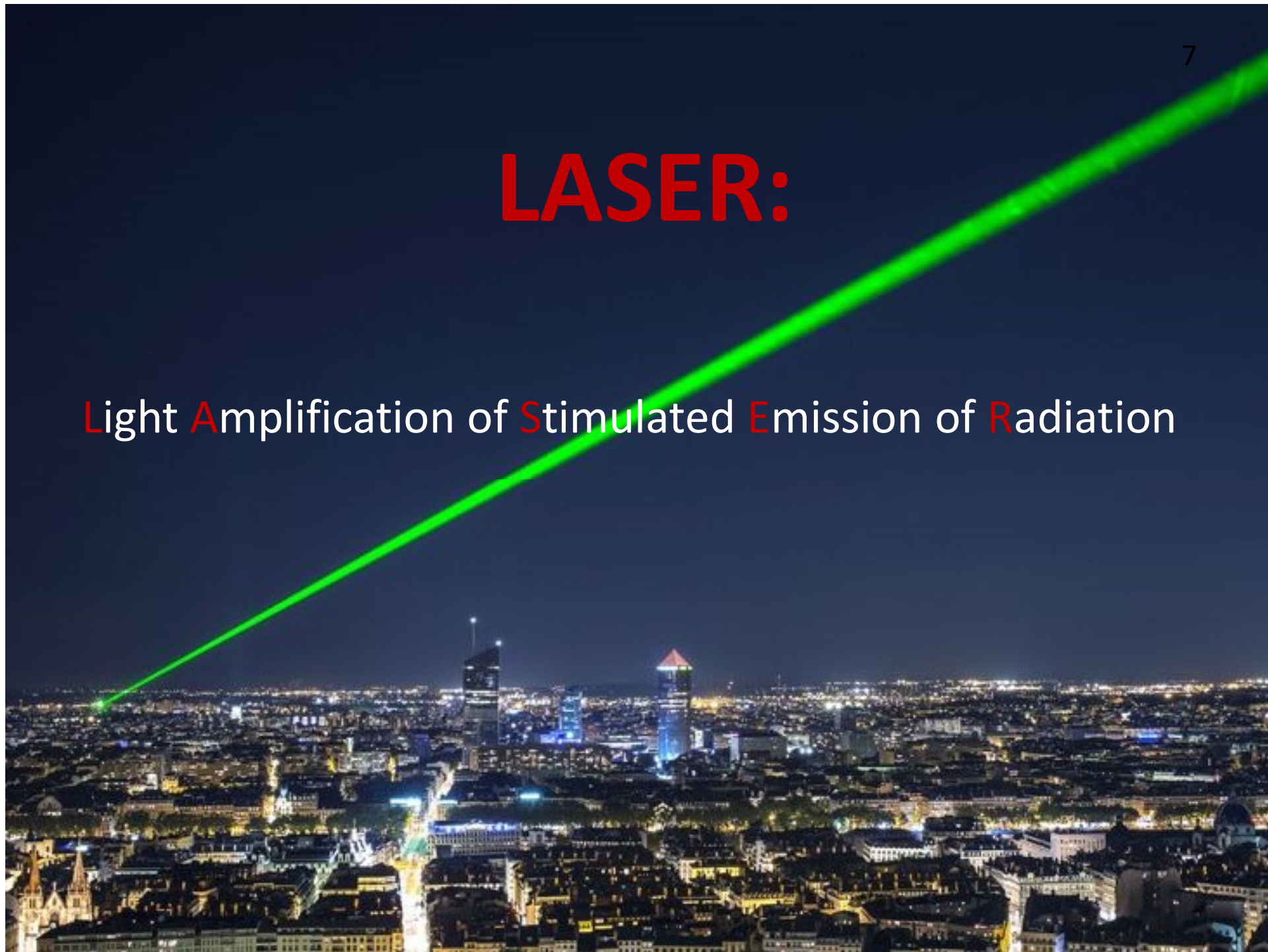
Wave number $k = 2\pi / \lambda$

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Photons

❑ Photon has two independent polarizations, corresponding to the two polarizations of optical waves.

❑ Photon has spin, and the quantum number of spin is integer.

❑ The statistics of a large number of photons obey the Bose-Einstein statistics, that is, for a certain quantum state, there is no limitation of the photon number.

- In contrast, electrons, protons, neutrons obey the Fermi Dirac statistics, that is, one quantum state can not have two identical fermions (Pauli exclusion principle)
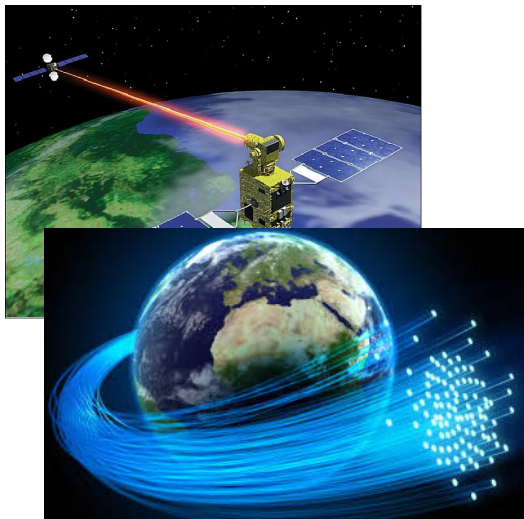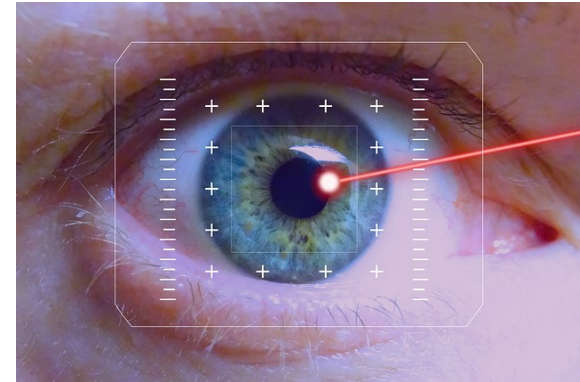
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LASER:

Light Amplification of Stimulated Emission of Radiation

Lasers in our life

Reflected Light from Surface

Laser Scans Terrain

https://youtu.be/D0DbgNju2wE
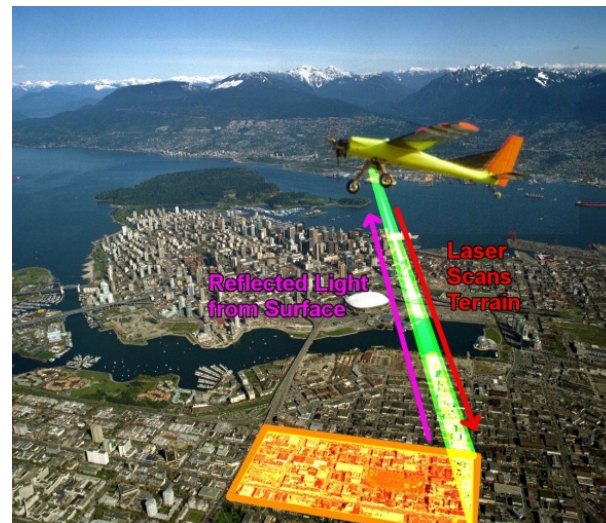
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Approaches

| Approach	Classical thoery	Semi-classical theory	Quantum theory (Quantum Electrodynamics)
Matter	Classical, Newtonian mechanics	Quantized, Quantum mechanics	Quantized, Quantum mechanics
Light	Classical, Maxwell's equations	Classical, Maxwell's equations	Quantized, Quantum field thoery
Complexity	Simple	middle	complex

❑ Laser history

❑ Laser concept

❑ Laser properties

Discovery of stimulated emission in 1917

Albert Einstein

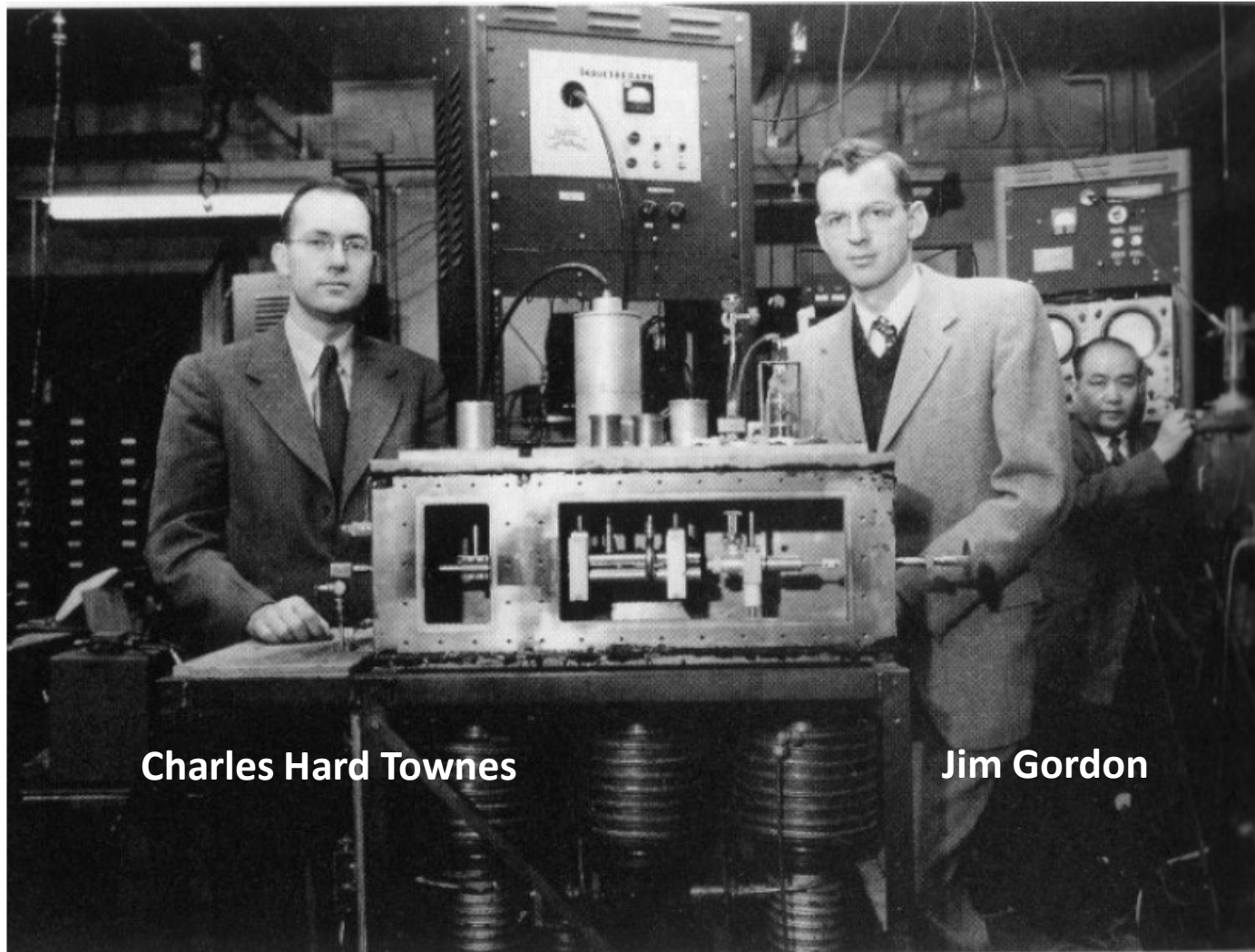* 14.3.1879 (Ulm, Germany) † 18.4.1955, (Princeton, USA)
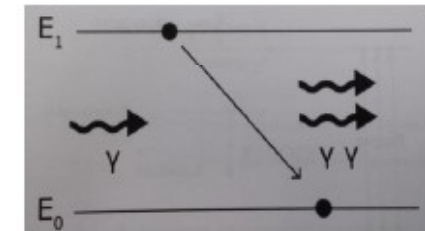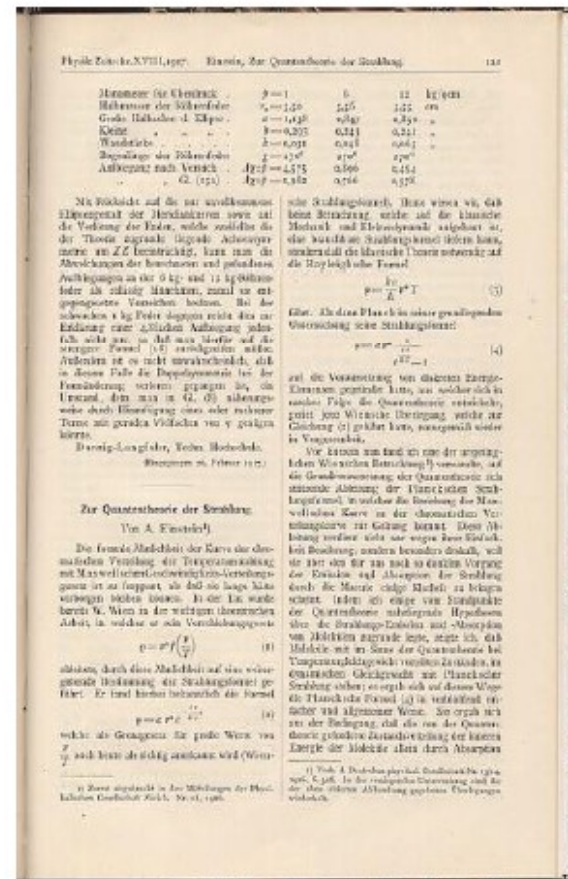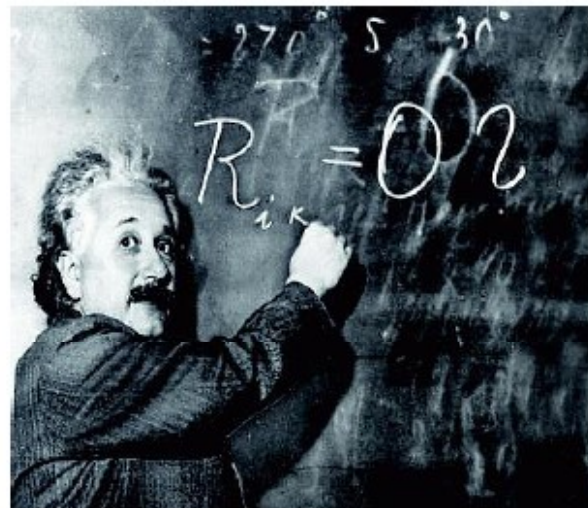
Ref: S. Domsch, *Basics of Laser Physics,* at Univeritaetsmedizin Mannheim
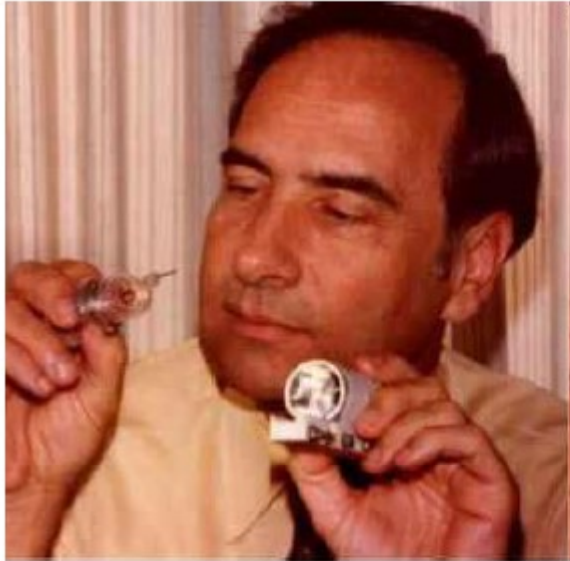
Charles Hard Townes

Jim Gordon

C. H. Townes, *How the Laser Happened*, (Oxford, 1999).

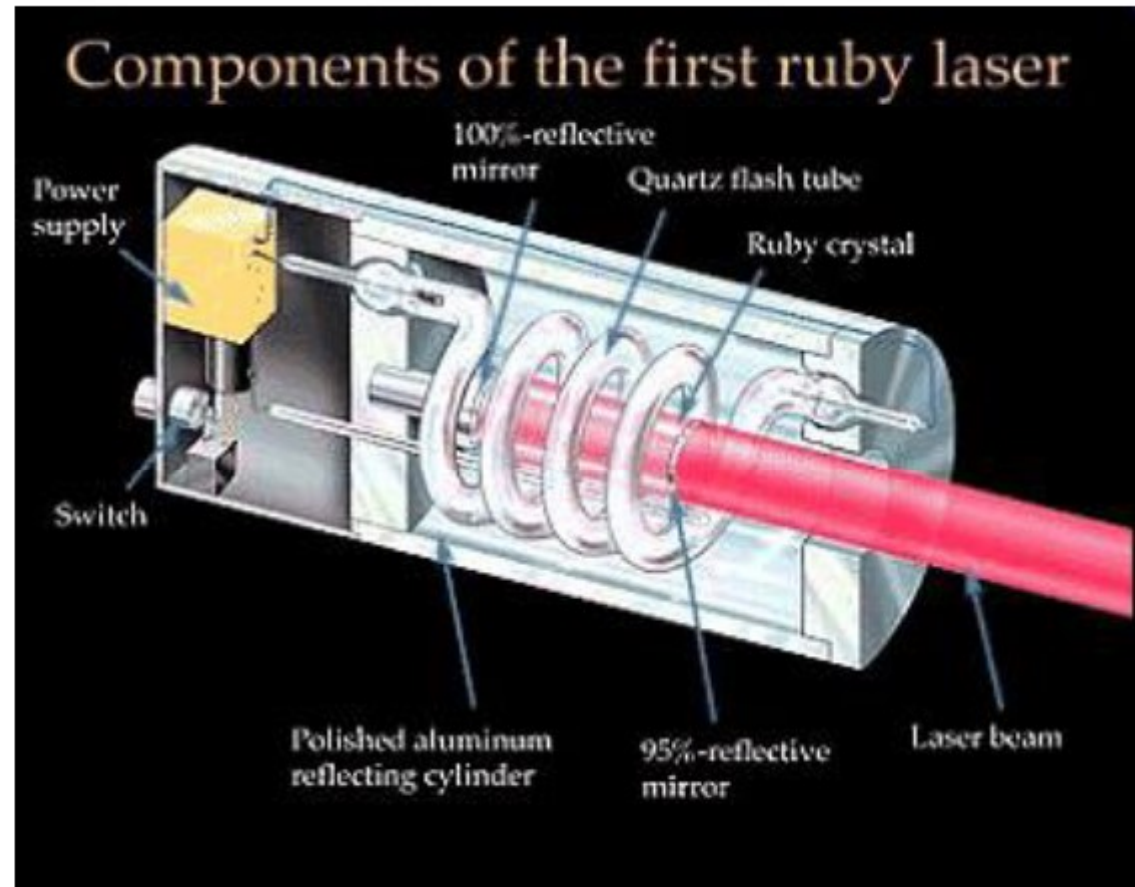上海科技大学
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First Laser in 1960 (Ruby)

Theodore Harold Maiman

* 11.7.1927, Los Angeles, USA

† 5.5.2007, Vancouver, Canada

@694 nm

Nobel prize in physics in 1964

„…for fundamental work in the field of quantum electronics, which has led to the construction of oscillators and amplifiers based on the maser-laser principle"

Charles Hard Townes
* 28.7.1915, Greenville, USA
† 27.1.2015, Oakland, USA

Nikolay Gennadiyevich Basow
* 14.12.1922, Usman, Russia
† 1.7.2001, Moscow, Russia

Aleksandr Mikhailovich Prokhorov
* 11.7.1916, Atherton, Australia
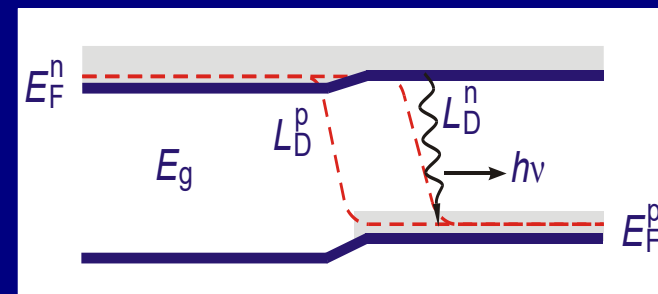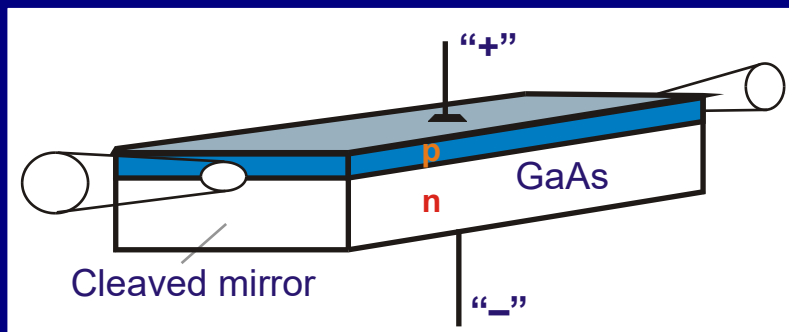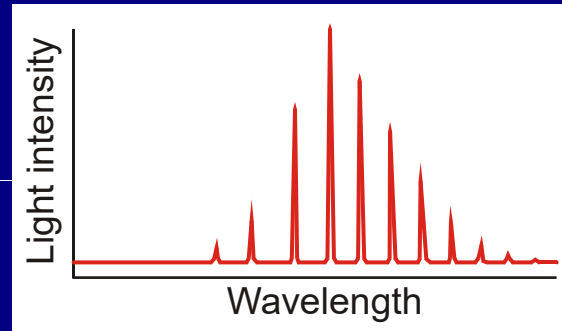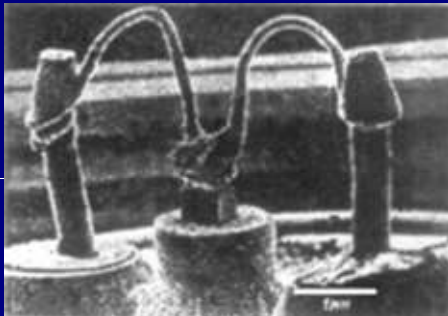† 8.1.2002, Moscow, Russia

Theoreticl work: MASER
principle -> LASER

Concept of optical pumping

Lasers and LEDs on p–n junctions

- **January 1962:** observations of superlumenscences in GaAs p-n junctions (Ioffe Institute, USSR).

- **Sept.-Dec. 1962:** laser action in GaAs and GaAsP p-n junctions (General Electric , IBM (USA); Lebedev Institute (USSR).

Condition of optical gain:

$$E^n_F - E^p_F > E_g$$

Ref: Z. Alferov, *Semiconductor Revolution in the 20th Century,* St Petersburg Academic University
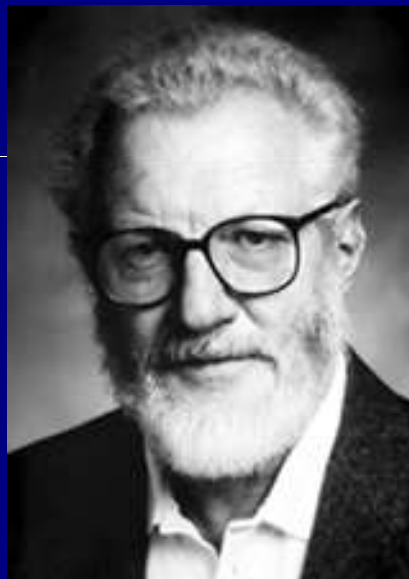
The Nobel Prize in Physics 2000
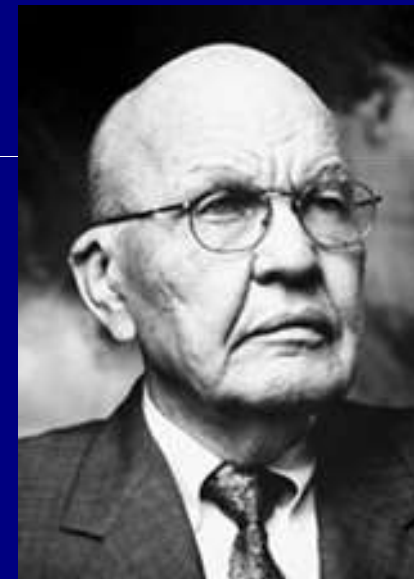
"for basic work on information and communication technology"

"for developing semiconductor heterostructures used in high-speed- and opto-electronics"

"for his part in the invention of the integrated circuit"

Zhores I.
Alferov
b. 1930
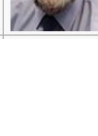
Herbert
Kroemer
b. 1928

Jack S.
Kilby
1923–2005

17

Laser-related Nobel prizes in Physics

1964	Nicolay Gennadiyevich Basov	Soviet Union	"for fundamental work in the field of quantum electronics, which has led to the construction of oscillators and amplifiers based on the maser–laser principle"[64]
	Alexander Prokhorov	Soviet Union	
	Charles Hard Townes	United States	
1981	Nicolaas Bloembergen	Netherlands United States	"for their contribution to the development of laser spectroscopy"[81]
	Arthur Leonard Schawlow	United States	
	Kai Manne Börje Siegbahn	Sweden	"for his contribution to the development of high-resolution electron spectroscopy"[81]
1997	Steven Chu	United States	"for development of methods to cool and trap atoms with laser light."[97]
	Claude Cohen-Tannoudji	France	
	William Daniel Phillips	United States	

Laser-related Nobel prizes in Physics

Year		Name	Country	Citation
2000		Zhores Ivanovich Alferov	Russia	"for developing semiconductor heterostructures used in high-speed- and optoelectronics"[100]
		Herbert Kroemer	Germany	
2005		Roy J. Glauber	United States	"for his contribution to the quantum theory of optical coherence"[105]
		John L. Hall	United States	"for their contributions to the development of laser-based precision spectroscopy, including the optical frequency comb technique"[105]
		Theodor W. Hänsch	Germany	
2009		Charles K. Kao	Hong Kong United Kingdom United States	"for groundbreaking achievements concerning the transmission of light in fibers for optical communication"[109]
		Willard S. Boyle	Canada United States	"for the invention of an imaging semiconductor circuit – the CCD sensor"[109]
		George E. Smith	United States	

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Laser-related Nobel prizes in Physics

2014		Isamu Akasaki	Japan	
		Hiroshi Amano	Japan	"for the invention of efficient blue light-emitting diodes which has enabled bright and energy-saving white light sources"[114]
		Shuji Nakamura	Japan United States	

Charles H. Townes, *How the Laser Happened*, Oxford, 1999

International Year of Light

In proclaiming an International Year focusing on the topic of light science and its applications, the UN has recognized the importance of raising global awareness about how light-based technologies promote sustainable development and provide solutions to global challenges in energy, education, agriculture and health. Light plays a vital role in our daily lives and is an imperative cross-cutting discipline of science in the 21st century. It has revolutionized medicine, opened up international communication via the Internet, and continues to be central to linking cultural, economic and political aspects of the global society.

❑ Laser history

❑ Laser concept

❑ Laser properties

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Interaction of light and material

When light passes through materials it is usually absorbed.
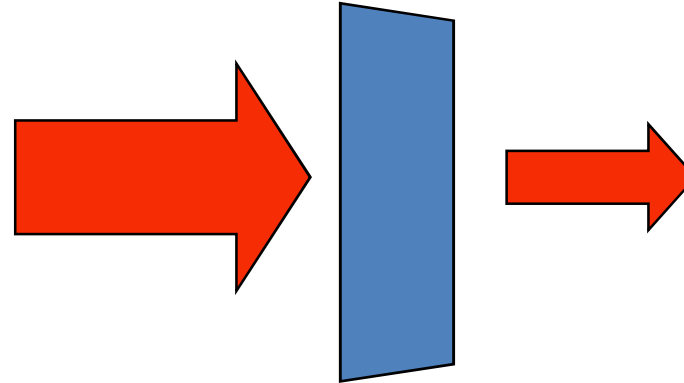
In certain circumstances light may be amplified.
This was called "gain"
(negative absorption)
It is the basis of laser action
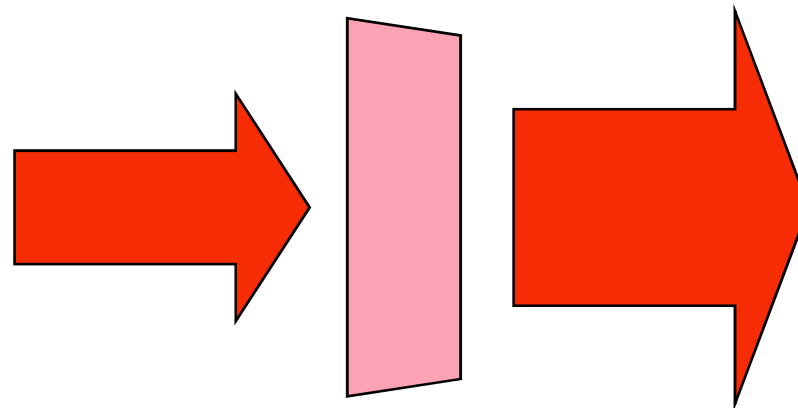
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Photon absorption process

Consider two energy levels of some atom (or molecule) of a material. Assume the atom is initially in level 1.

Absorption

hv_0

E_2

E_1

$$hv_0 = E_2 - E_1$$
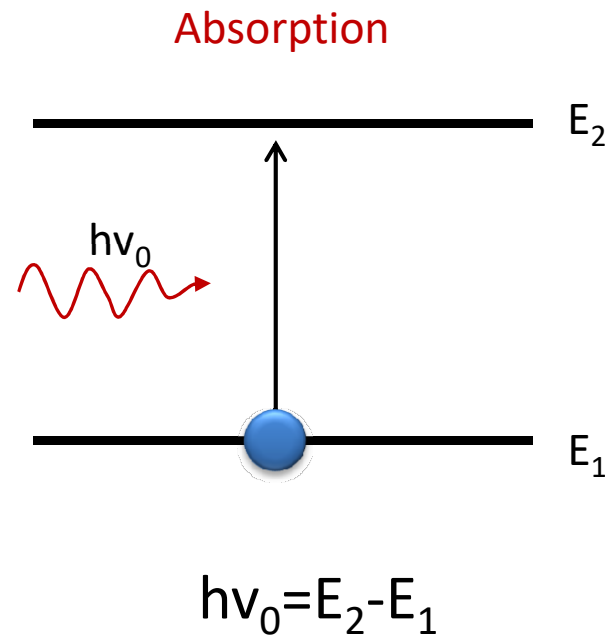
The photon energy is the same as the energy difference between the two energy levels.

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Spontaneous emission process

Assume the atom is initially in level 2.

Spontaneous emission

E_2

$h\nu_0$

E_1

$h\nu_0 = E_2 - E_1$
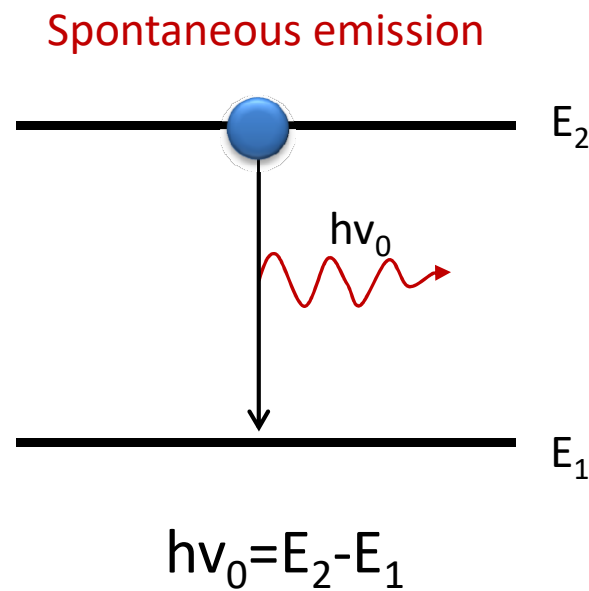
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Stimulated emission process

Assume the atom is initially in level 2.

Stimulated emission

E_2

$h\nu_0$

E_1

$$h\nu_0 = E_2 - E_1$$
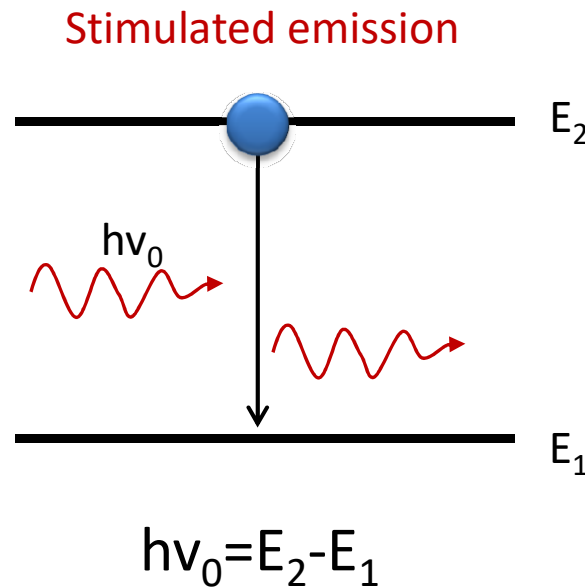
The emitted photon is <u>exactly the same</u> as the incident photon (in amplitude, frequency, phase, polarization, direction)

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Nonradiative transition

Assume the atom is initially in level 2.

Nonradiative transition

E_2

E_1

$$E_{non} = E_2 - E_1$$
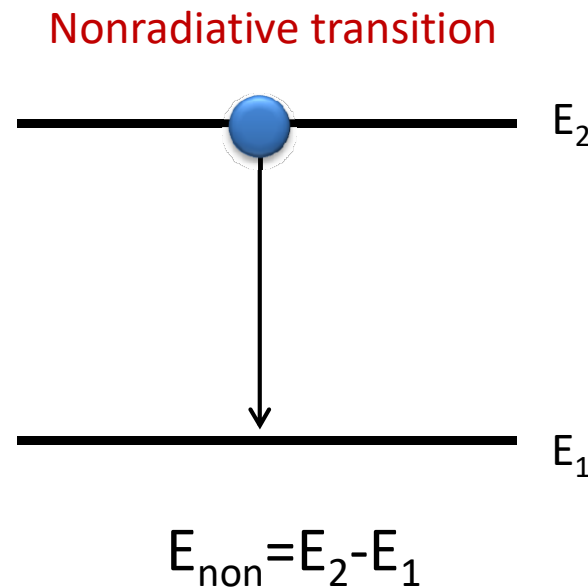
The lost energy is delivered in some form of energy other than e.m. radiation, such as kinetic energy, thermal energy, or phonon, etc.

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Probability of emission and absorption

Let N be the atom number per unit volume, at time t, are lying in a given energy level, N is called the population of the level.
The probability of the emission and absorption processes are described by a differential equation:

Spontaneous emission

$$\left(\frac{dN_2}{dt}\right)_{sp} = -AN_2$$

$$A = \frac{1}{\tau_{sp}}$$

A is spontaneous emission rate (unit /s)
τ_{sp} is spontaneous emission lifetime (unit s)
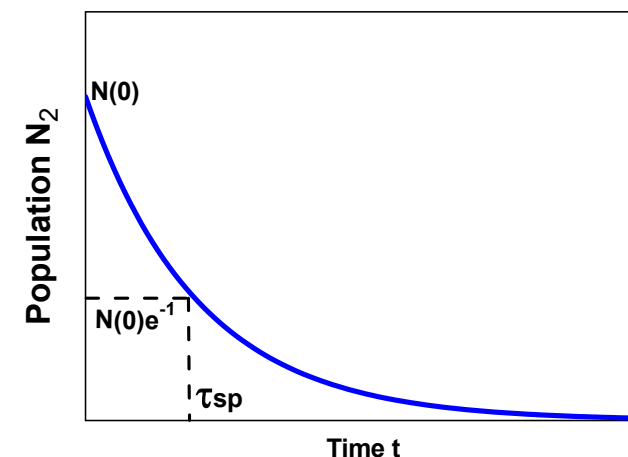
Non-radiative transition (no photon)

$$\left(\frac{dN_2}{dt}\right)_{nr} = -\frac{N_2}{\tau_{nr}}$$

τ_{nr} is non-radiative transition lifetime

$$\left[N_2(t)\right]_{sp} = N_2(0)e^{-t/\tau_{sp}}$$

N(0)

Population N_2

N(0)e^{-1}

τsp

Time t

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Probability of emission and absorption

Stimulated emission

$$\left(\frac{dN_2}{dt}\right)_{st} = -W_{21}N_2$$

$$W_{21} = \sigma_{21}F_s$$

$$F_s = I / h\nu$$

W_{21} is the stimulated emission rate (unit /s)

σ_{21} is the stimulated emission cross section (unit cm²)

F_s is the photon flux (unit s⁻¹cm⁻²)

I is the light intensity.

Absorption

$$\left(\frac{dN_1}{dt}\right)_a = -W_{12}N_1$$

$$W_{12} = \sigma_{12}F_s$$

W_{12} is the absorption rate
σ_{12} is the absorption cross section (unit cm²)

Rates relation $\quad g_2 W_{21} = g_1 W_{12}$

Cross section relation $\quad g_2 \sigma_{21} = g_1 \sigma_{12}$

❑ Degeneracy $g_{2,1}$: Particles of the same energy can be in different quantum state, that is, there can be several different quantum states for a certain energy level. If the number of quantum state is g, we say the degeneracy of the energy level is g. Usually, a higher energy level has a higher degeneracy.
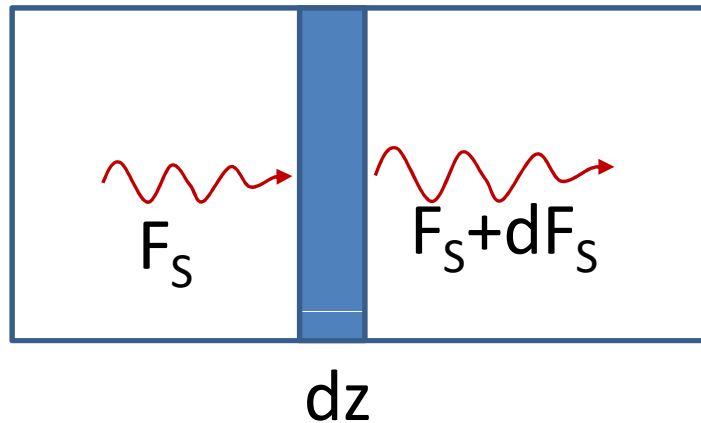
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Assume the light passes through a distance of dz in a given material per unit time, the light will experience both the absorption and the stimulated emission processes (others neglected), the photon difference is then,

$$dF_S = \left(W_{21} N_2 - W_{12} N_1 \right) dz$$

$$= \sigma_{21} \left(N_2 - \frac{g_2}{g_1} N_1 \right) F_S dz$$

$$\Rightarrow \frac{dF_S}{dt} = v_g \sigma_{21} \left(N_2 - \frac{g_2}{g_1} N_1 \right) F_S$$
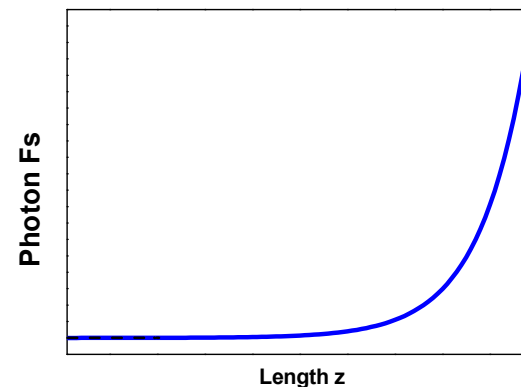
F_S $F_S + dF_S$

dz

❑ Gain coefficient g: Relative photon flux (light intensity) change per unit length

$$g = \frac{1}{F_s} \frac{dF_s}{dz} = \sigma_{21} \left(N_2 - \frac{g_2}{g_1} N_1 \right)$$

$$F_S(z) = F_S(0) e^{gz}$$

If g>0, the material is called amplifier.
If g<0, the material is called absorber.

Photon Fs

Length z

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The first question: can the material has a positive gain naturally?

❑ Thermal equilibrium: A system is said to be in thermal equilibrium if the temperature within the system is spatially and temporally uniform (constant), where the motion of atoms reach a steady state, and the atom fluctuations are, on average, invariant to time.

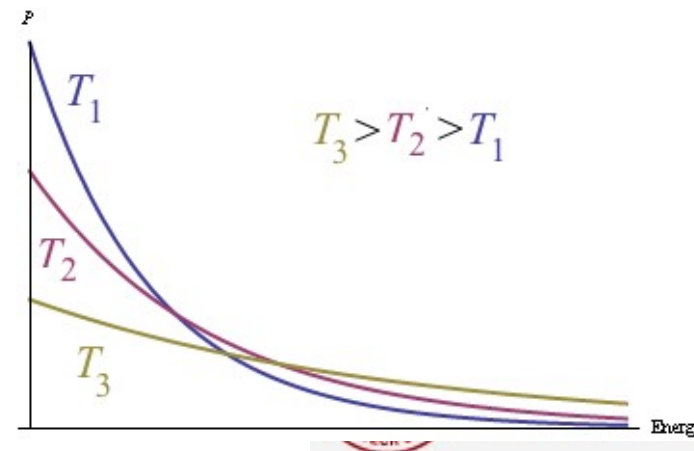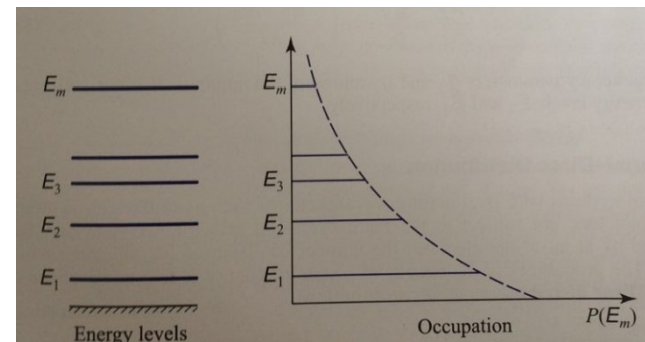❑ In thermal equilibrium, the populations in energy levels is determined by Boltzmann statistics.

$$P(E_m) = \frac{N_m}{N_T} \propto \exp(-E_m / kT)$$

$$\Rightarrow \frac{N_2}{N_1} = \frac{g_2}{g_1} \exp\left(-\frac{E_2 - E_1}{kT}\right)$$

kT = 25.7 meV @298 K

❑ Under thermal equilibrium, the material gain is always negative, as an absorber

$$N_2 < \frac{g_2}{g_1} N_1 \Rightarrow g < 0$$

$T_3 > T_2 > T_1$

The laser idea

The second question: how to have a positive gain?

❑ In the non-equilibrium condition, it is possible to achieve a positive gain, then the material will act as an amplifier.

❑ In this case, $N_2 > g_2 N_1 / g_1$, it is said there is population inversion in the material.

❑ The material with population inversion is called active material/medium.

❑ The population inversion can be achieved by the pumping process, either using electrical pumping scheme or the optical pumping scheme.

Pump
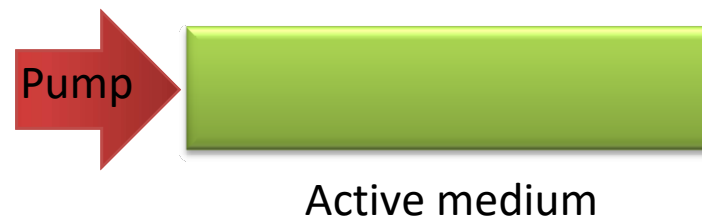
Active medium

The laser idea

The third question: how to make a laser oscillator?

❑ In order to achieve a laser oscillator from an amplifier, a suitable positive feedback is required. This feedback is usually achieved by placing the active medium between two highly parallel reflecting mirrors

Length L

Pump

Active medium

Mirror R1
R=100%
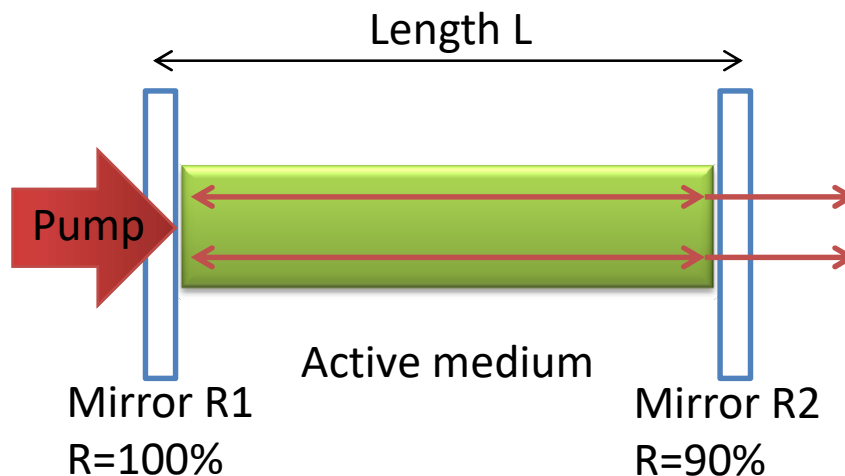
Mirror R2
R=90%

Laser components:

✓ Active medium

✓ Resonant cavity (mirrors)

✓ Pump source

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The way to laser

The fourth question: how to get laser emission from a laser oscillator?

❑ To achieve laser emission, the gain g of the medium has to overcome the loss α in the cavity (coupling, scattering loss, etc.)

❑ The critical pumping condition for g reaches $g = \alpha$ is called laser threshold.

❑ The critical population inversion condition is called critical inversion.

Once the critical inversion is achieved, oscillation will build up from spontaneous emission. The photons that are spontaneously emitted along the cavity axis will, in fact, initiate the amplification process.

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Two-level system

For two-level system, with $g_1=g_2$, that is, $W_{21}=W_{12}$
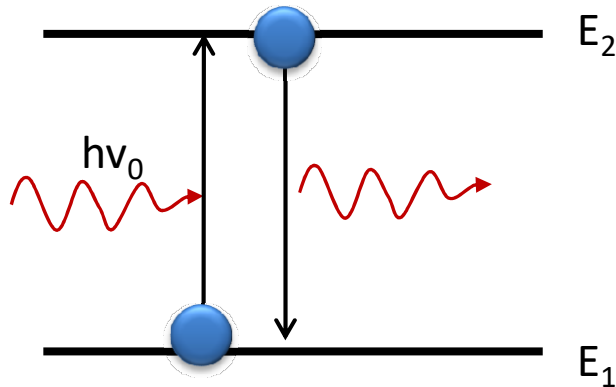
$$\left(\frac{dN_2}{dt}\right)_{st} = -W_{21}N_2$$

$$\left(\frac{dN_1}{dt}\right)_{a} = -W_{12}N_1$$

E₂, E₁, hv₀ labels on diagram

❑ Under equilibrium, $N_1>N_2$

❑ Under pumping of intense lamp at frequency $v=v_0$, transition 1-->2 is more than transition 2-->1

❑ When the condition $N_1=N_2$ can be reached, the absorption and stimulated emission processes will compensate on another, and the material will become transparent. This is referred to as two-level saturation.

❑ Therefore, it is impossible to achieve population inverion in a two-level system.

For three-level laser, population inversion is achieved at level 2 and level 1.
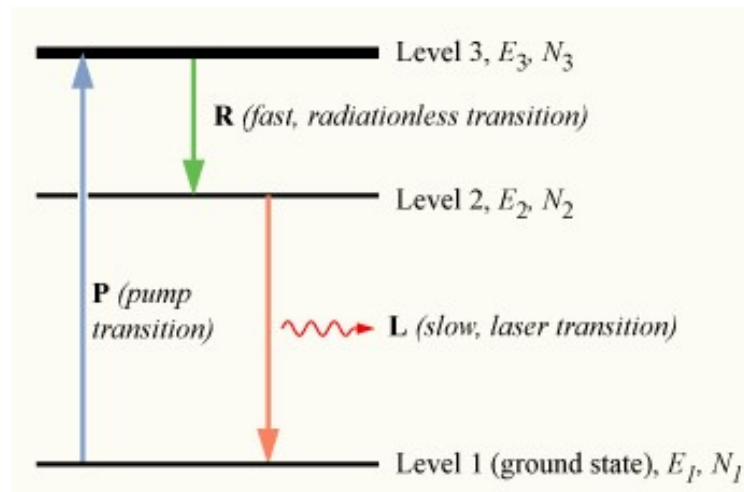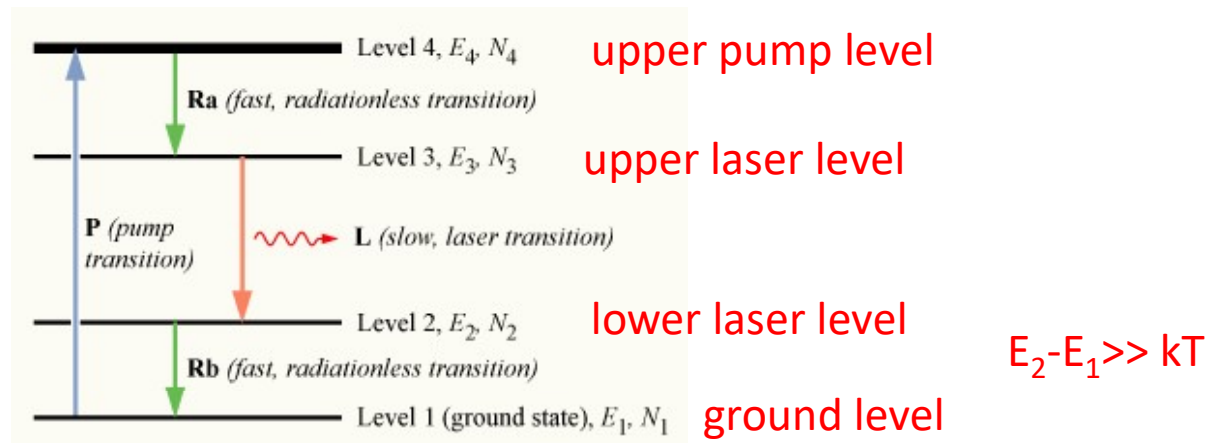

$E_2 - E_1 \gg kT$

❑ Because the energy difference between levels are >>kT, thus most all carriers are initially (at equilibrium) in the ground level (level 1).

❑ Assume the total carrier number is N_t, the transparency is achieved when $N_2 = N_1 = N_t/2$, that is, to realize population inversion, at least half of the total carriers in the ground state must be pumped up.

Four-level laser

For four-level laser, population inversion is achieved at level 3 and level 2.

Level 4, E_4, N_4 upper pump level

Ra *(fast, radiationless transition)*

Level 3, E_3, N_3 upper laser level

P *(pump transition)*

L *(slow, laser transition)*

Level 2, E_2, N_2 lower laser level

$E_2 - E_1 >> kT$

Rb *(fast, radiationless transition)*

Level 1 (ground state), E_1, N_1 ground level

❑ Since level 2 is almost empty, any atom raised to level 3 will contribute to the population inversion.

❑ The "four-level laser" is used for any laser in which the lower laser level is essentially empty, by virtue of being above the ground level by many kT.

❑ Note that materials have many energy levels, "three" or "four" only refers to those related to the laser emission.

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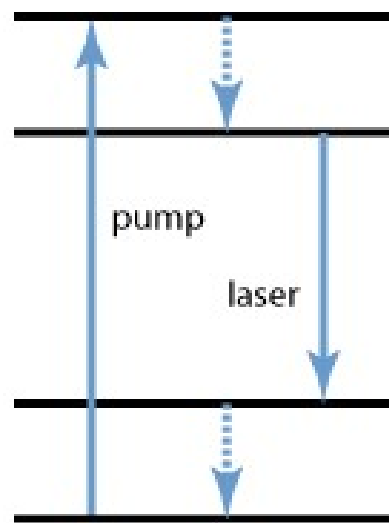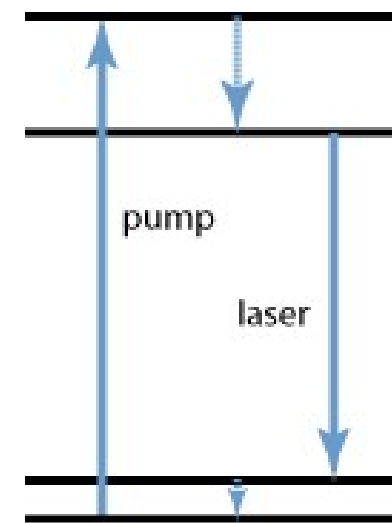# Quasi-three-level laser

For quasi-three-level laser, the ground state consists of many sublevels, the lower laser level being one of these sublevels.
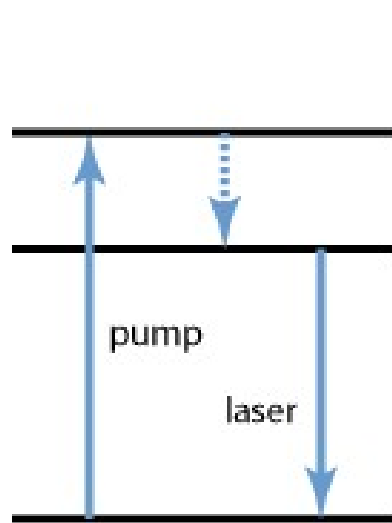
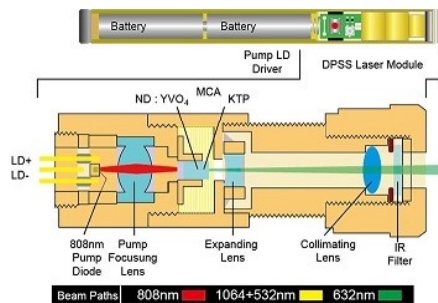three-level laser   four-level laser   quasi-three-level laser
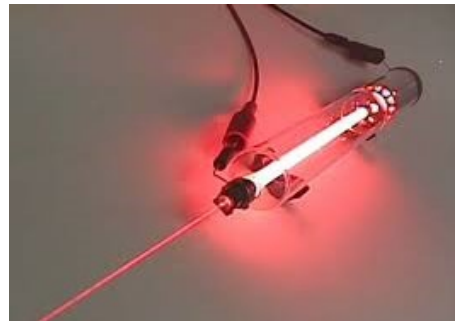
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Pumping schemes

❑ **Pumping** is the process by which atoms are raised from level 1 to level 3 (in three-level laser) or from level 1 to level 4 (in four level laser).
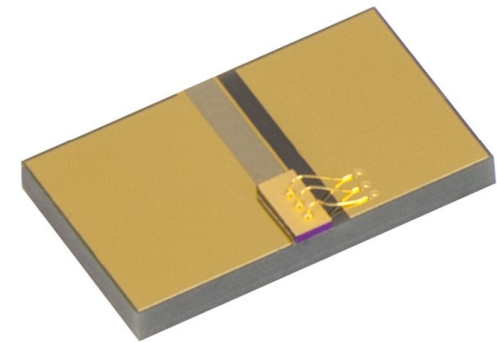
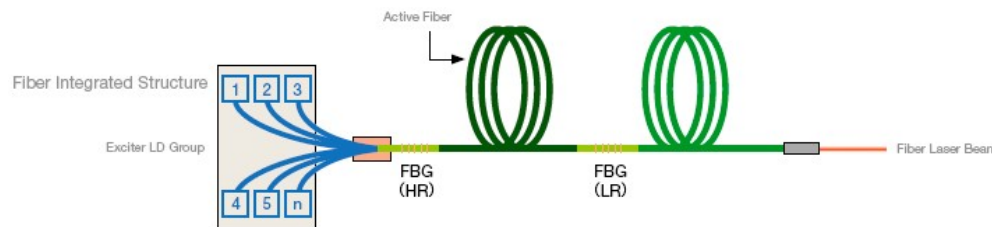❑ Pumping can use high intensity lamp, diode lasers, or electrical discharge in the active medium.

Diode pumped solid-state laser

Gas laser

Semiconductor laser

Fiber laser

Assume the carrier decay from upper pump level to the upper laser level is very fast, that is, the upper pump level is almost empty. Then, the carrier population in the upper laser level due to the pump is given by

$$\left(\frac{dN_{upl}}{dt}\right)_p = W_p N_g$$

W_p is a rate for pump, N_g is the population in the ground level.

For four-level and quasi-three-level lasers, the depletion of the ground level due to the pumping process, can be neglected, that is, N_g~ constant. Then,
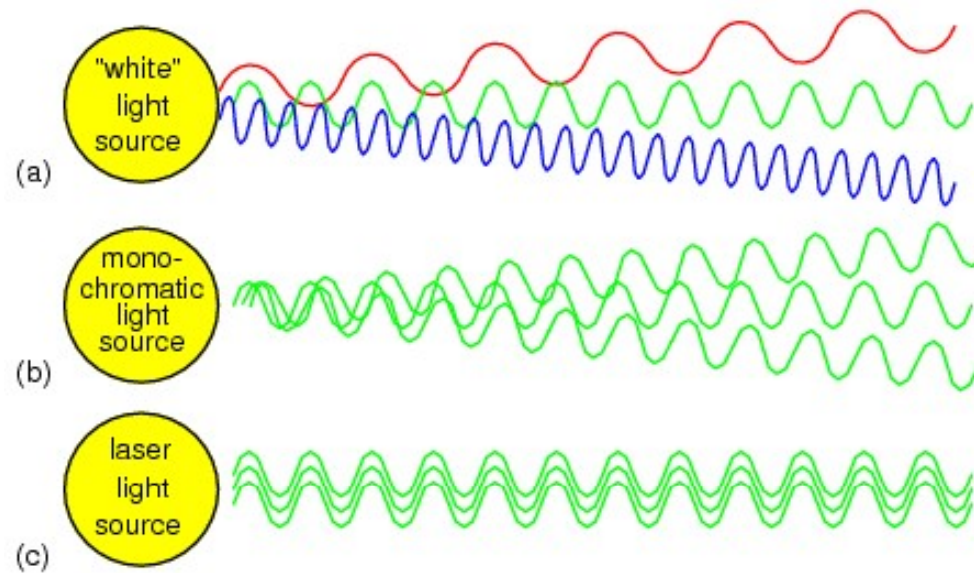
$$\left(\frac{dN_{upl}}{dt}\right)_p = R_p$$

R_p is the pump rate per unit volume.

❑ Laser history

❑ Laser concept

❑ Laser properties

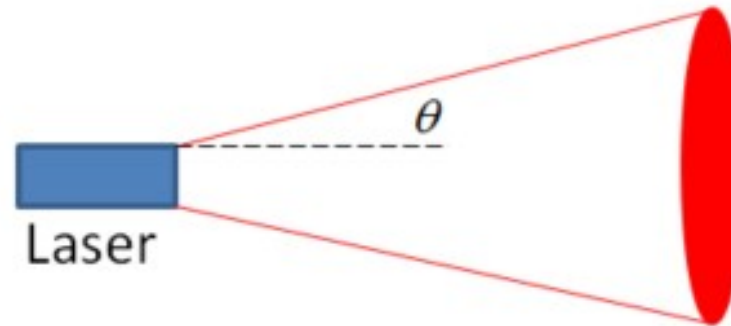# Properties of laser beams

❑ Directionality   ❑ Monochromaticity

❑ Coherence   ❑ Brightness

❑ short time duration (maybe not)

Directionality

Laser

❑ The directionality is described by the beam divergence, which is determined by the diffraction limit, assume the cavity mirror diameter is D,

$$\theta_d \approx \frac{\lambda}{D}$$

For a He-Ne laser at 0.63 μm, and a mirror of D=3 mm, θ_d=2e-4 rad=0.02°

❑ The directionality is related to the cavity type, cavity length, laser medium etc.

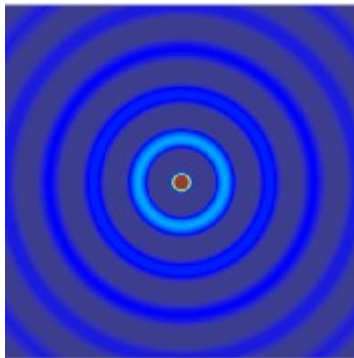❑ Gas laser is better than solid state laser, better than laser diode

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Note: Wave optics

✓ In the framework of wave optics, **Wavefront** is the collection of points characterized by propagation of position of the same phase: a propagation of a line in 1d, a curve in 2d or a surface for a wave in 3d.
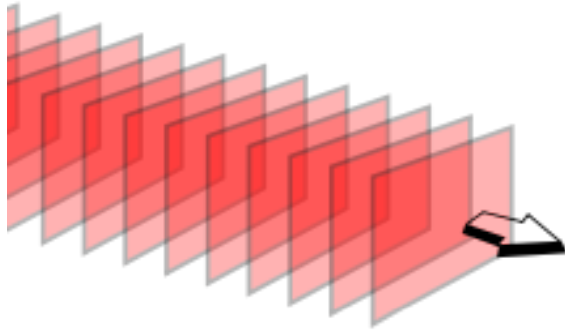
The wavefronts of a plane wave are planes.

The wavefronts of a spherical wave are circles.
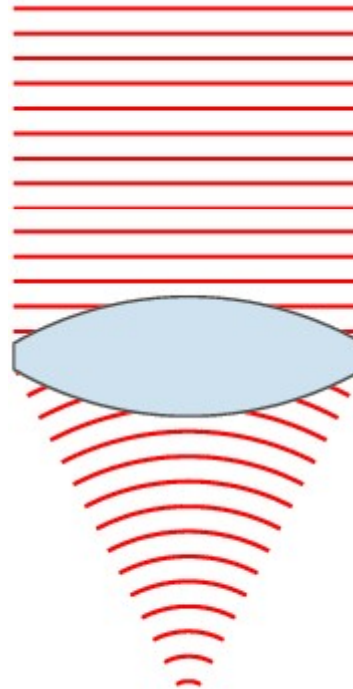
A lens can be used to change the shape of wavefronts. Here, plane wavefronts become spherical after going through the lens.

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Note: Huygens' principle

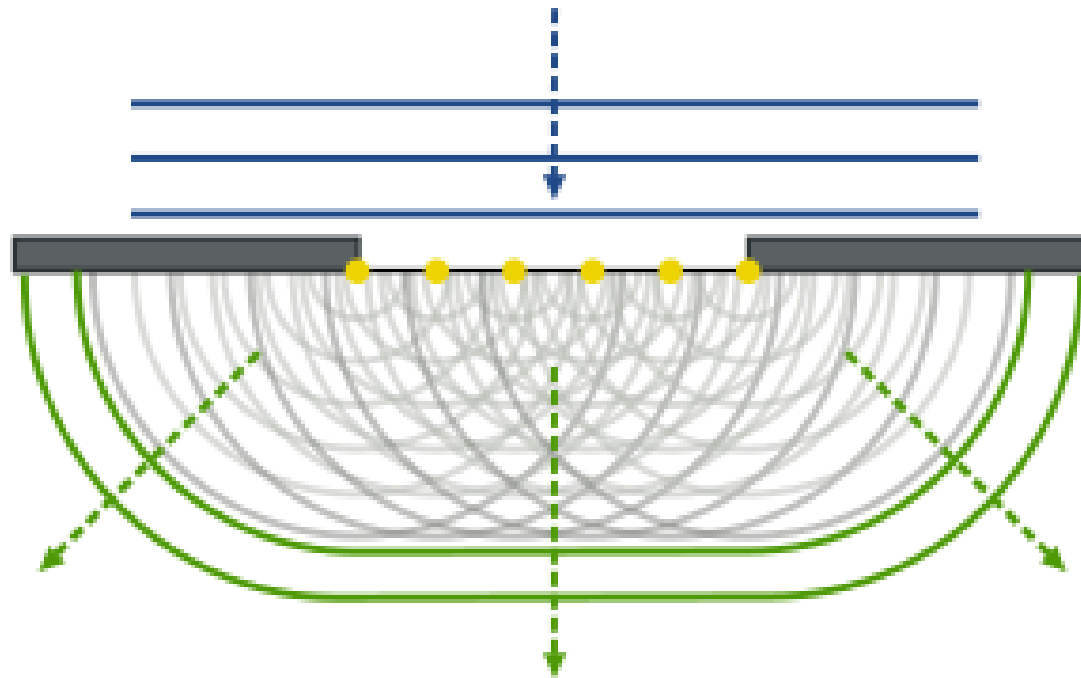❑ Huygens principle: Each point at the wavefront becomes a source for the secondary spherical wave. At any subsequent time, the wavefront can be determined by the sum of these secondary waves.
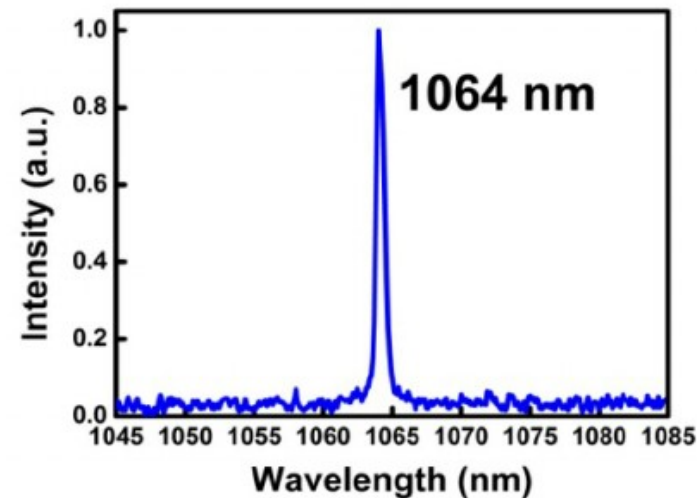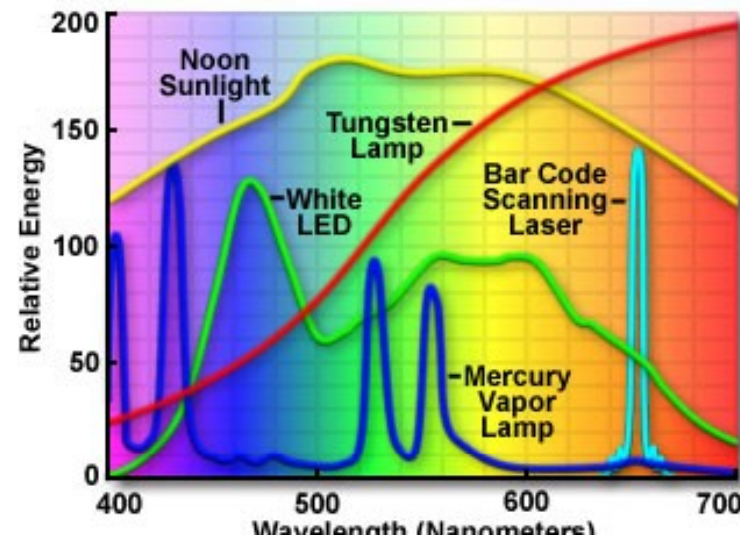
Monochromaticity

Nd:YAG laser

❑ The monochromaticity is described by the spectral linewidth.

❑ The spectral purity of lasers come from:

    1. The laser medium only emits light of frequency at $h\nu_0=E_2-E_1$.

    2. The laser cavity only allows light oscillation at the resonance frequencies of this cavity.

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Coherence

❑ Coherence means the phase correlation relation in the spatial

of temporal dimension.

    --- The spatial coherence is linked with the directionality

    --- The temporal coherence is linked with the monochromaticity

❑ The coherence is degraded by the discontinuity of the phase
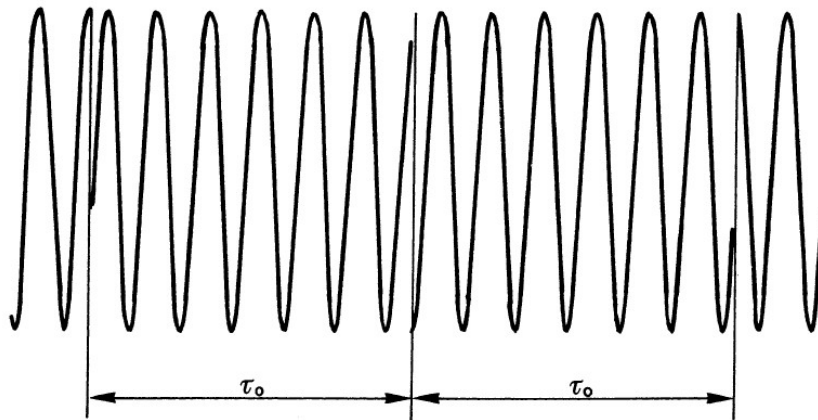
Coherence time vs. spectral linewidth

$$\Delta v \approx 1 / \tau_c$$

Coherence time vs. coherence length

$$L_c = c\tau_c$$

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Brightness of laser

❑ Brightness: The light power per unit surface area of the light source per unit solid angle.

Laser

θ

D
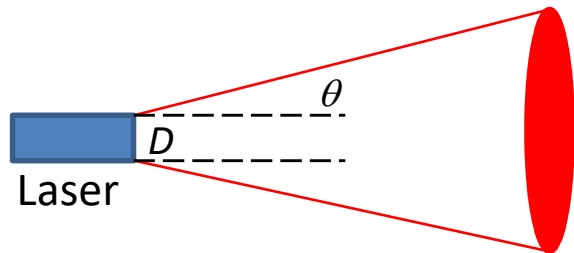
When normal to the emission surface, the solid angle (unit is steradian) is

$$\Omega = \pi \sin^2 \theta$$

$$B \equiv \frac{P}{S\Omega} \qquad \text{unit: } \mathrm{W}/\left(cm^2 sr\right)$$

❑ The brightness at difraction limit

$$= \frac{P}{\left(\pi R^2\right)\left(\pi \sin^2 \theta\right)}$$

$$\approx \frac{P}{\left(\pi \theta D / 2\right)^2}$$

$$B_d \approx \left(\frac{2}{\pi\lambda}\right)^2 P$$

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Note: Solid angle

Solid Angle

Excerpt from Field Guide to Illumination

The definition of intensity involves the concept of a solid angle. A solid angle is a 3D angular volume that is defined analogously to the definition of a plane angle in two dimensions.

A plane angle, θ, made up of the lines from two points meeting at a vertex, is defined by the arc length of a circle subtended by the lines and by the radius of that circle, as shown below. The dimensionless unit of plane angle is the radian, with 2π radians in a full circle.

$\theta = l/r$
(radians)

2π radians in a full circle
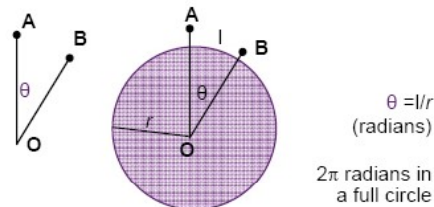
A solid angle, ω, made up of all the lines from a closed curve meeting at a vertex, is defined by the surface area of a sphere subtended by the lines and by the radius of that sphere, as shown below. The dimensionless unit of solid angle is the steradian, with 4π steradians in a full sphere.

Closed curve

area, a, on surface of sphere

$\omega = a/r^2$ (steradians)

4π steradians in a full sphere
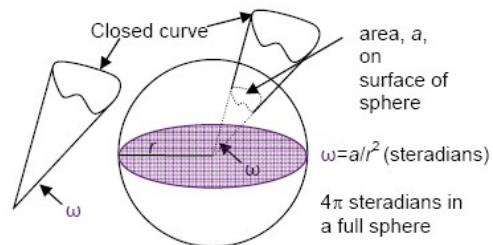
Solid Angle and Projected Solid Angle

Excerpt from Field Guide to Illumination

The relationship between solid angle and projected solid angle can be confusing. Projected solid angle has meaning primarily for a small Lambertian source, which has intensity that varies as the cosine of the angle with the surface normal. The projected solid angle, Ω, is the solid angle, ω, weighted by the cosine of the angle with the surface normal.

$\Omega = \omega \cos \varphi$
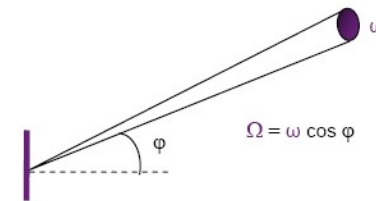
When the solid angle is large enough so that the angle with the surface normal is not the same over the entire solid angle, the total projected solid angle must be computed by integrating the incremental projected solid angles.

For some special cases, the integration results in simple expressions, such as for a large circular cone that is normal to a surface and subtends a half angle, θ.

$\Omega = \pi \sin^2 \theta$
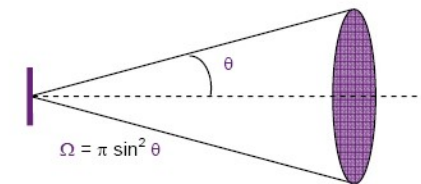
A hemisphere has 2π steradians (solid angle) but π projected steradians (projected solid angle).

Ref: A. V. Arecchin, T. Messadi; R. J. Koshe, *Field Guide to Illuminatin*, SPIE, 2007

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❑ Q-switch technique produces short pulses width duration

of ~ ns

❑ Mode locking technique produces ultrashot pulses with

duration of fs to ps

❑ Not all the lasers can be used to produce short pulses

Types of lasers

❑ Physical state: solid state, liquid, and gas lasers. (free electron lasers)

❑ Wavelength: infrared, visible, UV, and X-ray lasers. (1 mm---1 nm)

❑ Power: CW laser from nW to a few MW; Pulsed laser peak power up to PW (10^{15} W)

❑ Pulse duration: from ms down to fs (10^{-15} s)

❑ Cavity length: from nm up to km

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Homework

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