Principles of Lasers

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Chapter 2

Interaction of photons and atoms

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Chapter 2_L6

- Gain coefficient and gain saturation
  - Homogeneous broadening medium
  - Inhomogeneous broadening medium

Gain of inhomo broadening medium

- Assume the total small-signal inverted population is ΔN_0, then population and the gain with apparent resonance frequency in $[v_0', v_0'+dv_0']$ is

$$\Delta N_0(v_0')dv_0' = \Delta N_0 g^*(v_0' - v_0)dv_0'$$

$$g_0(v_0')dv_0' = \sigma_{21}^h(v_0' - v_0')\Delta N_0 g^*(v_0' - v_0)dv_0'$$

- Note that this part population has homogeneous broadening, aussume the homo broadening linewidth is Δv_0, this part population contributes to the gain at v_1, with incident light of v_1 and I_{v1},

$$dg = \frac{1}{1 + \left[2(v_1 - v_0')/\Delta v_0'\right]^2 + \frac{I_{v1}}{I_s(v_0')}} g_0(v_0')dv_0'$$

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Gain of inhomo broadening medium

- The total gain of the medium at v_1 is

$$g(v_1, I_{v1}) = \int dg = \int \frac{1}{1 + \left[2(v_1 - v_0')/\Delta v_0'\right]^2 + \dfrac{I_{v1}}{I_s(v_0')}} g_0(v_0')dv_0'$$

- Only populations with v_0' in a small range contribute to the gain at v_1,

$$\left|v_1 - v_0'\right| < \frac{\Delta v_0}{2}$$

- Then, the total gain is approximately

$$g(v_1, I_{v1}) \approx \int_{v_1 - \Delta v_0/2}^{v_1 + \Delta v_0/2} \frac{1}{1 + \left[2(v_1 - v_0')/\Delta v_0'\right]^2 + \dfrac{I_{v1}}{I_s(v_0')}} g_0(v_0')dv_0'$$

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Gain of inhomo broadening medium

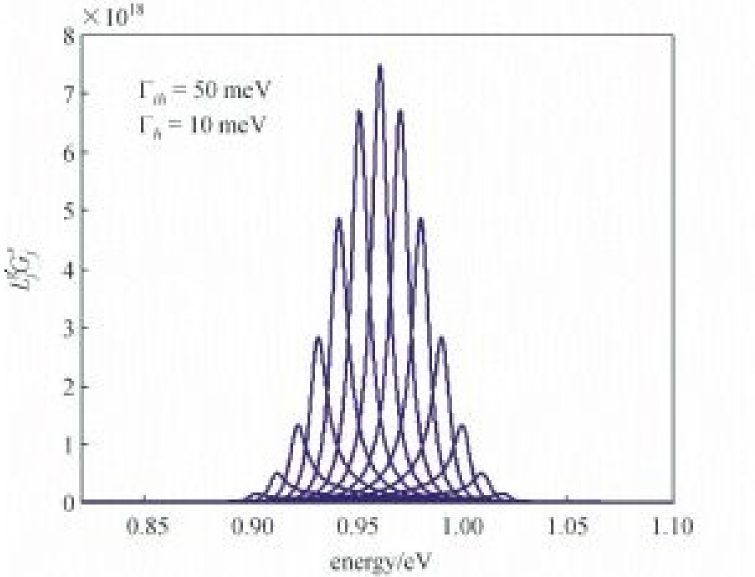
- Using the following relation, we obtain

$$\Delta\nu_0 << \Delta\nu_0^* \Rightarrow$$

$$g_0(\nu_0') \approx g_0(\nu_1),\ \ I_s(\nu_0') \approx\ I_s(\nu_1) \Rightarrow$$

$$g(\nu_1, I_{\nu 1}) \approx g_0(\nu_1) \int_{\nu_1-\Delta\nu_0/2}^{\nu_1+\Delta\nu_0/2} \frac{1}{1+\left[2(\nu_1-\nu_0')/\Delta\nu_0'\right]^2+\dfrac{I_{\nu 1}}{I_s(\nu_1)}} d\nu_0'$$

$$\approx g_0(\nu_1) \int_{-\infty}^{\infty} \frac{1}{1+\left[2(\nu_1-\nu_0')/\Delta\nu_0'\right]^2+\dfrac{I_{\nu 1}}{I_s(\nu_1)}} d\nu_0'$$



Gain of inhomo broadening medium

- Then, the gain of the inhomo broadening medium at v_1 becomes

$$\boxed{g(v_1, I_{v1}) = \frac{g_0(v_1)}{\sqrt{1 + I_{v1} / I_s(v_1)}}}$$

This is the gain saturation effect for the inhomo broadening medium.

- The small signal gain at v_1 is

$$g_0(v_1) = \sigma_{21}^{h}(v_1 - v_1)\Delta N_0 g^*(v_1 - v_0) = \sigma_{21}^{inh}(v_1 - v_0)\Delta N_0$$

- The saturation intensity is independent on the frequency v_1,

$$I_s(v_1) = \frac{hv_1}{\sigma_{21}^{h}(v_1 - v_1)\tau_{sp}} = \frac{hv_1}{\frac{2\pi^2}{3n\varepsilon_0 c_0 h}|\mu|^2 v_1 g(v_1 - v_1)} = \frac{h\Delta v_0}{\frac{4\pi}{3n\varepsilon_0 c_0 h}|\mu|^2} \qquad \boxed{I_s(v_1) = I_s}$$

- The saturation effect in inhomo broadening medium is independent on the frequency.

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Gain of inhomo broadening medium

- The small signal gain at v_1 is

$$g_0(v_1)=\frac{\sigma_{21}^{inh}(v_1-v_0)}{\sigma_{21}^{inh}(v_0-v_0)}g_0(v_0)=\frac{v_1 g^*\left(v_1-v_0\right)}{v_0 g^*\left(v_0-v_0\right)}g_0(v_0)\approx\frac{g^*\left(v_1-v_0\right)}{g^*\left(v_0-v_0\right)}g_0(v_0)$$

- Therefore, the small signal gain at v_1,

$$\boxed{g_0(v_1)=g_0(v_0)\exp\left[-4\ln 2\frac{(v_1-v_0)^2}{\Delta v_0^{*2}}\right]}$$

- Therefore, the gain at v_1 with incident light at v_1, for the inhomo broadening medium

$$\boxed{g(v_1,I_{v1})=\frac{g_0(v_0)}{\sqrt{1+I_{v1}/I_s}}\exp\left[-4\ln 2\frac{(v_1-v_0)^2}{\Delta v_0^{*2}}\right]}$$

$$g^*\left(v_0^{'}-v_0\right)=\frac{2}{\Delta v_0^*}\left(\frac{\ln 2}{\pi}\right)^{1/2}\exp\left[-4\ln 2\frac{(v_0^{'}-v_0)^2}{\Delta v_0^{*2}}\right]$$

Spectral hole burning

- For the inhomo broadening medium, with light injection at v_1, in addition to the gain at v_1, how about the gain at other frequencies, or the whole gain profile?

- For populations with resonance frequency at v_1,

$$\Delta N(v_1) = \frac{\Delta N_0(v_1)}{1 + I_{v1} / I_s(v_1 - v_1)}$$

- For populations with resonance frequency at v_2,

$$\Delta N(v_2) = \frac{\Delta N_0(v_2)}{1 + I_{v1} / I_s(v_1 - v_2)}$$

- The relation,

$$I_s(v_1 - v_2) > I_s(v_1 - v_1) \Rightarrow$$

$$\frac{\Delta N(v_2)}{\Delta N_0(v_2)} > \frac{\Delta N(v_1)}{\Delta N_0(v_1)}$$

- If the frequency difference,

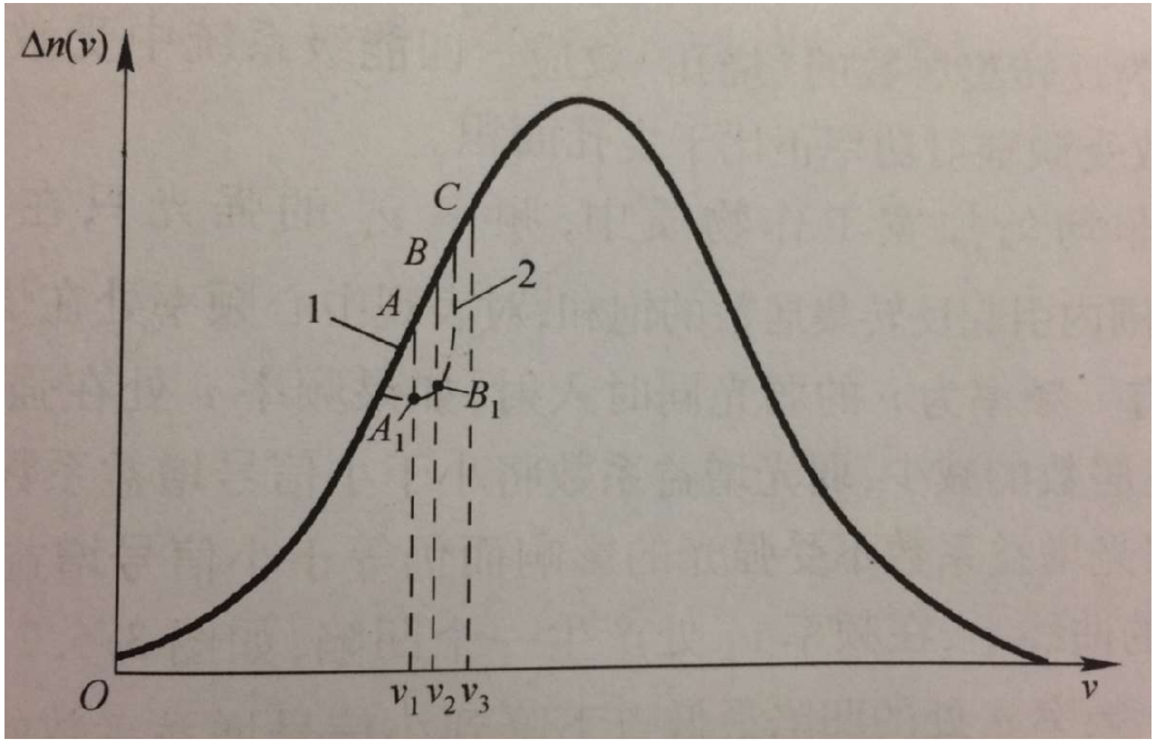
$$|v_1 - v_2| \to \infty \Rightarrow$$

$$I_s(v_1 - v_2) \to \infty \Rightarrow$$

$$\Delta N(v_2) \approx \Delta N_0(v_2)$$

Spectral hole burning

- Therefore, only populations with frequency close to the incident light have saturation effect. The result is that a “hole” is created in the inverted population spectrum, this is called spectral hole burning effect of the inverted population.



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Spectral hole burning

- The saturation range is determined by the homo broadening linewidth,

$$|\nu_2 - \nu_1| < \sqrt{1 + \frac{I_{\nu 1}}{I_s(\nu_1)}}\frac{\Delta\nu_0}{2}$$

- Therefore, the hole width is

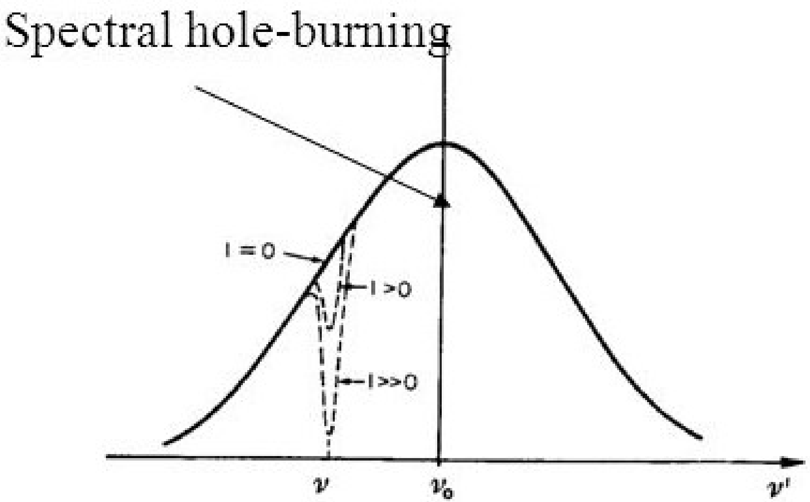
$$\delta\nu = \sqrt{1 + \frac{I_{\nu 1}}{I_s(\nu_1)}}\Delta\nu_0$$

- Therefore, the hole depth is

$$\Delta N_0(\nu_1) - \Delta N(\nu_1) = \frac{I_{\nu 1}}{I_{\nu 1} + I_s(\nu_1)}\Delta N_0(\nu_1)$$

Spectral hole burning

- Correspondingly, with incident light at v_1, the gain around v_1 saturates, and a hole is created in the gain spectrum, called spectral hole burning of the gain.



- The hole width of the gain is the same as the population,

$$\delta v = \sqrt{1 + \frac{I_{v1}}{I_s(v_1)}} \Delta v_0$$

- The hole deption of the gain is different to the population,

$$g_0(v_1) - g(v_1) = \left(1 - \frac{1}{\sqrt{1 + I_{v1}/I_s(v_1)}}\right) g_0(v_1)$$

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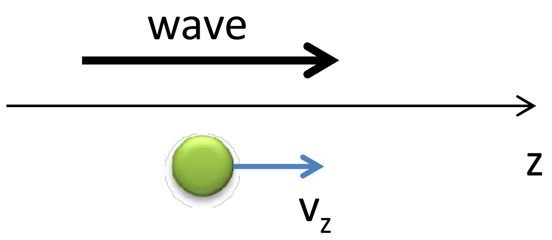
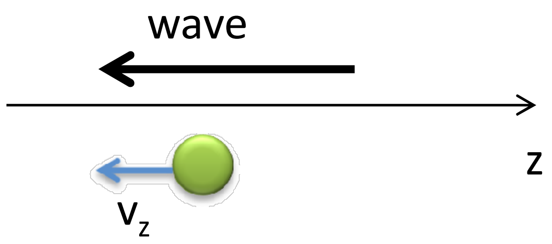
SHB in Doppler broadening medium

- For the gas laser with a laser cavity, the light at v_1 goes and forth. If the atoms with velocity v_z interacts with the light going forth, then the atoms with velocity $-v_z$ will interacts with the light going back.

- Atoms apparent frequencies

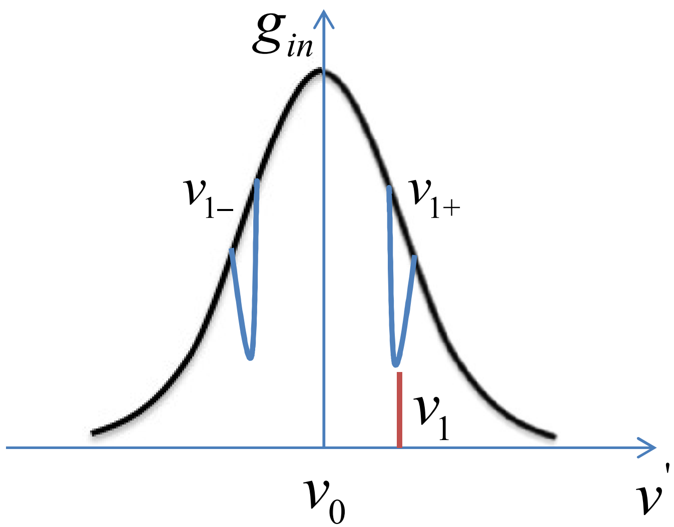


$$\nu_{1+} = \left(1 + \upsilon_z / c_n\right)\nu_0$$



$$\nu_{1-} = \left(1 - \upsilon_z / c_n\right)\nu_0$$

- In consequence, gain saturation will result in two holes on the inhomo. broadening gain profile



$$\nu_{1+} = \nu_1$$

$$\nu_{1-} = 2\nu_0 - \nu_1$$

$$v_z = \pm c \frac{\nu_1 - \nu_0}{\nu_0}$$

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Common lasers

TABLE 2.2. Emission wavelengths, peak transition cross sections, upper-state lifetime and transition linewidths for some of the most common gas and solid state lasers[(18–21)]

Transition	λ [nm]	σ_p [cm^2]	τ [μs]	$\Delta\nu_0$	Remarks
He-Ne	632.8	3×10^{-13}	150×10^{-3}	1.7 GHz	
Ar^+	514.5	2.5×10^{-13}	6×10^{-3}	3.5 GHz	
Nd:YAG	1,064	2.8×10^{-19}	230	120 GHz	
Nd:Glass	1,054	4×10^{-20}	300	5.4 THz	
Rhod. 6G	570	3.2×10^{-16}	5.5×10^{-3}	46 THz	
Alexandrite	704	0.8×10^{-20}	300	60 THz	T = 300 K
$Ti^{3+} : Al_2O_3$	790	4×10^{-19}	3.9	100 THz	E||c axis
Cr^{3+} : LiSAF	845	5×10^{-20}	67	84 THz	E||c axis

Refractive index

- The refractive index spectrum of a material can be determined by its absorption (or gain) spectrum.
- The electrical dipole moment of an atom induced by the electric field is

$$p(z,t) = \frac{e^2}{m} \frac{1}{2\omega_0(\omega_0 - \omega) + j\gamma\omega_0} E(z,t)$$

m electron mass, e electron charge

ω_0 resonance frequency, γ damping factor

- The (macroscopic) polarization and (microscopic) dipole moment relation

$$P(z,t) = Np(z,t) = \varepsilon_0 \chi E(z,t)$$

N atom density

χ the susceptibility (极化系数)

Refractive index

- Thus, the susceptibility of the medium is, which is a complex value

$$\chi(\omega)=\frac{e^2}{Nm}\frac{1}{2\omega_0(\omega_0-\omega)+j\gamma\omega_0}$$

- It can be divided into

$$\chi=\chi'+j\chi''$$

$$\chi'=\frac{Ne^2}{m\omega_0\varepsilon_0\gamma}\frac{2(\omega_0-\omega)/\gamma}{1+\left[(\omega-\omega_0)/\gamma\right]^2}$$

$$\chi''=-\frac{Ne^2}{m\omega_0\varepsilon_0\gamma}\frac{1}{1+\left[(\omega-\omega_0)/\gamma\right]^2}$$

- The relation between susceptibility and relative permittivity (relative dielectric coefficient)

$$\varepsilon_r=1+\chi=1+\chi'+j\chi''$$

- The relation between relative permittivity and refractive index, gain

$$\sqrt{\varepsilon_r}=\mathrm{n_r}+\mathrm{j}\frac{cg}{2\omega}$$

$$=\sqrt{1+\chi}\approx 1+\frac{\chi}{2}=1+\frac{\chi'}{2}+j\frac{\chi''}{2}$$

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Refractive index

- The refractive index, and the gain

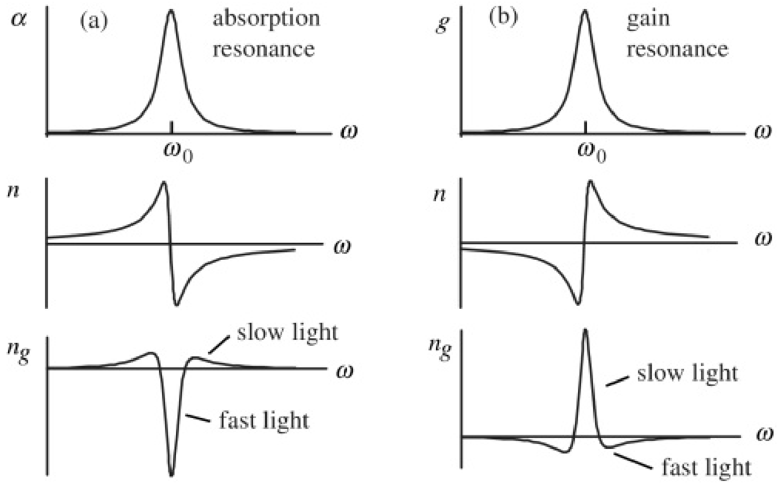
$$n_r = 1 + \frac{\chi'}{2}$$

$$g = \frac{\omega}{c}\chi''$$

- The refractive index, and the gain

$$n_r = 1 + \frac{Ne^2}{m\omega_0\varepsilon_0\gamma}\frac{(\omega_0-\omega)/\gamma}{1+\left[(\omega-\omega_0)/\gamma\right]^2}$$

$$g = -\frac{Ne^2}{mc\varepsilon_0\gamma}\frac{1}{1+\left[(\omega-\omega_0)/\gamma\right]^2}$$



Kramers-Kronig relation

- The refractive index and the gain relation

$$n_r = 1 - \frac{\omega_0-\omega}{\gamma}\frac{cg}{\omega_0}$$

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Homework

- In homogeneous broadening medium, derive the relation between the transition cross section at the resonance peak and the spontaneous emission rate.
- In homogeneous broadening medium, the small-signal gain at the peak v_0 is $g_0(v_0)$, the broadening linewidth is Δv_0. Now, a light at v_1, with intensity I_{v1} is injected. Calculate the gain broadening linewidth under this light injection.
- In homogeneous broadening medium, the small signal gain at v is $g_0(v)$, now two light with (v_1, I_{v1}) and (v_2, I_{v2}) are injected simultaneously. Calculate the saturated (large-signal) gain at v.

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Homework

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