Principles of Lasers

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Chapter 7

Continuous Wave Laser Behavior

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Chapter 7_L17

- Rate equations

✓ Four-level laser

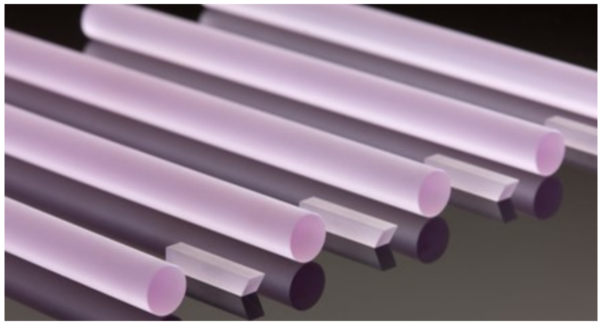
✓ Three-level laser

Laser types

❑ In the aspect of energy levels involved in the laser emission, lasers can be generally divided into two types:

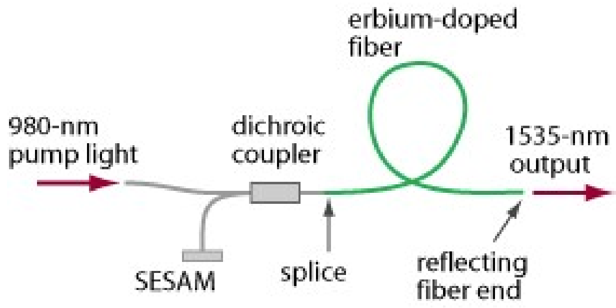
✓ Four-level lasers: Ionic crystal lasers like Nd^{3+}:YAG (Neodymium-doped Yttrium Aluminium Garnet 掺钕钇铝石榴石 1064 nm); Gas lasers like CO_2 (10.6 um), He-Ne (632.5 nm)

✓ Three-level lasers: ruby laser (694 nm), fiber lasers like Er (铒1550 nm), Yb (镱1030 nm) , Ho (钬2000 nm) , Tm (铥2000 nm) doped fiber lasers



Nd:YAG laser

He-Ne laser



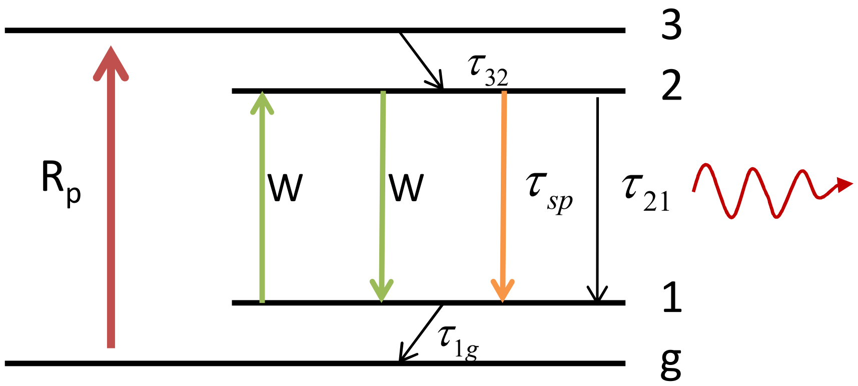
Fiber laser

Rate equations

- Rate equations are a set of coupled differential equations, describing the temporal and spatial behavoirs of the carrier populations and the photons. It is a phenomenonlogical description of the Bloch-Maxwell's equations. It is a semi-classical description of laser dynamics.
- Space indepdendent rate equation: we assume that the laser is oscillating on a single mode and that pumping and mode energy densities are uniform within in the laser material. This means that the mode transverse profile must be uniform and that we are neglecting the effects of the standing wave character of the mode. In this case, the carrier population and the photon change are only dependent on time.

Four level laser

❑ The rate equation describes the populaiton and the photon change owing to all the processes, including the pumping, the stimulated emission, the spontaneous emission, the nonradiative decay, and the photon decay (lifetime) in the cavity.



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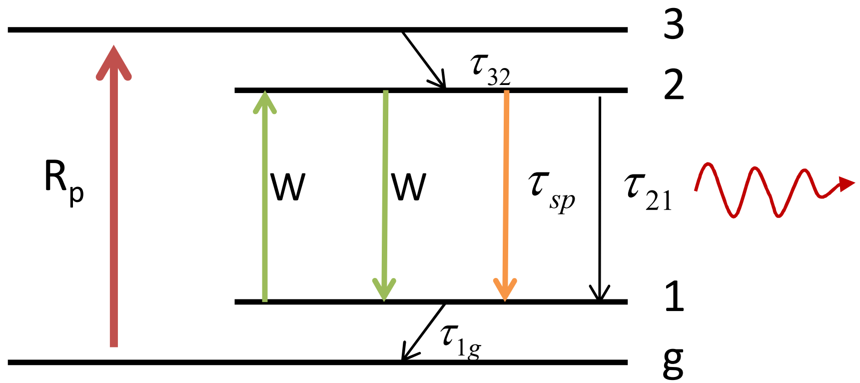
Four level laser

❑ For the populations (carrier density)



$$\frac{dN_3}{dt} = R_p - \frac{N_3}{\tau_{32}}$$

$$\frac{dN_2}{dt} = \frac{N_3}{\tau_{32}} - W(N_2 - N_1) - \frac{N_2}{\tau_{21}} - \frac{N_2}{\tau_{sp}}$$

$$\frac{dN_1}{dt} = W(N_2 - N_1) + \frac{N_2}{\tau_{21}} + \frac{N_2}{\tau_{sp}} - \frac{N_1}{\tau_{1g}}$$

$$\frac{dN_g}{dt} = \frac{N_1}{\tau_{1g}} - R_p$$

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Four level laser

- For the photons (photon density)
  - ✓ The stimulated emission generates photons
  - ✓ The absorption kills photons
  - ✓ The spontaneous emission generates photons
  - ✓ The cavity loss kills photons through the finite photon lifetime

$$\boxed{\frac{dN_P}{dt} = W(N_2 - N_1) - \frac{N_P}{\tau_P} + \beta \frac{N_2}{\tau_{sp}}}$$

Four-level laser

- The stimulated emission term,

$$\boxed{\begin{aligned} W(N_2 - N_1) &= \sigma F_s (N_2 - N_1) \\ &= \sigma N_P v_g (N_2 - N_1) \\ &= v_g g N_P \end{aligned}}$$

- The carrier lifetime of level 2

$$\boxed{\frac{1}{\tau_2} = \frac{1}{\tau_{21}} + \frac{1}{\tau_{sp}}}$$

- The photon decay term,

$$\boxed{\frac{1}{\tau_P} = v_g \alpha = v_g \frac{\delta}{L}}$$

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Four level laser

- The spontaneous emission term,

$$\boxed{\frac{N_2}{\tau_{sp}} \quad \text{vs.} \quad \beta \frac{N_2}{\tau_{sp}}}$$

- From quantum mechanics theory, the laser action is initiated by the spontaneous emission. Indeed, if the rate equation of the photon does not include the spontaneous emission, the integral of photon will be always N_p=0.
- Every spontaneous emission process kills a carrier, but not contribute to one single mode. The spontaneously emitted light is distributed over the entire frequency range corresponding to the gain bandwidth. Therefore, only a fraction of the spontaneously emitted light contributes to the given mode. This fraction is described by the spontaneous emission factor beta.

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Four level laser

- The full set of coupled rate equations (densities),

$$\boxed{\begin{aligned}
\frac{dN_3}{dt} &= R_p - \frac{N_3}{\tau_{32}} \\
\frac{dN_2}{dt} &= \frac{N_3}{\tau_{32}} - v_g g N_P - \frac{N_2}{\tau_{21}} - \frac{N_2}{\tau_{sp}} \\
\frac{dN_1}{dt} &= v_g g N_P + \frac{N_2}{\tau_{21}} + \frac{N_2}{\tau_{sp}} - \frac{N_1}{\tau_{1g}} \\
\frac{dN_g}{dt} &= \frac{N_1}{\tau_{1g}} - R_p \\
\frac{dN_P}{dt} &= v_g g N_P - \frac{N_P}{\tau_P} + \beta \frac{N_2}{\tau_{sp}}
\end{aligned}}$$

- Once the total population N_t is known, the ground state population can be replace by

$$\boxed{N_3 + N_2 + N_1 + N_g = N_t}$$

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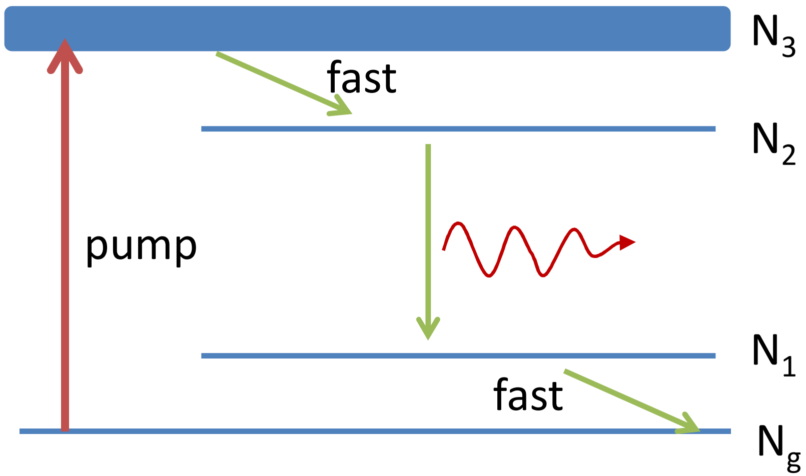
Four level laser

- The rate equations can be simplified through approximations,



- The carrier decay level 3 and level 1 is very fast

$$\boxed{N_3 \approx 0, \quad N_1 \approx 0}$$

- Only very small part of carriers is pumped up

$$\boxed{N_g \approx constant}$$

- The gain

$$\boxed{g = \sigma\left(N_2 - N_1\right) \approx \sigma N_2}$$

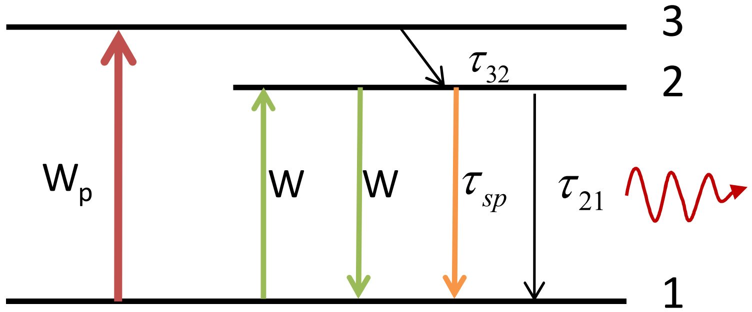
Four level laser

- The rate equations are simplified to

$$\boxed{\begin{aligned}
&\frac{dN_2}{dt} = R_p - v_g g N_P - \frac{N_2}{\tau_{21}} - \frac{N_2}{\tau_{sp}} \\
&\frac{dN_P}{dt} = v_g g N_P - \frac{N_P}{\tau_P} + \beta \frac{N_2}{\tau_{sp}} \\
&N_3 \approx 0 \\
&N_1 \approx 0 \\
&N_g \approx cons\tan t
\end{aligned}}$$

Three level laser

- This model is suitable for both three and quasi-three level laser



$$\frac{dN_3}{dt} = W_p N_1 - \frac{N_3}{\tau_{32}}$$

$$\frac{dN_2}{dt} = \frac{N_3}{\tau_{32}} - v_g g N_P - \frac{N_2}{\tau_{21}} - \frac{N_2}{\tau_{sp}}$$

$$\frac{dN_1}{dt} = -W_p N_1 + v_g g N_P + \frac{N_2}{\tau_{21}} + \frac{N_2}{\tau_{sp}}$$

$$\frac{dN_P}{dt} = v_g g N_P - \frac{N_P}{\tau_P} + \beta \frac{N_2}{\tau_{sp}}$$

$$g = \sigma\left(N_2 - N_1\right)$$

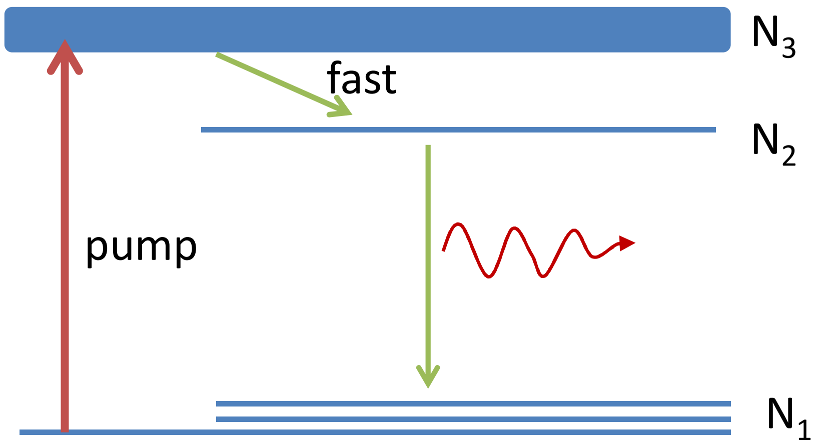
- Once the total population N_t is known, the level-1 population can be replace by

$$N_1 = N_t - N_3 - N_2$$

Three level laser

- The rate equations can be simplified through approximations,

$$\boxed{N_3 \approx 0}$$



- The rate equations are simplified to

$$\boxed{\begin{aligned}
&\frac{dN_2}{dt} = W_p N_1 - v_g g N_P - \frac{N_2}{\tau_{21}} - \frac{N_2}{\tau_{sp}} \\
&\frac{dN_1}{dt} = -W_p N_1 + v_g g N_P + \frac{N_2}{\tau_{21}} + \frac{N_2}{\tau_{sp}} \\
&\frac{dN_P}{dt} = v_g g N_P - \frac{N_P}{\tau_P} + \beta \frac{N_2}{\tau_{sp}} \\
&N_3 \approx 0
\end{aligned}}$$

$$\boxed{N_2 + N_1 \approx N_t}$$

Multimode laser

- For a multi-longitudinal mode four level laser, each mode has a different frequency, different photon density and, different photon lifetime.
- Thus, the rate equations are given by

$$\boxed{\begin{aligned} \frac{dN_2}{dt} &= R_p - v_g \sum_{l=1}^{l=n} g_l N_{Pl} - \frac{N_2}{\tau_{21}} - \frac{N_2}{\tau_{sp}} \\ \frac{dN_{Pl}}{dt} &= v_g g_l N_{Pl} - \frac{N_{Pl}}{\tau_{Pl}} + \beta \frac{N_2}{\tau_{sp}} \\ N_P &= \sum_{l=1}^{l=n} N_{Pl} \end{aligned}}$$

$$\boxed{g_l\left(v_l\right) = \sigma(v_l - v_0)\left(N_2 - N_1\right)}$$

- Usually, the photon lifetime of different modes are similar.

Argument

- One remark is that the rate equations assume that the laser has uniform pump and mode energy distributions in the active medium. This is helpful to understand the basic aspects of laser behaviors. Besides, the results are similar to the compilicated space-dependent rate equations, at least for c.w. Laser behavior.
- A second remark is that the rate equations only consider a single mode or multimode without interactions. However, once different modes have certain phase relationship, the beating terms among the various modes should be included.
- A third remark is tat the laser gain and the electric field amplitude is dependent on the longitudinal coordinate along the cavity axis due to the standing wave.

The output power

- The toal energy inside the cavity

$$\boxed{E = N_P h\upsilon V_p}$$

- The total loss coefficient

$$\alpha = \alpha_m + \alpha_i; \quad \alpha_m = \frac{1}{2L}\ln\frac{1}{R_1R_2}$$

$$\alpha_{m1} = \frac{1}{2L}\ln\frac{1}{R_1}; \quad \alpha_{m2} = \frac{1}{2L}\ln\frac{1}{R_2}$$

- The energy output (loss) rate of the two mirrors

$$v_g\alpha_m = \frac{v_g}{2L}\ln\frac{1}{R_1R_2}$$

If $R_1=R_2 \Rightarrow$

$$v_g\alpha_m = \frac{v_g}{L}\ln\frac{1}{R}$$

- The total output power from the two mirrors is

$$P_{out} = E\left(v_g\alpha_m\right)$$

$$= \left(N_P h\upsilon V_p\right)\left(v_g\alpha_m\right)$$

Photon flux ($cm^{-2}s^{-1}$)

$F = v_g N_P$

Energy density (J/cm^3)

$\rho = N_P h\upsilon$

Light intensity(mW/cm^2)

$I = v_g N_P h\upsilon$

$= F h\upsilon = \rho v_g$

Examples 7.1

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Threshold conditions and output power

- Four-level laser
- Three-level laser

Four level laser

- To achieve lasing, the population must be inverted firstly, that is, $N_2>N_1$. Therefore, the lifetime of the upper lasing level must be longer than the lifetime of the lower lasing level.

$$\boxed{\begin{array}{l}\frac{1}{\tau_{21}}+\frac{1}{\tau_{sp}}<\frac{1}{\tau_{1g}}\Rightarrow \\ \tau_2>\tau_1\end{array}}$$

- In our discussions, we assume,

$$\boxed{\tau_2 >> \tau_1}$$

- If this condition is not satisfied, the laser action is onlu possible on a pulsed basis provided that the pumping pulse is shorter than or comparable to the lifetime of the upper laser level.

$$\boxed{\begin{aligned}\frac{dN_2}{dt}&=R_p-v_g g N_P-\frac{N_2}{\tau_{21}}-\frac{N_2}{\tau_{sp}}\\ \frac{dN_1}{dt}&=v_g g N_P+\frac{N_2}{\tau_{21}}+\frac{N_2}{\tau_{sp}}-\frac{N_1}{\tau_{1g}}\\ \frac{dN_P}{dt}&=v_g g N_P-\frac{N_P}{\tau_P}+\beta\frac{N_2}{\tau_{sp}}\end{aligned}}$$

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Threshold condition

- The laser emission is initiated from the spontaneous emission. The laser osciilation can only occur when the gain of the active medium overcomes the loss of the cavity. This critical condition for the laser oscillation is the lasing threshold.

$$\boxed{g \geq \alpha \Rightarrow v_g g \geq \frac{1}{\tau_P}}$$

- This condition gives the threshold population inversion and threshold gain,

$$\boxed{\Delta N_t \approx N_{2t} = \frac{1}{\sigma v_g \tau_P}} \qquad \boxed{g_t = \frac{1}{v_g \tau_P} = \alpha}$$

- The threshold pump rate can be obtained from the steady-state condition,

$$\boxed{\begin{aligned} \frac{dN_2}{dt} &= 0 \\ N_P &\approx 0 \end{aligned}} \qquad \boxed{R_{pt} = \frac{N_{2t}}{\tau_{21}} + \frac{N_{2t}}{\tau_{sp}}} \qquad \boxed{\begin{aligned} \frac{dN_2}{dt} &= R_p - v_g g N_P - \frac{N_2}{\tau_{21}} - \frac{N_2}{\tau_{sp}} \\ \frac{dN_P}{dt} &= v_g g N_P - \frac{N_P}{\tau_P} + \beta \frac{N_2}{\tau_{sp}} \end{aligned}}$$

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Below threshold

- When the laser reaches the threshold condition, it is at threshold; or else, it is below threshold or above threshold.
- Below threshold, the spontaneous emission produces incoherent photons, and the photon number is small. The stimulated emision does not produces coherent photons because the gain is negative or smaller than the loss.
- Therefore, the stimulated emission term is negligible, and the rate equations become

$$\boxed{v_g g N_P \approx 0}$$

$$\boxed{\begin{aligned} \frac{dN_2}{dt} &= R_p - \frac{N_2}{\tau_{21}} - \frac{N_2}{\tau_{sp}} \\ \frac{dN_P}{dt} &= -\frac{N_P}{\tau_P} + \beta\frac{N_2}{\tau_{sp}} \end{aligned}}$$

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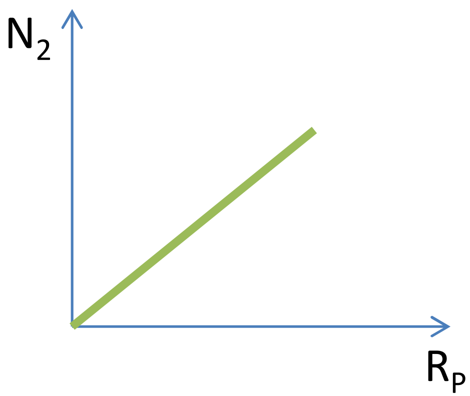
Below threshold

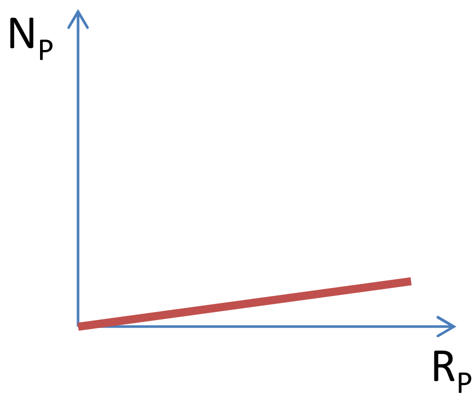
- The steady-state carriers and photons below threshold can be obtained by setting the differential equations equal to zero.

$$\boxed{\begin{aligned} 0 &= R_p - \frac{N_2}{\tau_{21}} - \frac{N_2}{\tau_{sp}} \\ 0 &= -\frac{N_P}{\tau_P} + \beta \frac{N_2}{\tau_{sp}} \end{aligned}}$$

$$\boxed{\begin{aligned} N_2 &= \frac{R_p}{1/\tau_{21} + 1/\tau_{sp}} \\ N_P &= \tau_P \frac{\beta}{\tau_{sp}} \frac{R_p}{1/\tau_{21} + 1/\tau_{sp}} \end{aligned}}$$

- Below threshold, both carrier and photon increase linearly with increasing pump rate





Above threshold

- Above threshold, the stimulated emission produce coheren photons, while the spontaneous emission remains produce incoherent photons which is much fewer than the coherent photon numbers, and thus negligible. That is, the stimulated emission dominates the photon generations.

$$\boxed{\beta \frac{N_2}{\tau_{sp}} \approx 0}$$

$$\boxed{\begin{aligned} \frac{dN_2}{dt} &= R_p - v_g g N_P - \frac{N_2}{\tau_{21}} - \frac{N_2}{\tau_{sp}} \\ \frac{dN_P}{dt} &= v_g g N_P - \frac{N_P}{\tau_P} \end{aligned}}$$

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Above threshold

- The steady-state carriers and photons above threshold are also obtained by setting the differential equations equal to zero.

$$\boxed{\begin{aligned} 0 &= R_p - v_g g N_P - \frac{N_2}{\tau_{21}} - \frac{N_2}{\tau_{sp}} \\ 0 &= v_g g N_P - \frac{N_P}{\tau_P} \end{aligned}}$$

- From the photon equation, we obtain the population inversion

$$\boxed{\Delta N \approx N_2 = \frac{1}{\sigma v_g \tau_P}}$$

- **Gain clamping**: the gain and the population inversion above threshold does not increase with the pump rate, but remains the same as that at the threshold.

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Above threshold

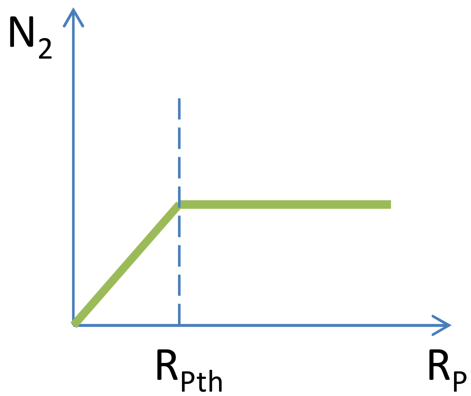
- From the carrier equation, we obtain the photon

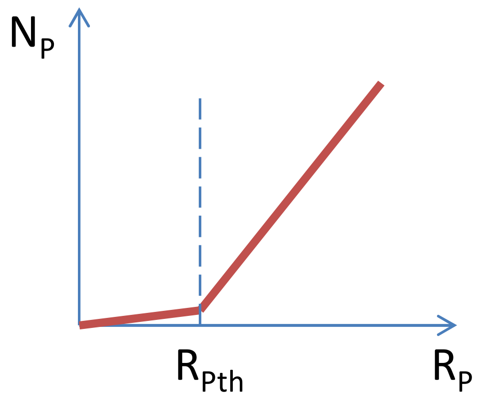
$$\boxed{N_P = \tau_P R_p - \frac{1}{\sigma v_g}\left(\frac{1}{\tau_{21}} + \frac{1}{\tau_{sp}}\right)}$$

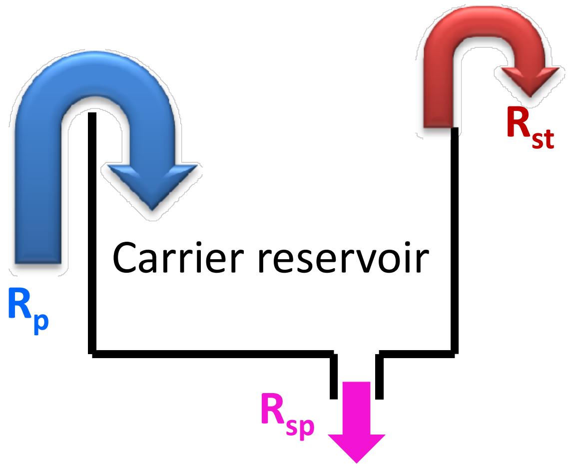
- Above threshold, the carrier population inversion remains the same as the threshold, while the photon increases with the pump rate.

- Therefore, the pump mainly increases the carrier population below threshold, while increase the photon above threshold.

Below and above the threshold







- Below the threshold

$$N_2 = \frac{R_p}{1/\tau_{21} + 1/\tau_{sp}}$$

$$N_P = \tau_P \frac{\beta}{\tau_{sp}} \frac{R_p}{1/\tau_{21} + 1/\tau_{sp}}$$

- At the threshold

$$\Delta N_t = \frac{1}{\sigma v_g \tau_P}$$

$$R_{pt} = \frac{1}{\sigma v_g \tau_P}\left(\frac{1}{\tau_{21}} + \frac{1}{\tau_{sp}}\right)$$

$$N_p = \tau_P \frac{\beta}{\tau_{sp}} \frac{R_{pt}}{1/\tau_{21} + 1/\tau_{sp}} \approx 0$$

- Above the threshold

$$\Delta N = \frac{1}{\sigma v_g \tau_P}$$

$$N_P = \tau_P\left(R_p - R_{pt}\right)$$

$$= \tau_P R_p - \frac{1}{\sigma v_g}\left(\frac{1}{\tau_{21}} + \frac{1}{\tau_{sp}}\right)$$

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The output power

- The photon density inside the cavity

$$\boxed{N_P = \tau_p \left(R_p - R_{pth} \right)}$$

$$\boxed{\begin{aligned} N_P &= R_{pth}\tau_p \left(\frac{R_p}{R_{pth}} - 1 \right) \\ &= \frac{1}{\sigma v_g \tau_P} \left(\frac{1}{\tau_{21}} + \frac{1}{\tau_{sp}} \right) \left(\frac{R_p}{R_{pth}} - 1 \right) \end{aligned}}$$

- The output power from both mirrors

$$\boxed{\begin{aligned} P_{out} &= \left(N_P h\upsilon V_p \right)\left(v_g \alpha_m \right) \\ &= \tau_p h\upsilon V_p \left(v_g \alpha_m \right)\left(R_p - R_{pth} \right) \end{aligned}}$$

$$\boxed{\begin{aligned} P_{out} &= \left(N_P h\upsilon V_p \right)\left(v_g \alpha_m \right) \\ &= \alpha_m \frac{h\upsilon}{\sigma \tau_2} V_p \left(\frac{R_p}{R_{pth}} - 1 \right) \end{aligned}}$$

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The output power

- The pump can be either optical pumping or electrical pumping

- For the optical pump, the pump power P_p

$$\boxed{\begin{aligned} P_{out} &= \left(N_p h\upsilon V_p\right)\left(v_g \alpha_m\right) \\ &= \alpha_m \frac{h\upsilon}{\sigma\tau_2} V_p \left(\frac{P_p}{P_{pth}} - 1\right) \end{aligned}}$$

- For the electrical pump, the pump current I

$$\boxed{\begin{aligned} P_{out} &= \left(N_p h\upsilon V_p\right)\left(v_g \alpha_m\right) \\ &= \alpha_m \frac{h\upsilon}{\sigma\tau_2} V_p \left(\frac{I}{I_{th}} - 1\right) \end{aligned}}$$

The output power

- The laser slope efficiency: the output laser power versus the input light power or injected current

$$\boxed{\begin{aligned}\eta_{so} &= \frac{dP_{out}}{dP_p} = \frac{h\upsilon}{\sigma\tau_2}\frac{\alpha_m V_p}{P_{pth}} \quad \text{(W/W)} \\ \eta_{se} &= \frac{dP_{out}}{dI} = \frac{h\upsilon}{\sigma\tau_2}\frac{\alpha_m V_p}{I_{th}} \quad \text{(W/A)}\end{aligned}}$$

The output power of Nd: YAG laser

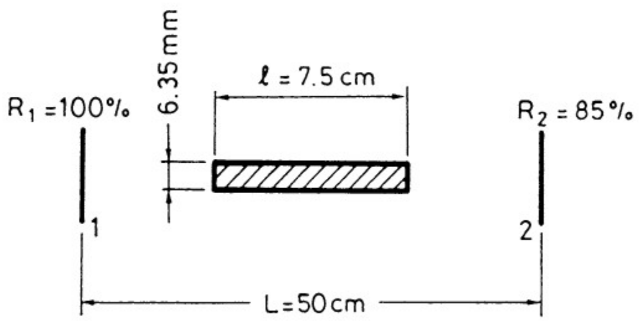

FIG. 7.4. Possible cavity configuration for a, lamp-pumped, cw Nd:YAG Laser.

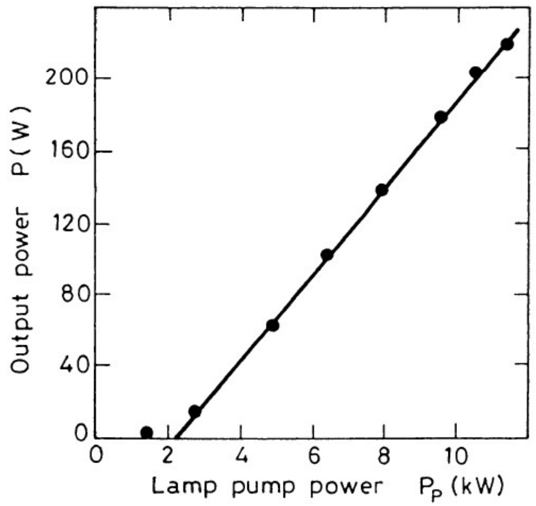

FIG. 7.5. Output power vs lamp input power for a powerful Nd:YAG laser (after Koechner,[7] by permission).

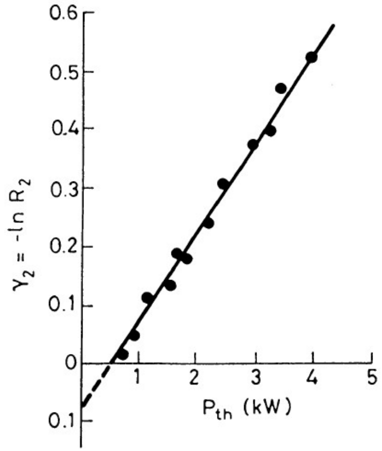

FIG. 7.6. Threshold pump power as a function of mirror reflectivity (after Koechner,[8] by permission).

Examples 7.2

The output power of CO_2 laser

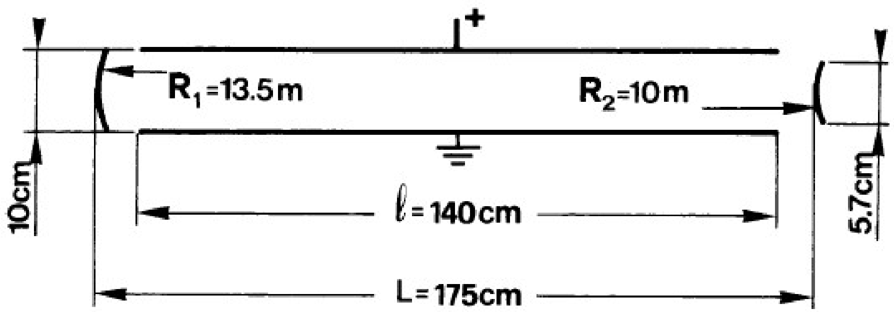

FIG. 7.7. Possible cavity configuration for a powerful cw CO_2 laser.

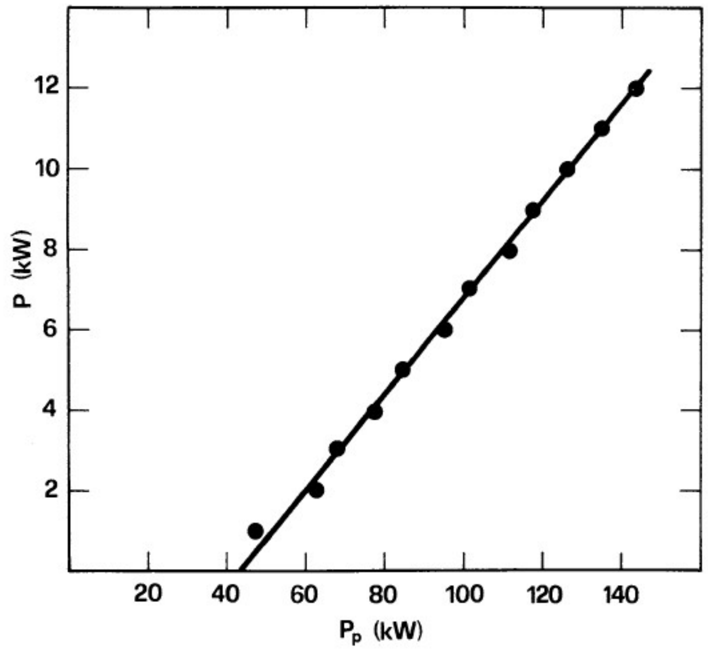

FIG. 7.8. Output power, P, versus electrical discharge power, P_p, for a powerful cw CO_2 laser.

Examples 7.3

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The output power of LD pumped Nd:YAG laser

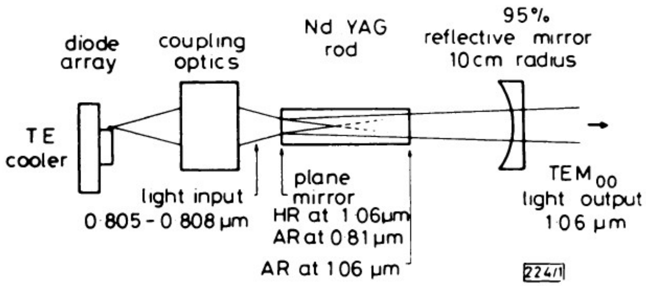

FIG. 7.12. Schematic illustration of the experimental set-up of a Nd:YAG laser, longitudinally pumped by a diode array (after ref.,[15] by permission).

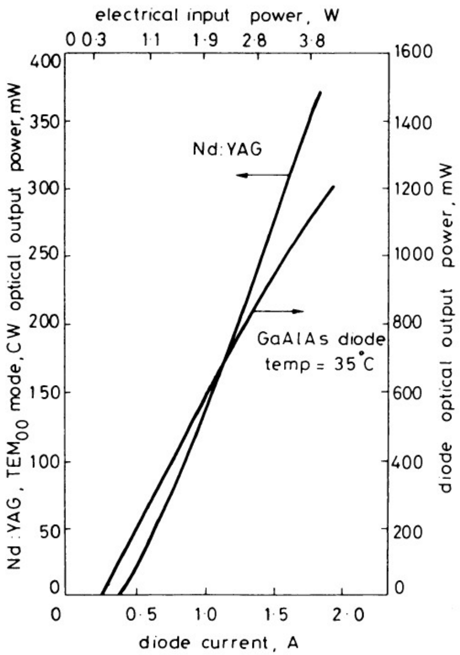

FIG. 7.13. Output power vs diode current for the Nd:YAG laser of Fig. 7.12. In the same figure the output power vs current of the laser diode array is also shown (after ref.,[15] by permission).

Examples 7.4

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Three level laser

- Assume the total carrier density is N_t, the rate equation can be equivalently writen as

$$\boxed{\begin{aligned} &\frac{dN_2}{dt} = \mathrm{R_p} - v_g g N_P - \frac{N_2}{\tau_{21}} - \frac{N_2}{\tau_{sp}} \\ &\frac{dN_P}{dt} = v_g g N_P - \frac{N_P}{\tau_P} + \beta \frac{N_2}{\tau_{sp}} \\ &N_1 = N_t - N_2 \\ &N_3 \approx 0 \end{aligned}}$$

- The population inversion is

$$\boxed{\Delta N = N_2 - N_1 = 2N_2 - N_t} \qquad \boxed{\frac{d\Delta N}{dt} = 2\frac{dN_2}{dt}}$$

$$\boxed{\begin{aligned} &\frac{dN_2}{dt} = W_p N_1 - v_g g N_P - \frac{N_2}{\tau_{21}} - \frac{N_2}{\tau_{sp}} \\ &\frac{dN_1}{dt} = -W_p N_1 + v_g g N_P + \frac{N_2}{\tau_{21}} + \frac{N_2}{\tau_{sp}} \\ &\frac{dN_P}{dt} = v_g g N_P - \frac{N_P}{\tau_P} + \beta \frac{N_2}{\tau_{sp}} \\ &N_3 \approx 0 \end{aligned}}$$

- The rate equation in terms of population inversion becomes

$$\boxed{\begin{aligned} &\frac{d\Delta N}{dt} = 2\mathrm{R_p} - 2v_g g N_P - \frac{\Delta N + N_t}{\tau_2} \\ &\frac{dN_P}{dt} = v_g g N_P - \frac{N_P}{\tau_P} + \beta \frac{\Delta N + N_t}{2\tau_{sp}} \end{aligned}}$$

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Three level laser

- The threshold gain and population inversion is obtained as

$$\boxed{\frac{dN_p}{dt}=0} \qquad \boxed{g_{th}=\frac{1}{v_g\tau_p};\ \Delta N_{th}=\frac{1}{\sigma v_g\tau_p}} \qquad \boxed{N_{2th}=\frac{N_t+\Delta N_{th}}{2}}$$

✓ The threshold gain and population inversion is the same as four level laser

- The threshold pump rate is obtained as

$$\boxed{N_p=0;\ \frac{d\Delta N}{dt}=0} \qquad \boxed{\begin{aligned} R_{pth} &= \frac{1}{2\tau_2}\left(\Delta N_{th}+N_t\right) \\ &= \frac{1}{2\tau_2}\left(\frac{1}{\sigma v_g\tau_p}+N_t\right) \\ &> \frac{1}{\tau_2\sigma v_g\tau_p} \end{aligned}}$$

✓ The threshold pump rate is larger than four level laser

$$\boxed{\begin{aligned} \frac{d\Delta N}{dt} &= 2\mathrm{R}_\mathrm{p}-2v_g g N_P-\frac{\Delta N+N_t}{\tau_2} \\ \frac{dN_P}{dt} &= v_g g N_P-\frac{N_P}{\tau_P}+\beta\frac{\Delta N+N_t}{2\tau_{sp}} \end{aligned}}$$

Three level laser

- The above threshold photon density is obtained as

$$\boxed{\frac{dN_p}{dt}=0;\quad \frac{d\Delta N}{dt}=0}$$

$$\boxed{N_P=\tau_p\left(R_p-R_{pth}\right)}$$

✓ The photon density and the output power is the same as four level laser

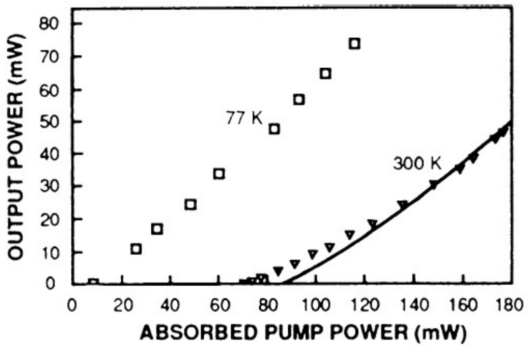

FIG. 7.14. Plots of the output power vs absorbed pump power for a Ti:sapphire-pumped Yb:YAG laser, for liquid nitrogen cooled operation (77 K) and at room temperature operation (300 K) (after ref.,[18] by permission).

$$\boxed{\begin{aligned}\frac{d\Delta N}{dt}&=2\mathrm{R_p}-2v_g gN_P-\frac{\Delta N+N_t}{\tau_2}\\ \frac{dN_P}{dt}&=v_g gN_P-\frac{N_P}{\tau_P}+\beta\frac{\Delta N+N_t}{2\tau_{sp}}\end{aligned}}$$

Examples 7.5

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Optimum output coupling

- For a fixed pump rate, to obtain a maximum output power, there is an optimum transmission T of the laser mirror. On one hand, the output power tends to increase due to the increased T; on the other hand, the output power tends to decrease since the increased cavity loss leads to the decrease of the intracavity photons.

- The mirror output coupling (loss) for $R_1=R_2$,

$$\boxed{\alpha_m = \frac{1}{L}\ln\left(\frac{1}{R}\right)}$$

- For the four-level lasers

$$\boxed{\begin{aligned} R_{p\mathrm{th}} &= \frac{1}{\sigma \mathrm{v_g} \tau_p \tau_2} \\ \tau_p &= \frac{1}{\mathrm{v_g}\left(\alpha_i + \alpha_m\right)} \end{aligned}}$$

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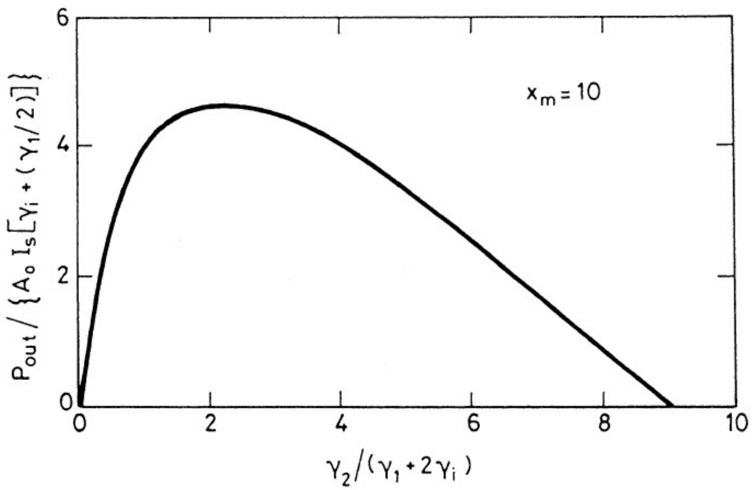
Optimum coupling (max power)

- The output power is given by

$$
\boxed{
\begin{aligned}
P_{out} &= \tau_p h\upsilon V_p \left(v_g \alpha_m\right)\left(R_p - R_{pth}\right) \\
&= \alpha_m h\upsilon V_p \left(\frac{R_p}{\alpha_i + \alpha_m} - \frac{1}{\sigma \tau_2}\right)
\end{aligned}
}
$$

- For a fixed pump rate, the maximum power is obtained at



$$
\boxed{
\begin{aligned}
&\frac{d}{dt} P_{out} = 0 \Rightarrow \\
&\alpha_m = \sqrt{R_p \sigma \tau_2 \alpha_i} - \alpha_i
\end{aligned}
}
$$

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Chapter 7_L19

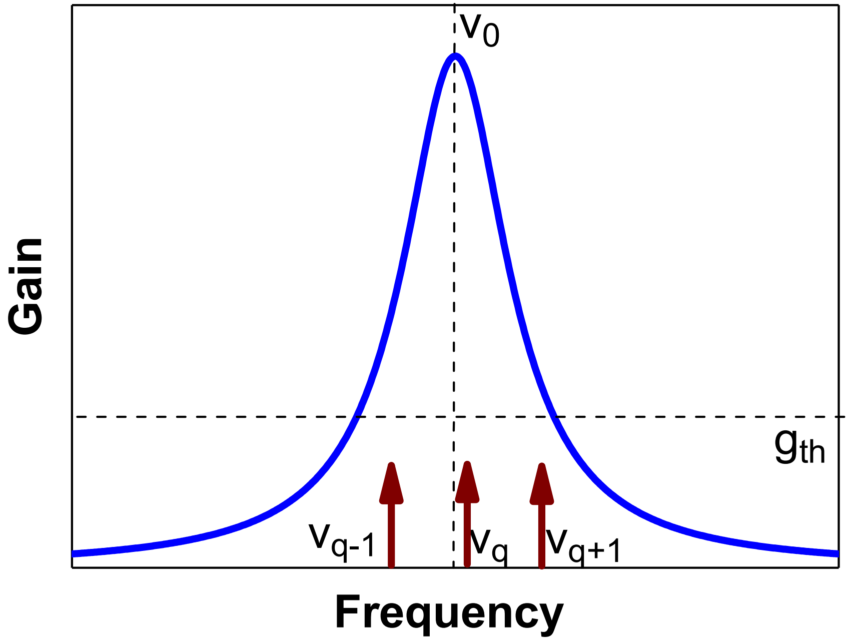
- Multimode laser oscillation

✓ Homogeneous broadening medium

✓ Inhomogeneous broadening medium

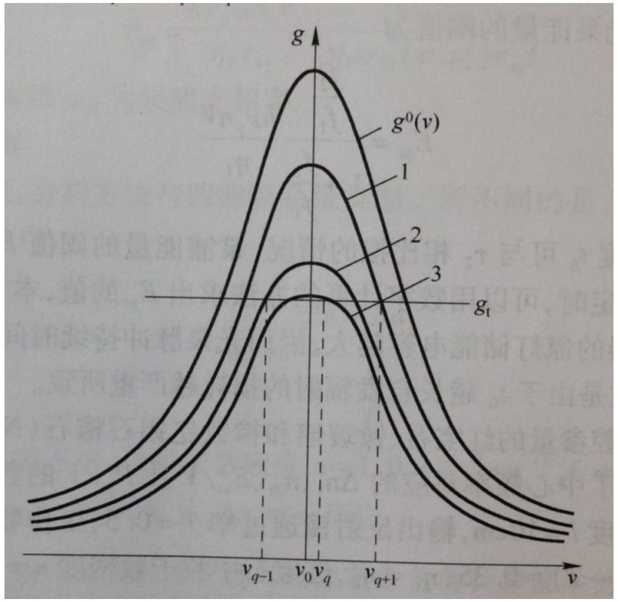
Homo broadening medium

- For homo broadening medium, the gain profile bandwidth is usually larger than the free spectral range FSR.
- Assume every longitudinal mode has the same loss (photon lifetime), that is, the lasing threshold is the same for all the modes.
- It seems that all the modes can oscillates once above threshold.
- Assume the small-signal gain of the medium peaks at v_0, and the threshold gain is g_{th}, the resonance frequencies of gain above the threshold gain are v_{q-1}, v_q, v_{q+1}, the gain profile $g_0(v-v_0)$ is as follows,



Homo. Broadening medium

- Now, assume the initial light intensities of each mode in the cavity are I_{q-1}, I_q, I_{q+1}, Because the small signal gains of each mode are larger than the threshold gain, all the mode intensities increase.
- On the other hand, due to the gain saturation effect, the whole gain profiles decreases with the increase of intensity.



(1) When the gain profile decreases to line 1,

$$g(\nu_{q+1})=g_{th}$$

I_{q+1} stops increasing. But I_q and I_{q-1} remains increasing, further reduces the gain profile, resulting in $g(\nu_{q+1})<g_{th}$. Then, I_{q+1} decreases to zero.

(2) When the gain profile decreases to line 2,

$$g(\nu_{q-1})=g_{th}$$

I_{q-1} stops increasing. But I_q remains increasing, further reduces the gain profile, resulting in $g(\nu_{q-1})<g_{th}$. Then, I_{q-1} decreases to zero.

(3) When the gain profile decreases to line 3,

$$g(\nu_q)=g_{th}$$

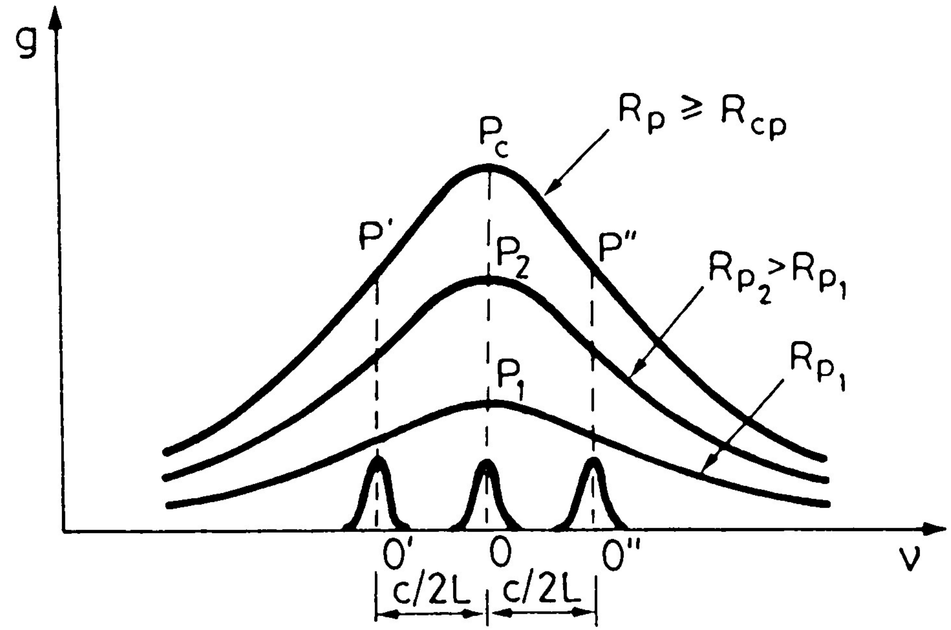
I_q stops increasing, and reaches the steady state.

Homo. Broadening medium

- Therefore, although three modes can oscillates temporally, only one mode of the highest gain can survive to reach the steady state, due to the gain clamping effect. This phenomenon is called mode competition effect. It is always the mode closest to the gain central frequency wins the competition, while all other modes turn off.
- Ideally, the homo broadening laser only outputs a single longitudinal mode, close to the peak frequency of the gain medium.

Homo broadening medium

❑ Another viewpoint is that below threshold, the whole gain profile increases with the pump. Once maximum gain reaches the threshold and begins to lasing, the whole gain profile is clamped, even the pump further increases. Thus, modes at other points of the gain profile can never reach threshold, leaving a single longitudinal mode oscillation.



$R_{p1}<R_{p2}<R_{pc}<=R_p$

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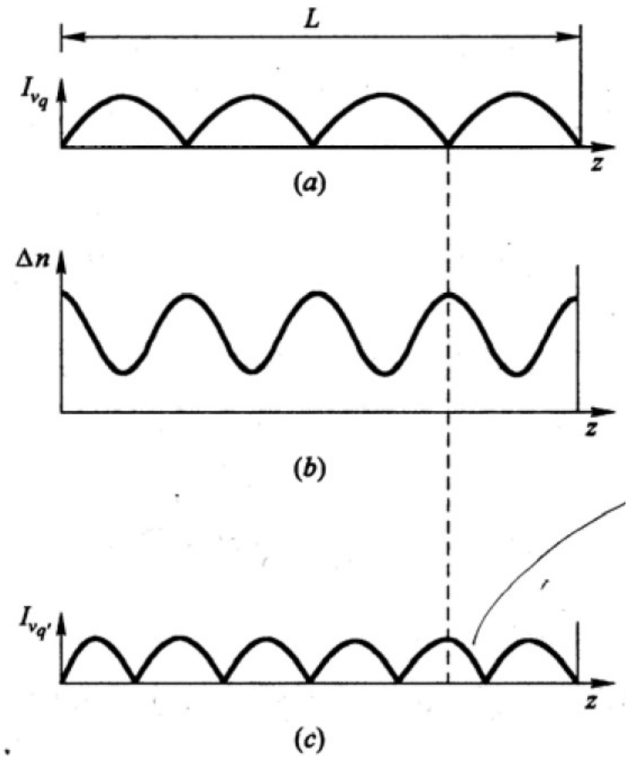
Homo. Broadening medium

❑ In practice, homo broadening medium usually lases on many longitudinal modes, and the lasing mode number increases with the pump rate.



❑ This is because the standing wave effect. For a certain mode, the field intensity is largest at the antinode while smallest at the node. Correspondingly, the population inversion/gain is the smallest at the antinode while largest at the node. This phenomenon is called spatial hole burning of the gain.

❑ Note that the average gain of this mode equals to the threshold gain.

❑ Meanwhile, another mode can use the population inversion at the antinode for lasing, which leads to multimode oscillation.

❑ Thus, due to the spatial hole burning effect, different longitudinal modes can use different spatial populations for lasing oscillation. This is called the spatial competition of longitudinal mode.

Homo. Broadening medium

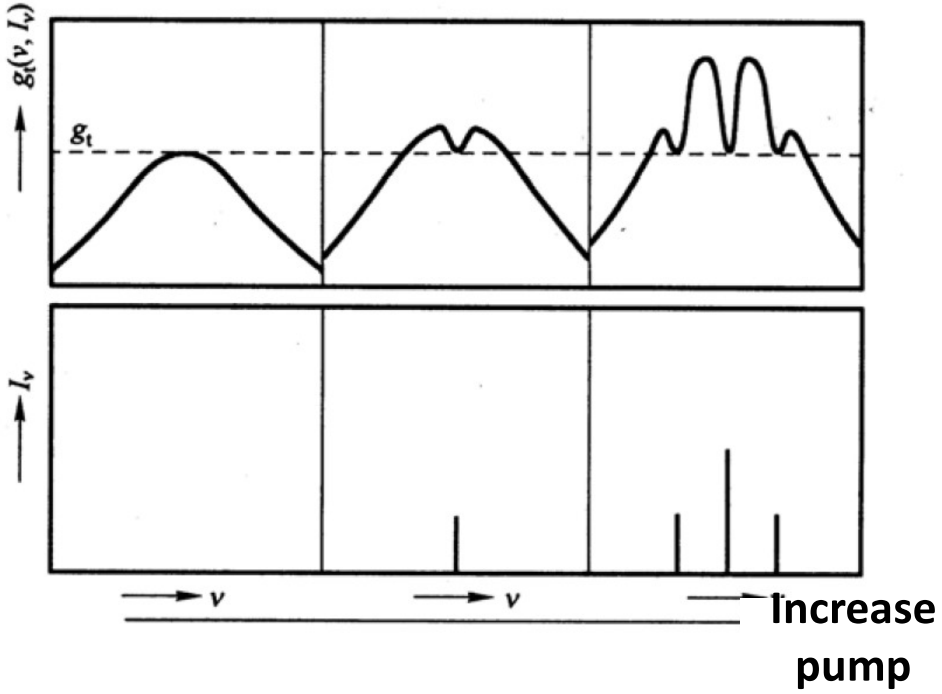
- The spatial hole burning usually occurs in solid state lasers, therefore are of multi longitudinal modes, **around the gain peak**.
- In gas lasers, the particles move fast, thus the spatial hole burning can hardly form, so usually have one single mode lasing.
- In ring lasers with isolator, the electric field is traveling wave rather than standing wave, so there is no spatial hole burning, leading to one single mode lasing as well.

Homo. Broadening medium

- Different transverse modes compete to lasing as well. Note that the threshold gain of each transverse mode is different.
- There is transverse spatial hole burning as well. Because the spatial distribution of different transverse mode is different, it uses different spatial carrier populations. When the pump is strong enough, there will be many modes oscillating simultaneously.
- Both longitudinal and transverse multi-modes compete for carrier populations, **leading to the output power fluctuations.**

Inhomo. Broadening medium

- In inhomo broadening medium, the lasing on one longitudinal mode only saturates/clamps the gain at its own mode frequency, and does not impact the gain at other frequencies. This allows the inhomo broadening medium oscillates on multi-mode almost independently. Therefore, spectral hole burning will be formed on the gain profile.

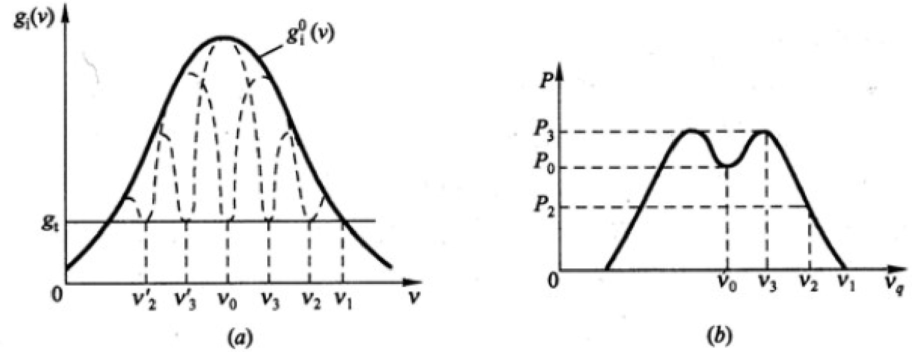


- ✓ If the mode spacing is too small, the holes will have some overlap with each other, leading to the mode competition due to the sharing of the same group carriers.

Lamb dip

❑ In the laser with Doppler inhomo. broadening medium, the output power increases when the longitudinal mode frequency is tuned towards the central frequency (gain peak). However, when the mode frequency is close to the central frequency, the power will decrease instead. Then, a dip is formed on the power spectrum, which is called **Lamb dip**.

✓ When the mode frequency $v_m != v_0$, the light interacts with two groups of carriers with velocity $+v_z$ and $-v_z$. Both groups contribute to the lasing power.



✓ When the mode frequency $v_m = v_0$, only one group of carrier with velocity $v_z=0$ interacts with the light, and contribute to the lasing power. Although the gain is highest, but the contributed carriers are the smallest. Therefore, the lasing power decreases, leading to the Lamb dip on the power spectrum.

✓ Lamb dip is a famous technique to stablize the laser frequency.

✓ The Lam dip disappears when the gas pressure increases, due the homo broadening from particle collisions.