Principles of Lasers

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Chapter 2

Interaction of photons and atoms

- Line broadening
  - ✓ Homogeneous broadening
  - ✓ Inhomogeneous broadening

Line broadening

- In the previous discusssions, the photon emission and absorption only occur at the frequency $v=v_0$.

$$W_{21} \equiv W_{12} = \frac{2\pi^2}{3n\varepsilon_0 c_0 h^2} \left|\mu_{21}\right|^2 I\delta(v - v_0)$$

$$v = v_0 \Rightarrow W = \infty$$

$$v \neq v_0 \Rightarrow W = 0$$

- The reason for this unphysical result is due to the assumption that the interaction with the em wave with the atom could continue undisturbed for an infinite time. Actually, a number of perturbation sources prevent this kind interaction, such as atom collisions or photon collisions.
- Consequently, atoms with energy difference hv_0 do not only emit or absorb photons at $v=v_0$, but also those around $v \sim v_0$. This phenonmenon is called (spectral) line broadening

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Line broadening

- In the case of spontaneous emission, assume the total emission power is P, the power within [v, v+dv] is P(v)dv, then

$$P = \int_{-\infty}^{+\infty} P(v) dv$$

- Define the lineshape function as

$$g(v - v_0) = P(v) / P \Rightarrow$$

$$\int_{-\infty}^{+\infty} g(v - v_0) dv = 1$$

- The FWHM linewith Δv of the lineshape is defined as

$$g\left(\pm \frac{\Delta v}{2}\right) = \frac{g(0)}{2}$$

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Homogeneous broadening

❑ Homogeneous broadening : Every atom has the same resonance frequency v_0 and the same broadening lineshape. This lineshape follows the Lorentzian function:

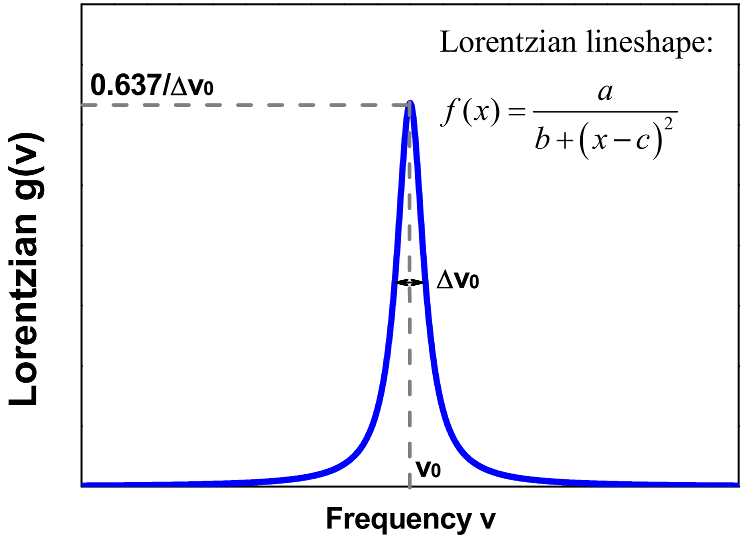
$$\boxed{g(v-v_0)=\frac{2}{\pi\Delta v_0}\frac{1}{1+\left[2(v-v_0)/\Delta v_0\right]^2}}$$

$$Note: \int g(v-v_0)dv = 1$$

Δv_0 is the broadening linewidth (FWHM)

❑ The maximum of g(v-v_0) occurs at v=v_0, and its value is given by

$$\boxed{g(0)=\frac{2}{\pi\Delta v_0}=\frac{0.637}{\Delta v_0}}$$



Rates of absorption/stimulated emission

- Due to the homogeneous broadening, the absorption and stimulated emission rates are corrected to

$$
\boxed{
\begin{aligned}
W_{21} &= \mathrm{W}_{12} \\
&= \frac{\pi^2}{3h^2}\left|\mu_{21}\right|^2 E_0^2\, g(\nu-\nu_0) \\
&= \frac{2\pi^2}{3n^2\varepsilon_0 h^2}\left|\mu\right|^2 \rho\, g(\nu-\nu_0) \\
&= \frac{2\pi^2}{3n\varepsilon_0 c_0 h^2}\left|\mu\right|^2 I\, g(\nu-\nu_0)
\end{aligned}
}
$$

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Homogeneous broadening

- The transition cross section is the transition rate per photon flux

$$\boxed{\sigma = \frac{W}{F_s} = h\nu\frac{W}{I}}$$

- The transition cross section of homogeneous broadening medium is

$$\boxed{\sigma_h(\nu-\nu_0) = \frac{2\pi^2}{3n\varepsilon_0 c_0 h}|\mu|^2 \nu g(\nu-\nu_0)}$$

$$W_h = \frac{2\pi^2}{3n\varepsilon_0 c_0 h^2}|\mu|^2 I g(\nu-\nu_0)$$

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Homogeneous broadening mechanisms

- Collision broadening: collision with atoms, with phonons (quasi-particle, quantized vibration of lattices, wave-particle duality) → Phase interruption

- Suppose the average time of collisions between two particles is τ_{cl}, then the broadening linewidth is given by:

$$\boxed{\Delta\nu_0^{\ cl} = \frac{1}{\pi\tau_{cl}}}$$

$$\boxed{g_{cl}(\nu-\nu_0) = 2\tau_{cl}\frac{1}{1+\left[2\pi\tau_{cl}(\nu-\nu_0)\right]^2}}$$

$$g(\nu-\nu_0) = \frac{2}{\pi\Delta\nu_0}\frac{1}{1+\left[2(\nu-\nu_0)/\Delta\nu_0\right]^2}$$

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Homogeneous broadening mechanisms

- Natural broadening: spontaneous emission → Phase interruption

- Suppose the sponteneous emission lifetime is τ_{sp}, then the broadening linewidth is given by:

$$\boxed{\Delta v_0{}^{sp} = \frac{1}{2\pi\tau_{sp}}}$$

$$\boxed{g_{sp}(v - v_0) = 4\tau_{sp}\frac{1}{1+\left[4\pi\tau_{sp}(v-v_0)\right]^2}}$$

$$g(v - v_0) = \frac{2}{\pi\Delta v_0}\frac{1}{1+\left[2(v-v_0)/\Delta v_0\right]^2}$$

Note: Examples 2.2---2.4 on page 45

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- Line broadening
  - ✓ Homogeneous broadening
  - ✓ Inhomogeneous broadening

Inhomogeneous broadening

- Inhomogeneous broadening medium: Atoms have different resonance frequencies v_0', and are distributed around a central frequency v_0, and the distribution of the resonance frequency usually follows the Gaussian lineshape:

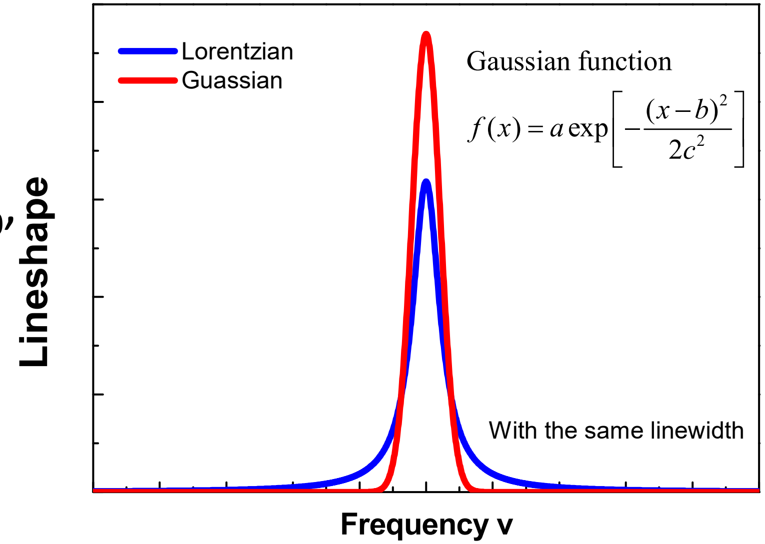
$$g^*\left(v_0' - v_0\right) = \frac{2}{\Delta v_0^*}\left(\frac{\ln 2}{\pi}\right)^{1/2} \exp\left[-4\ln 2 \frac{(v_0' - v_0)^2}{\Delta v_0^{*2}}\right]$$

$$Note: \int g^*(v_0' - v_0) dv_0' = 1$$

Δv_0^* is the inhomo. broad. linewidth

- The maximum of $g^*(v_0'-v_0)$ occurs at $v_0'=v_0$, and its value is given by

$$g^*(0) = \frac{2}{\Delta v_0^*}\sqrt{\frac{\ln 2}{\pi}} = \frac{0.939}{\Delta v_0^*}$$



Inhomogeneous broadening

- For an atom with resonance frequency ν_0', the transition rate is

$$W_h\left(\nu-\nu_0'\right)$$

- The existence probability of atoms with frequencies in the range $\nu_0' \sim \nu_0' + d\nu_0'$ is

$$g^*\left(\nu_0'-\nu_0\right)d\nu_0'$$

- The total transition rate for atoms within the whole frequency range

$$W_{in}=\int W_h\left(\nu-\nu_0'\right)g^*\left(\nu_0'-\nu_0\right)d\nu_0'$$

Inhomogeneous broadening

- The inhomogeneous broadening transition cross section is

$$\sigma_{in} = \frac{W_{in}}{F_s} = \int \sigma_h\left(v - v_0^{'}\right) g^*\left(v_0^{'} - v_0\right) dv_0^{'}$$

$$= \left(\frac{2\pi^2}{3n\varepsilon_0 c_0 h}|\mu|^2 v\right)\int\left[g(v - v_0 - x)g^*\left(x\right)\right]dx$$

$$= \frac{2\pi^2}{3n\varepsilon_0 c_0 h}|\mu|^2 v g_t(v - v_0)$$

with

$$g_t(v - v_0) = \int\left[g(v - v_0^{'})g^*\left(v_0^{'} - v_0\right)\right]dv_0^{'}$$

$$\text{Note:}\int g_t(v - v_0)dv = 1$$

Convolution function

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau$$

$$\int_{-\infty}^{\infty} \delta(\tau)g(t-\tau)d\tau = g(t)$$

Inhomogeneous broadening

- If $\Delta\nu_0 << \Delta\nu_0^*$, we have $g(\nu-\nu_0^{'}) \approx \delta(\nu-\nu_0^{'})$, then

$$\boxed{\begin{aligned} g_{in}(\nu-\nu_0) &= g^*(\nu-\nu_0) \\ &= \frac{2}{\Delta\nu_0^*}\left(\frac{\ln 2}{\pi}\right)^{1/2}\exp\left[-4\ln 2\frac{(\nu-\nu_0)^2}{\Delta\nu_0^{*2}}\right] \end{aligned}}$$

This is pure inhomogeneous broadening

- For the pure inhomogeneous broadening, the transition cross section is

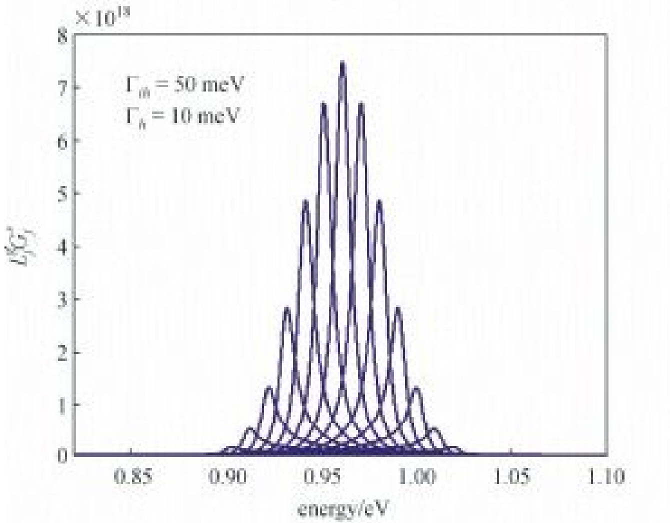
$$\boxed{\sigma_{in} = \frac{2\pi^2}{3n\varepsilon_0 c_0 h}|\mu|^2 \nu g^*(\nu-\nu_0)}$$

Transition cross section (homo & Inhomo)

❑ For medium with both homo & inhomo broadenings, the total lineshape and the total cross section are

$$g_t(\nu-\nu_0)=\int\left[g(\nu-\nu_0^{'})g^*\left(\nu_0^{'}-\nu_0\right)\right]d\nu_0^{'}$$

$$\sigma_t(\nu-\nu_0)=\frac{2\pi^2}{3n\varepsilon_0 c_0 h}|\mu|^2\,\nu g_t(\nu-\nu_0)$$



Ref: Taleb, Frontiers of Optoelectronics 5, 445 (2012)

❑ For medium with purely homo broadening:

$$g_t(\nu-\nu_0)=g(\nu-\nu_0)$$

$$\sigma_h(\nu-\nu_0)=\frac{2\pi^2}{3n\varepsilon_0 c_0 h}|\mu|^2\,\nu g(\nu-\nu_0)$$

❑ For medium with purely inhomo broadening:

$$g_t(\nu-\nu_0)=g^*(\nu-\nu_0)$$

$$\sigma_{in}(\nu-\nu_0)=\frac{2\pi^2}{3n\varepsilon_0 c_0 h}|\mu|^2\,\nu g^*(\nu-\nu_0)$$

$g(\nu-\nu_0^{'})$ Lorentzian lineshape

$g^*\left(\nu_0^{'}-\nu_0\right)$ Guassian lineshape

ν Photon frequency

$\nu_0^{'}$ Resonance frequency

ν_0 Central frequency

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Multi-mechanism broadening

- When more than two mechanisms contribute to line broadening, the overall lineshape is given by the convolution of the two lineshape functions:

$$g_t(\nu-\nu_0)=\int\left[g_1(\nu-\nu_0-x)g_2\left(x\right)\right]dx$$

- ✓ Both mechanisms are homo, the overall is homo, and the FWHM is

$$\boxed{\Delta\nu_h=\Delta\nu_{h1}+\Delta\nu_{h2}}$$

- ✓ Both mechanisms are inhomo, the overall is inhomo, and the FWHM is

$$\boxed{\Delta\nu_{in}=\sqrt{\Delta\nu_{in1}{}^2+\Delta\nu_{in2}{}^2}}$$

- ✓ One mechanism is homo, the other is inhomo

$$g_t(\nu-\nu_0)=\int\left[g(\nu-\nu_0{}')g^*\left(\nu_0{}'-\nu_0\right)\right]d\nu_0{}'$$

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Inhomogeneous broadening mechanisms

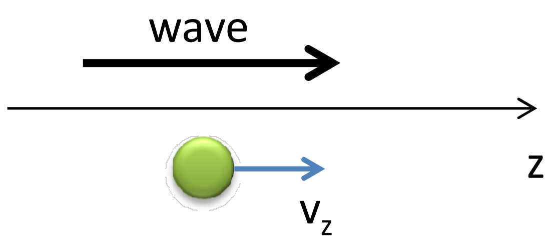
❑ Material inhomogeneity (Local field inhomogeneous broadening): It occurs for ions in ionic crystals or glasses (Nd:glass). Ions will experience a local electric field produced by the surrounding atoms of the material, and due to material inhomogeneities which are particularly significant in glass medium, these field will be difference from ion to ion. These local field variations will produce local variation of the energy levels and thus of the transition frequencies of the ions.

The FWHM of the inhomogeneous broadening Gaussian lineshape depends on the extent of variation of transition frequencies in the material and hence upon the amount of field inhomogeneity within the crystal or glass.

Inhomogeneous broadening mechanisms

❑ Doppler broadening: In gases, atomic motions induce the Doppler inhomogeneous broadening.

Assume the electric field frequency is ν, but seen from the atom with a velocity v_z (positive if it is the same direction as the electric field, and vice versa), the apparent frequency of the electric field is:



$$\nu' = \frac{c - \upsilon_z}{c}\nu$$

When this frequency equals to the atom resonance (transition) frequency ν_0, both can interact with each other

$$\nu = \frac{c}{c - \upsilon_z}\nu_0$$

The interaction is equivalent to atoms without movement with an apparent resonance frequency, equaling to the light frequency:

$$\boxed{\nu_0' = \frac{c}{c - \upsilon_z}\nu_0 \approx \left(1 + \upsilon_z / c\right)\nu_0; \quad \nu = \nu_0'}$$

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Inhomogeneous broadening mechanisms

- The probability of atoms at a velocity of v_z is given by the Maxwell distribution

$$p_{vz} = \sqrt{\frac{M}{2\pi kT}} \exp\left(-\frac{M}{2kT} v_z^2\right)$$

M is mass of atom，T is temperature

- The Doppler broadening lineshape can be derived by

$$g^*\left(v_0^{'} - v_0\right) dv_0^{'} = p_{vz} d_{vz} \Rightarrow$$

$$g^*\left(v_0^{'} - v_0\right) = \frac{c}{v_0}\sqrt{\frac{M}{2\pi kT}} \exp\left[-\frac{Mc^2}{2kT}\frac{(v_0^{'} - v_0)^2}{v_0^2}\right]$$

- Therefore, the Doppler broadening linewidth is

$$\boxed{\Delta v_0^* = 2v_0\sqrt{2\ln 2\frac{kT}{Mc^2}}}$$

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Typical linewidth broadenings

TABLE 2.1. Typical magnitude of frequency broadening for the various line-broadening mechanisms

	Type	Gas	Liquid	Solid
Homogeneous	Natural	1 kHz ÷ 10 MHz	Negligible	Negligible
	Collisions	5 ÷ 10 MHz/Torr	~300 cm^{-1}	–
	Phonons	–	–	~10 cm^{-1}
Inhomogeneous	Doppler	50 MHz ÷ 1 GHz	Negligible	–
	Local field	–	~500 cm^{-1}	1 ÷ 500 cm^{-1}

He-Ne laser (632.8 nm)

Nd: YAG (掺钕钇石榴石, 1064 nm)

Note: Examples 2.5---2.6 on page 48

Chapter 2_L5

- Gain coefficient and gain saturation
  - ✓ Homogeneous broadening medium
  - ✓ Inhomogeneous broadening medium

Gain coefficient

- Atom density in state 1 is N_1, and atom density in state 2 is N_2

  - ✓ If $N_1>N_2$, the absorption coefficient is

    Macroscopy $$\boxed{\alpha = -\frac{1}{I}\frac{dI}{dz} \quad (/\mathrm{cm})}$$

    Microscopy $$\boxed{\alpha = \sigma(N_1 - N_2)}$$

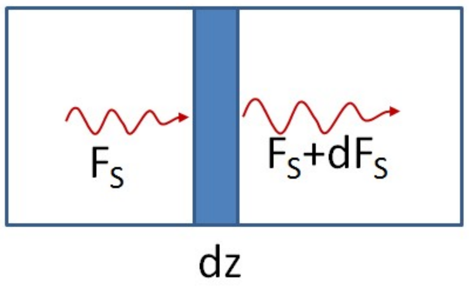
    $$\boxed{I(z) = I(0)e^{-\alpha z}}$$ Absorber

  - ✓ If $N_2>N_1$, the gain coefficient is

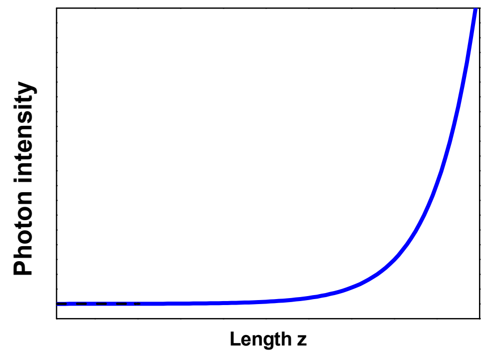
    $$\boxed{g = \frac{1}{I}\frac{dI}{dz} \quad (/\mathrm{cm})}$$

    $$\boxed{g = \sigma(N_2 - N_1)}$$

    $$\boxed{I(z) = I(0)e^{gz}}$$ Amplifier/ Laser



$$\boxed{\alpha = -g}$$



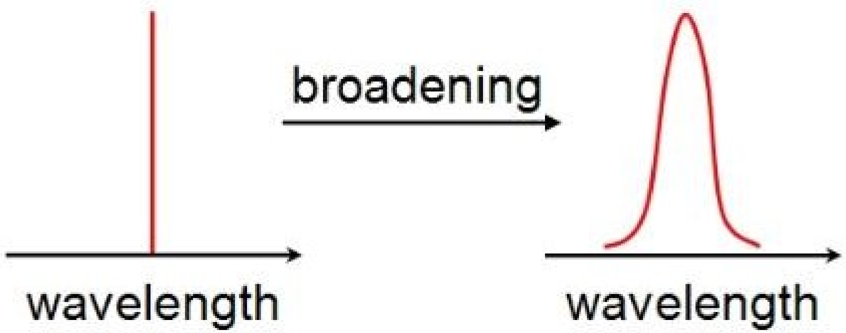
Gain coefficient

- Considering the degeneracy, the gain coefficient

$$g=\sigma_{21}N_2-\sigma_{12}N_1=\sigma_{21}\left(N_2-\frac{g_2}{g_1}N_1\right)=\sigma_{21}\Delta N$$

- Considering the line broadening, the gain coefficient

$$g=\sigma_t(v-v_0)\Delta N=\frac{2\pi^2}{3n\varepsilon_0 c_0 h}|\mu|^2 v\Delta N g_t(v-v_0)$$



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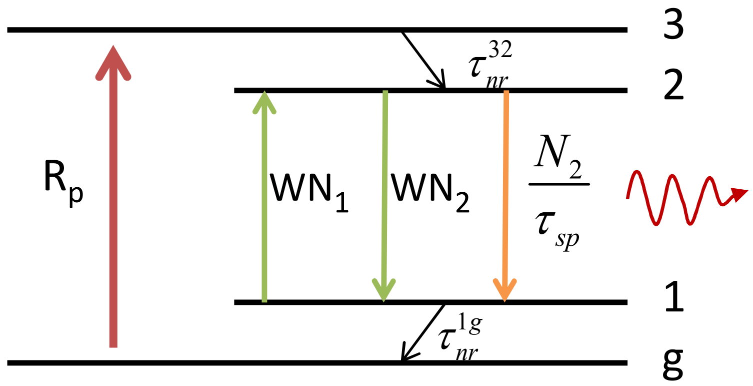
Inverted population saturation

- Consider a four-level laser system, the nonradiative transtion from 3 to 2, and from 1 to g is very fast, such that N_3~0 and N_1~0. The population in level 2 is



$$\boxed{\begin{aligned} &\tau_{nr}^{32},\ \tau_{nr}^{1g} \text{ very short} \Rightarrow N_3 \approx 0,\ N_1 \approx 0 \\ &\Delta N = N_2 - N_1 \approx N_2 \end{aligned}}$$

$$\frac{dN_2}{dt} = \frac{N_3}{\tau_2^3} + WN_1 - WN_2 - \frac{N_2}{\tau_{sp}}$$

$$\approx R_p - WN_2 - N_2/\tau_{sp}$$

- Under steady-state,

$$\frac{dN_2}{dt} = 0 \Rightarrow N_2 = \frac{R_p \tau_{sp}}{1 + W\tau_{sp}} = \frac{R_p \tau_{sp}}{1 + \sigma_{21} I \tau_{sp} / (hv)}$$

Inverted population saturation

- In the absence of light (I=0), we get the population in level 2,

$$N_{20} = R_p \tau_{sp}$$

- With the presence of light with intensity I, the expression of the population in level 2 can be rewritten as

$$N_2 = \frac{N_{20}}{1 + I / I_s} \quad with \quad I_s = \frac{hv}{\sigma_{21}\tau_{sp}}$$

- I_s is defined as the saturation intensity: The physical meaning is that only when the light intensity is comparable to I_s, the stimulated emission induced population reduction can be comparable to that of spontaneous emission (and nonraditive transition).

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Inverted population saturation

- The inverted popuplation is given by, (for four level system)

$$\Delta N = \frac{\Delta N_0}{1 + I / I_s}, \quad (\Delta N \approx N_2)$$

- When the light intensity $I \ll I_s$, the small-signal inverted population is

$$\Delta N \approx \Delta N_0 = R_p \tau_{sp}$$

- When the light intensity I becomes comparable to I_s, the inverted population

$$\Delta N < \Delta N_0$$

- Inverted population saturation: The phenomenon that the inverted population decreases with the light intensity.

- When the light intensity $I = I_s$,

$$\Delta N = \Delta N_0 / 2$$

Gain saturation

- Analog to the inverted population saturation, the gain saturation is given by

$$g = \frac{g_0}{1 + I / I_s}$$

$$g_0 = \sigma \Delta N_0 = \sigma R_p \tau_{sp}$$

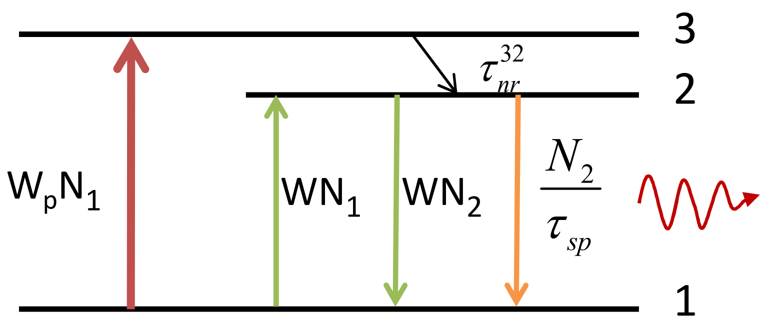
- Small-signal gain: the gain that without light, or with weak light $I \ll I_s$.

- Gain saturation: the phenomenon that the gain decreases with the light intensity.

Homework

- Calculate the small-signal inverted population, and saturation intensity in three-level laser system.

The total atom number is N, the pump rate is W_pN_1, tau_32_nr is very fast that N_3~0.



$$\Delta N = \frac{\Delta N_0}{1 + I / I_s}$$

Note:

$$\Delta N = N_2 - N_1$$

$$N \approx N_2 + N_1$$