Principles of Lasers

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Chapter 5

Passive Optical Resonators

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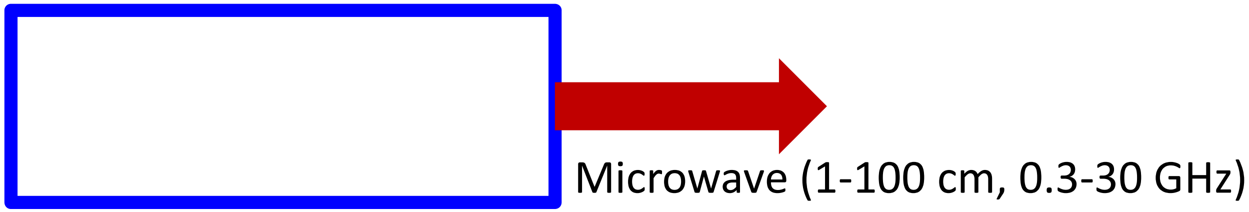
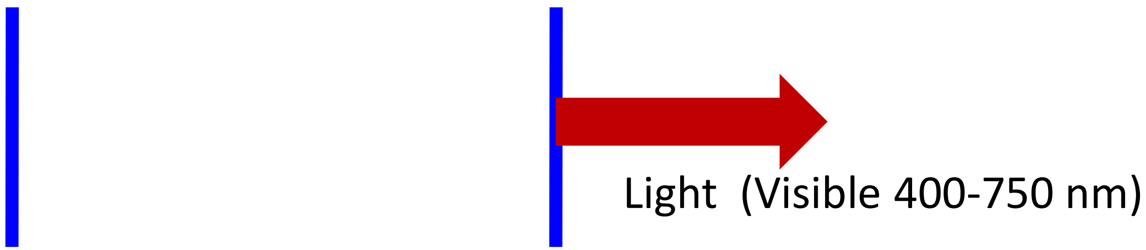
- Optical resonator

Optical resonator

- ❑ Maser oscillation relies on a closed cavity



- ❑ Laser oscillation relies on a open cavity formed by plane or spherical mirrors



✓ Open resonator only allows a few modes travelling nearly parrallel to the resonator axis to oscillates in the cavity due to losses, the mode number is much fewer than in a closed cavity (with the same size).

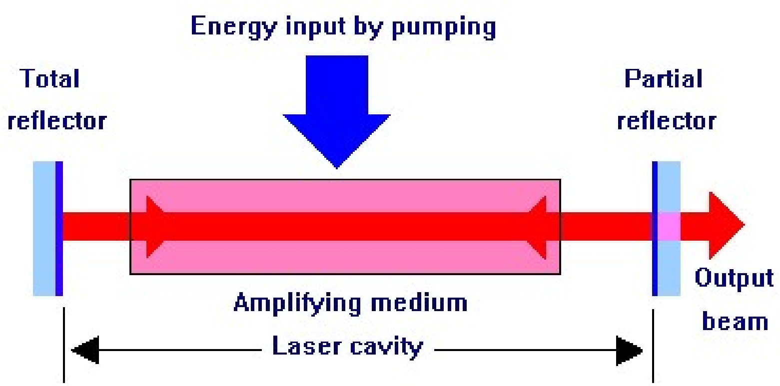
✓ The resonator dimensions are much greater than the laser wavelength. A laser cavity with length comparable to the laser wavelength have too low gain to allow laser oscillation.

Optical resonator

- For the laser generation, the optical resonator,

  (1) On one hand, provides feedback to the optical mode, which can pass through the gain medium for multiple times.

  (2) On the other hand, select longitudinal mode to enhance the coherence of laser field.



- In the open optical cavity, standing wave of low loss exists, and the electric field is described as (different to the closed cavity for blackbody study in pp. 20).

$$\boxed{\mathrm{E}(r,t) = E_0 u(r) \exp\left(j\omega t + \phi - \frac{t}{2\tau_p} \right)}$$

- τ_p is the photon lifetime in the cavity or cavity photon decay time

Mode in a resonator

- Either in the closed cavity or in the open cavity, only e.m. field of some certain frequencies and certain spatial distributions can exist. These discrete eigensolutions of the e. m. field is called (optical) mode of the resonator.
- From the viewpoint of photon, different mode corresponds to different photon state.
- The mode is determined by the resonator structure, such as the cavity length and the mirror.
- The mode is mathematically solved by the Maxwell's equations.
- The mode in the open cavity (optical resonator) is characterized by TEM_{mnq}.

TEM_{mnq}
q is the longitudinal mode index
m,n is the transverse mode index

- Longitudinal mode stands for the frequency (spectral) feature of the mode.
- Transverse mode stands for the spatial feature of the mode, the fundamental transverse mode is Gaussian mode.
- One mode owns a certain longitudinal and transverse mode.

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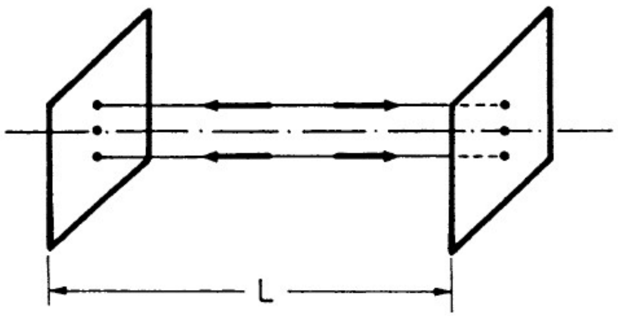
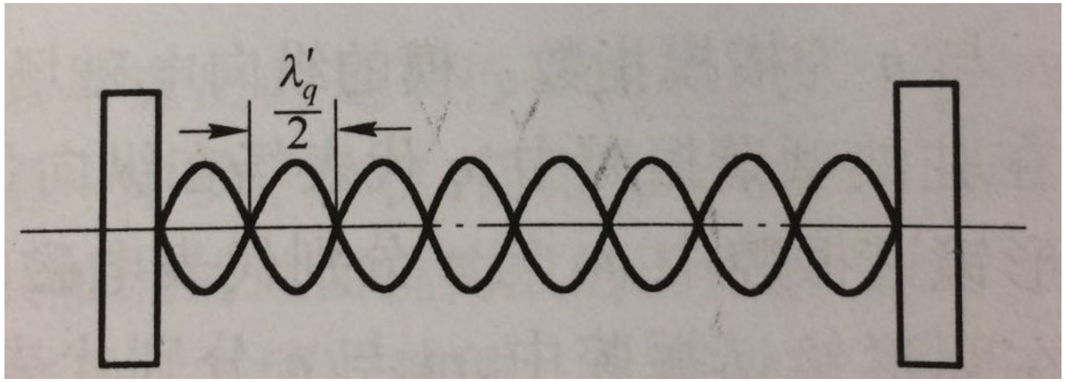
Fabry-Perot resonator

- Fabry-Perot resonator/cavity is also called plane-parallel resonantor, it consists of two plane mirrors set parallel to one another.

- The modes of the cavity can be regarded as the superposition of two plane waves (approxiamately) propagating in opposite directions along the cavity axis.
- The modes are standing waves inside the cavity.





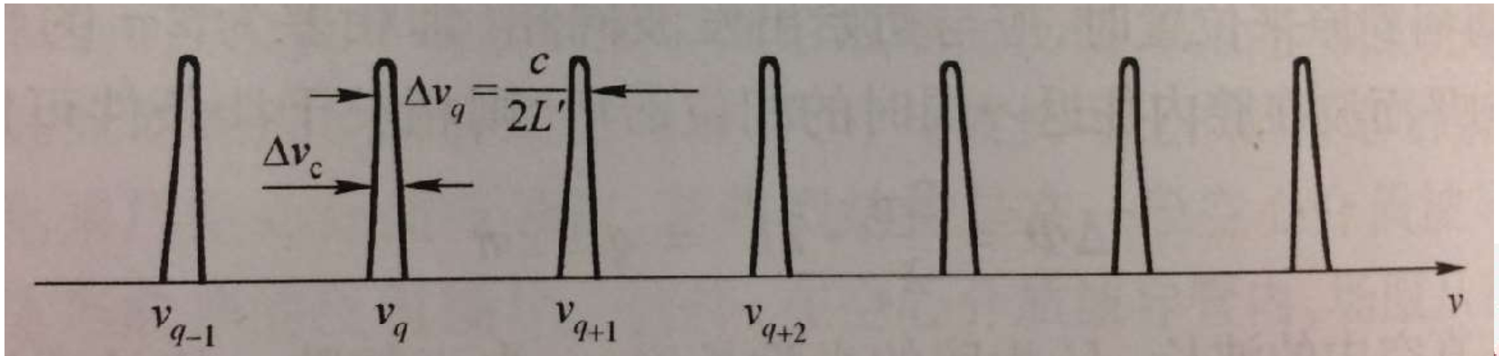
Fabry-Perot resonator

- Because the electric field of the standing wave at the mirrors of the cavity must be zero, the optical length of the cavity must be integral numbers of the half wavelength:

$$\boxed{2n_r L = q\lambda}$$

- q indicates the number of half-wavelengths of the longitudinal mode.
- Thus, the resonant frequency of the cavity is

$$\boxed{\nu_q = q\frac{c}{2n_r L}}$$



Fabry-Perot resonator

- This can be also reached by the condition that the phase shift of a plane wave passing through a round trip must be integral numbers of 2pi:

$$\boxed{2kn_r L = q2\pi}$$

- The frequency difference between adjacent longitudinal modes (free spectral range) is

$$\boxed{\Delta v_{\mathrm{q}} = \frac{c}{2n_r L}}$$

- In maser, the oscillation mode usually has only q=1 because the cavity length is comparable to the microwave wavelength.
- In laser, the oscillation mode usually has q=10^4-10^6, because the cavity length is much longer than the optical wavelength.

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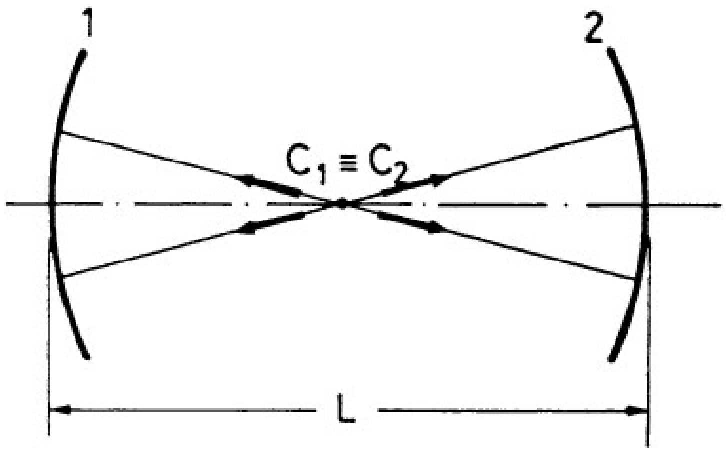
Concentric resonator

- A concentric or spherical resonator consists of two spherical mirrors having the same radius R and separated by a distance L, such that the mirror centers of curvature C1 and C2 are coincident, that is, L=2R.
- The modes of this resonator are approximately a superposition of two oppositely traveling spherical waves originating from the coincident point.
- The resonant frequencies and the FSR are the same as the Fabry-Perot laser



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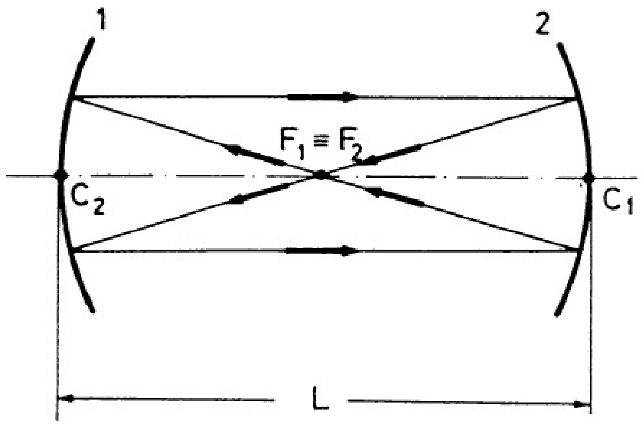
Confocal resonator

- A confocal resonator consists of two spherical mirrors having the same radius R and separated by a distance L, such that the mirror foci F1 and F2 are coincident, that is, L=R. Then, the center of the curvature C of one mirror lies on the surface of the second mirror.
- The modes of this resonator can not be described by a purely plane or a purely spherical wave.
- The resonant frequencies and the FSR can not be determined by the ray-optics framework.



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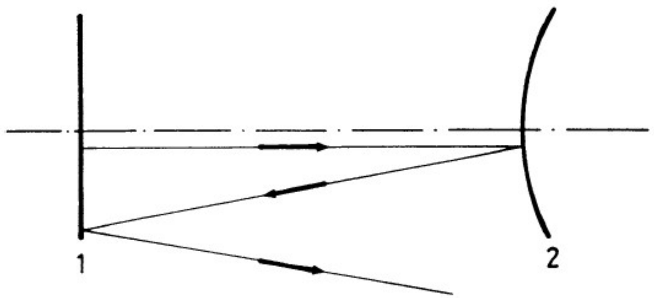
General resonators

- A general resonator can be formed by any mirrors of curvature R_1 and R_2, and any cavity length L.
- If any paraxial ray does not diverge away from the resonator with infinite times of bouncing back and forth, it is a stable resonator, e. g. confocal resonator.
- If some paraxial rays are always in the resonator with infinite round trips, while others escape the resonator with finite round trips, it is a critial stable resonator, e.g. FP and concentric resonators.
- If any paraxial ray (except several) diverges away from the resonator with finite times of bouncing back and forth, it is an unstable resonator.



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Ring resonator

❑ **Ring resonator** provides a closed loop path for the optical rays.

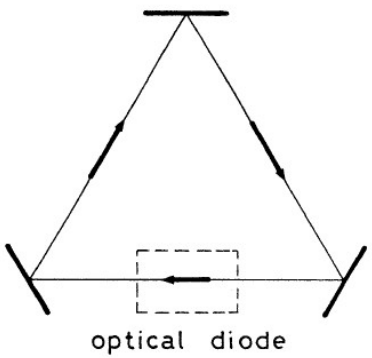
❑ It allows both clockwise and anti-clockwise propagation of waves, and thus the superposition is a standing wave as well.

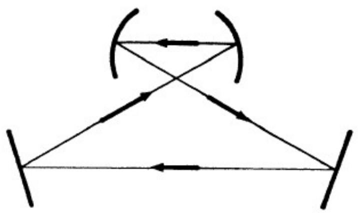
❑ The phase shift after a closed loop path is an integral number of 2pi, so the resonance frequencies are

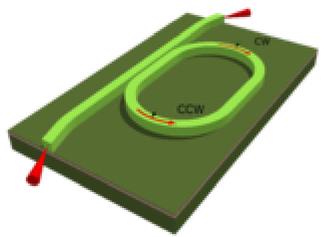
$$\boxed{\nu_q = q\frac{c}{n_r L}}$$

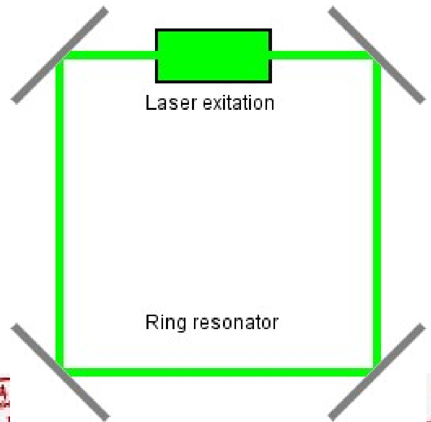
❑ The ring cavity can be stable or unstable

❑ If an optical isolator is inserted, only one direction propagation is allowed, then the electric field in the cavity becomes traveling wave. The concepts of cavity mode and resonance frequency are not confined to the standing wave configuration.









Cavity loss

- The laser cavity has some losses, and the main loss types are

1. Mirror reflection loss: this loss includes the mirror absorption, scattering, and transmission, the transmission provides the laser output.

2. Diffraction loss: The cavity mirrors and active medium have finite size, leading to diffrations. Smaller mirror size results in higher diffration loss.

3. Geometry loss: The optical ray can diverge away from the laser cavity after rounds of trip. Thus, unstable cavity has a higher geometry loss.

4. Inserted device loss (polarizer, Q-switch, waveplate), scattering and impurity of gain meidum

Cavity loss

- The cavity loss can be quantified by the single-pass loss factor delta or the loss coefficient alpha (Note: L is the geometry length rather than the optical length)

$$\boxed{\delta = \alpha L}$$

- If the laser cavity has several types of loss mechanism, the total single-pass loss factor is

$$\boxed{\delta = \delta_1 + \delta_2 + \delta_3 + \dots}$$

- The cavity loss can be divided into mirror loss and internal loss

$$\boxed{\begin{aligned} \delta &= \delta_m + \delta_i \\ \alpha &= \alpha_m + \alpha_i \end{aligned}}$$

Cavity loss

- Assume the initial light intensity in the cavity is I_0, after one round trip, the intensity I_1

$$\boxed{I_1 = I_0 e^{-2\delta}}$$

- The mirror loss

$$\boxed{\begin{aligned} &I_1 = I_0 e^{-2\delta_m} = I_0 R_1 R_2 \Rightarrow \\ &\delta_m = -\frac{1}{2}\ln\left(R_1 R_2\right) \\ &\alpha_m = -\frac{1}{2L}\ln\left(R_1 R_2\right) \end{aligned}}$$

- After m round-trip,

$$\boxed{I_m = I_0 e^{-2m\delta}}$$

Photon lifetime

- m round-trip needs a time of light propagation

$$\boxed{t = m\frac{2n_r L}{c}}$$

- Therefore, the photon lifetime or photon decay time in the cavity

$$\boxed{I(t) = I_0 \exp\left(-\frac{t}{\tau_p}\right)}$$

$$\boxed{N_p(t) = N_{p0} \exp\left(-\frac{t}{\tau_p}\right)}$$

$$\boxed{\begin{aligned}\tau_p &= \frac{n_r L}{\delta c} = \frac{L}{\delta v_g} \\ &= \frac{n_r}{\alpha c} = \frac{1}{\alpha v_g}\end{aligned}}$$

Photon lifetime

❑ The physical meaning of photon lifetime is that when t=tau$_p$, the light intensity in the cavity reduces to

$$\boxed{t = \tau_p \Rightarrow I(\tau_p) = I_0 / e}$$

❑ It can be regarded that the average time of photons staying in the cavity is tau$_p$, after that, the photons get out of the cavity.

❑ The longer the cavity and the smaller the loss lead to a longer photon lifetime.

Examples 5.2

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Q factor

- For any resonator, including an optical cavity, the Q factor (quality factor) of the resonator is defined as

$$\boxed{Q = 2\pi \frac{\text{Total stored energy}}{\text{Energy lost in one period of oscillation}}}$$

$$\text{Total stored energy } E(t) = N_p(t) h\nu$$

$$\text{Lost energy per unit time } P_0(t) = -\frac{dE}{dt} = -h\nu \frac{dN_p}{dt} = h\nu \frac{N_p(t)}{\tau_p}$$

$$\text{Lost energy per period of oscillation } P_p(t) = P_0(t)\frac{1}{\nu} = h\frac{N_p(t)}{\tau_p}$$

- The Q factor of the laser cavity is obtained as

$$\boxed{Q = 2\pi \nu \tau_p}$$

- The Q factor indicates the capability of a resonator for storing energy.

Examples 5.4

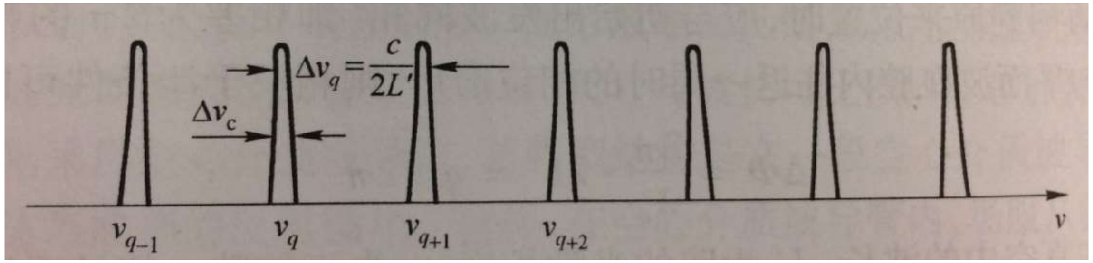
Spectral linewidth of the cavity

- The electric field in the optical cavity is given by

$$\boxed{E(t) = E_0 \exp\left(j\omega t + \phi - \frac{t}{2\tau_p} \right)}$$

- The Fourier transform of the electric field gives a Lorenzian lineshape of the optical power spectrum, and the FWHM is

$$\boxed{\Delta v_c = \frac{1}{2\pi\tau_{\mathrm{p}}} = \frac{v}{Q}}$$



- The above expression is different to the spectral linewidth of FP interferometer due to the assumption of any time t is the same as t_m.

- A high Q factor implies a low loss of the cavity, and a narrow spectral linewidth.

Examples 5.3

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Homework

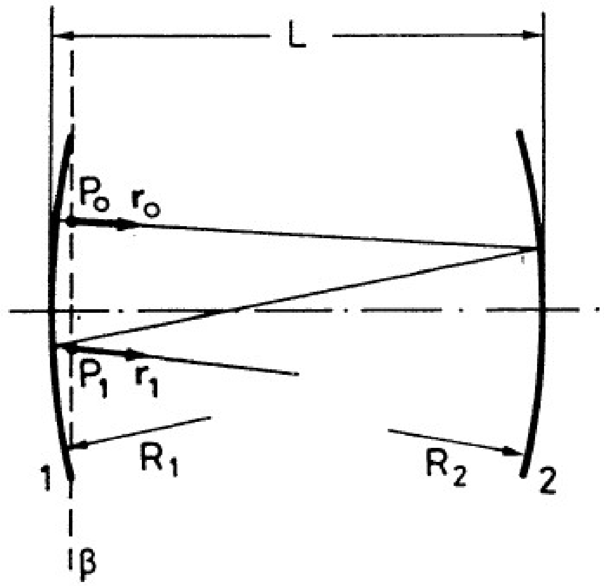
- A laser has a cavity length of 10 cm. The active medium inside the cavity has a length of 1 cm, and its refractive index is 3.0. The internal loss coefficient inside the whole cavity is 5/cm, the front mirror reflectivity is 60%, and the rear mirror reflectivity is 100%. Calculate the round-trip propagation time of the light and the photon lifetime of the laser cavity.

- Stability condition
- Eigenmodes and Eigenvalues

Stability condition

- Assume the round-trip transfer matrix of the optical cavity is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$$



- After one-round trip propagation

$$\begin{bmatrix} r_1 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} r_0 \\ \theta_0 \end{bmatrix}$$

- After n-round trip propagation

$$\begin{bmatrix} r_n \\ \theta_n \end{bmatrix} = \begin{bmatrix} A_n & B_n \\ C_n & D_n \end{bmatrix} \begin{bmatrix} r_0 \\ \theta_0 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^n \begin{bmatrix} r_0 \\ \theta_0 \end{bmatrix}$$

Stability condition

- To make the resonator stable, the position and the angle must be finite after infinite round trips, to keep the paraxial ray remains in the cavity (as long as the mirror size is large enough). That is, the n-round trip transfer matrix must not diverge.
- From the Sylvester's theorem,

$$\begin{bmatrix} A_n & B_n \\ C_n & D_n \end{bmatrix} = \frac{1}{\sin\theta}\begin{bmatrix} A\sin(n\theta)-\sin[(n-1)\theta] & B\sin(n\theta) \\ C\sin(n\theta) & D\sin(n\theta)-\sin[(n-1)\theta] \end{bmatrix}$$

$$\cos\theta = \frac{A+D}{2}$$

Stability condition

- To make each element in the matrix finite for any n, theta must be a real number, that is

$$\boxed{-1 < \frac{A+D}{2} < 1}$$

- If theta is a complex number,

$$\theta = a + jb \Rightarrow$$

$$\sin(n\theta) = \frac{\exp(jn\theta) - \exp(-jn\theta)}{2j} = \frac{\exp(jna - nb) - \exp(-jna + nb)}{2j} \Rightarrow$$

$$\left|\sin(n\theta)\right|^2 = \frac{\exp(-2nb) - \exp(2nb)}{4} \text{ becomes infinite}$$

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Stability condition

- The round-trip matrix for the optical cavity is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2/R_1 & 1 \end{bmatrix} \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2/R_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-\dfrac{2L}{R_2} & \left(1-\dfrac{2L}{R_2}\right)L+L \\ -\dfrac{2}{R_1}+\dfrac{4L}{R_1R_2}-\dfrac{2}{R_2} & \left(-\dfrac{2}{R_1}+\dfrac{4L}{R_1R_2}-\dfrac{2}{R_2}\right)L-\dfrac{2L}{R_1}+1 \end{bmatrix}$$

- Note: R is positive for concave mirrors, while negative for convex mirrors.

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Stability condition

- Then, we obtain the value,

$$\boxed{\frac{A+D}{2}=1-\frac{2L}{R_1}-\frac{2L}{R_2}+\frac{2L^2}{R_1R_2}=2\left(1-\frac{L}{R_1}\right)\left(1-\frac{L}{R_2}\right)-1}$$

- Define g parameters of the optical resonator

$$\boxed{g_1=1-\frac{L}{R_1};\quad g_2=1-\frac{L}{R_2}}$$

$$\boxed{\frac{A+D}{2}=2g_1g_2-1}$$

- The stable condition of optical resonators becomes

$$\boxed{0<g_1g_2<1}$$

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Stability condition

- The unstable condition of optical resonators is

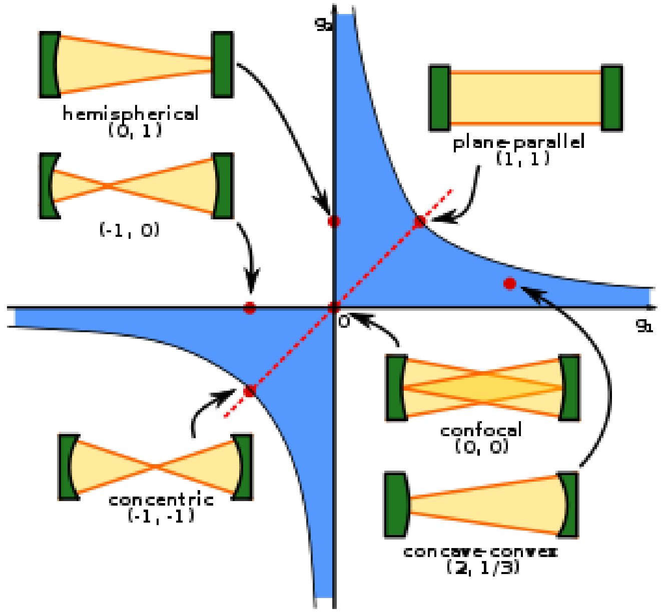
$$\boxed{\begin{aligned} &g_1 g_2 < 0,\ \text{i.e.}\ \frac{\mathrm{A}+D}{2} < -1 \quad or \\ &g_1 g_2 > 1,\ \text{i.e.}\ \frac{\mathrm{A}+D}{2} > 1 \end{aligned}}$$

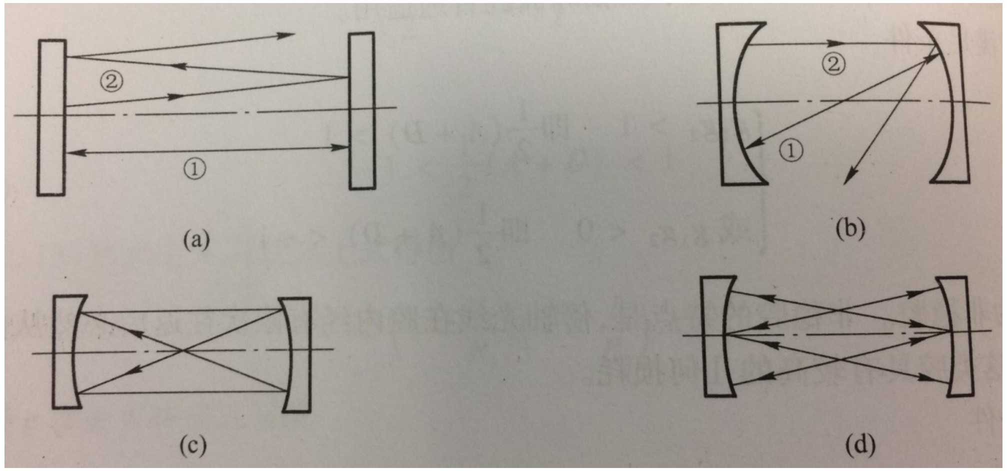
- The critically/marginally stable condition of optical resonators is

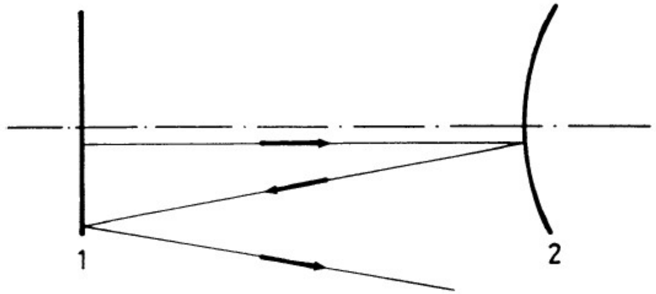
$$\boxed{\begin{aligned} &g_1 g_2 = 0,\ \text{i.e.}\ \frac{\mathrm{A}+D}{2} = -1 \quad or \\ &g_1 g_2 = 1,\ \text{i.e.}\ \frac{\mathrm{A}+D}{2} = 1 \end{aligned}}$$

Stability condition

- The blue region is stable, the boundary is critically stable, and the outside region is unstable
- Fabry-Perot cavity (a) and concentric cavity (b) is critically stable, while confocal cavity (c)&(d) is stable.







Unstable

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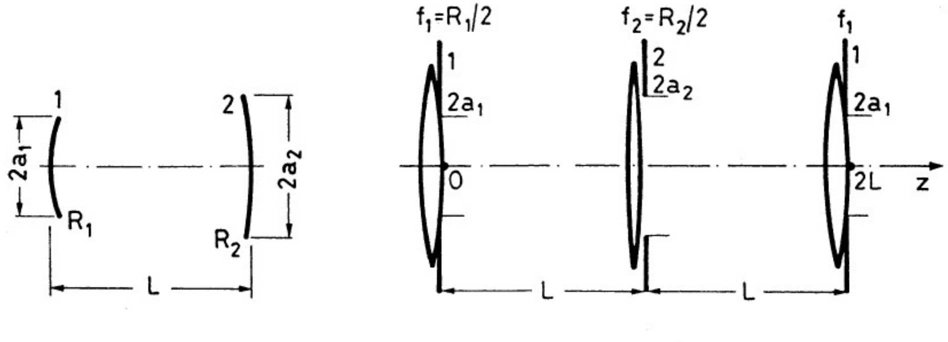
Eigenmodes and eigenvalues

- A light beam travels back and forth in a optical resonantor, is equivalent to that passes through a periodic lens-guide structure in one direction. Both have the same focal length, the same length, the same aperture size.



(a) (b)

- Using this equivalent configuration, the electric field after one-round trip propagation can be calculated from the Fresnel-Kirchoff equation. Assuming mirror 1 is at position z=0,

$$\boxed{E(x,y,2L)=\exp\left(-jk2L\right)\iint_1 K(x,y,x_1,y_1)E(x_1,y_1,0)dx_1dy_1}$$

Eigenmodes and eigenvalues

- The propagation Kernel $K(x,y,x_1,y_1)$ is a function of the transverse coordinates of both input (z=0) and output (z=2L) planes.

- If $E(x_1, y_1, 0)$ is a bidimensional delta-function at x_1', y_1',

$$\boxed{\begin{aligned} &E(x_1, y_1, 0) = \delta\left(x_1 - x_1', y_1 - y_1'\right) \Rightarrow \\ &E(x, y, 2L) = \exp(-jk2L)K(x, y, x_1', y_1') \end{aligned}}$$

That is, the propagation kernal K represents the electric field at the output plane generated by a point-loke source located at coordinates x_1,y_1, in the input plane.

Eigenmodes and eigenvalues

- In the laser cavity, the electric field must have self-consistency, that is, the electric field must reproduce its shape after one round trip (one lens-guide period). It requires,

$$\boxed{\begin{array}{l} E(x,y,2L) = \tilde{\sigma} E(x,y,0)\exp(-j2kL) \\ \text{with the constant} \\ \tilde{\sigma} = |\tilde{\sigma}|\exp(j\phi_0) \end{array}}$$

$|\tilde{\sigma}| < 1$ represents the beam amplitude attenuation, due to cavity (diffraction) losses

ϕ_0 represents additional phase change (induced by the mirrors) except 2kL

- The total phase shift of the electric field after one round trip is

$$\boxed{\Delta\phi = -2kL + \phi_0}$$

Eigenmodes and eigenvalues

- Then, the F-K equation becomes Fredholm homogeneous integral equation,

$$\boxed{\tilde{\sigma}E(x,y,0)=\iint_1 K(x,y,x_1,y_1)E(x_1,y_1,0)dx_1dy_1}$$

- Its electric field solution is named eigensolution, which are self-reproducing after each round trip propagation inside the cavity. Correspondingly, the constant coefficient is named eigenvalue,

$$\boxed{\begin{array}{l}\text{Eigensolution}: E_{lm}(x,y,0)\\ \text{Eigenvalue}: \tilde{\sigma}_{lm}\\ l,m \text{ stand for mode order for x,y directions}\end{array}}$$

Eigenmodes and eigenvalues

- The eigensolution E_{lm} gives the field of the eigenmodes at all point in a given plane.

- The eigenvalue's amplitude describes the attenuation of the beam intensity after one round trip, through its square value. That is, it gives the round trip (diffraction) loss.

$$\boxed{\delta_{lm} = 1 - \left|\tilde{\sigma}_{lm}\right|^2}$$

- The eigenvalue's phase describes the additional phase shift of the laser field, the total phase shift of one round trip must be integers of 2pi.

$$\boxed{\Delta\phi_{lm} = -2kn_r L + \phi_{lm} = -2\pi n}$$

- From the above phase condition, the resonance frequency of the modes in the cavity becomes

$$\boxed{v_{lmn} = \frac{c}{2n_r L}\left[n + \frac{\phi_{lm}}{2\pi}\right]}$$

Examples 5.1

That is, the resonant frequencies of different transverse modes are different.

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