Principles of Lasers

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Chapter 7

Continuous Wave Laser Behavior

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Chapter 7_L20

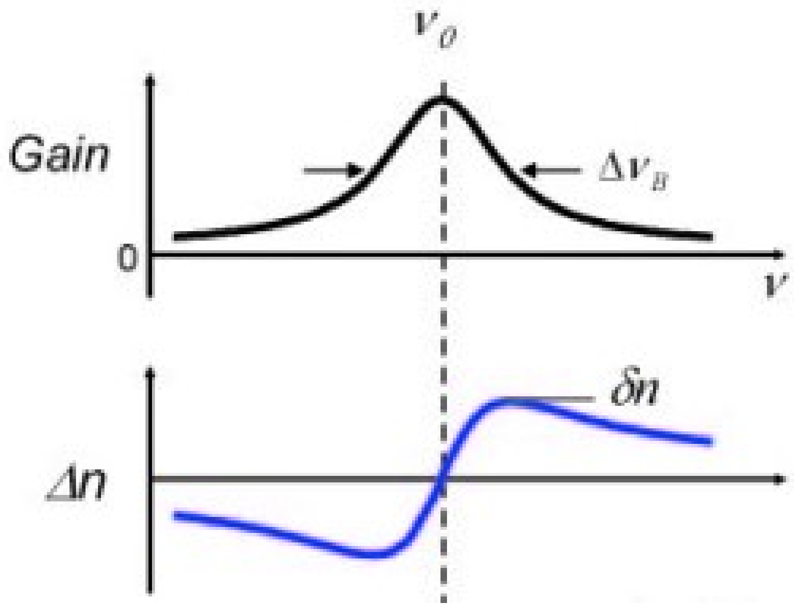
- Frequency pulling effect
- Laser linewidth and noise

Frequency pulling effect

❑ From the Kromer-Kronig relation, the refractive index around the gain peak varies considerably. That is, the dispersion (different frequency has different refractive index) is strong, and this dispersion increases with increasing gain.

$$\mathrm{n_r}(v) = n_{r0} + \Delta n_r(v)$$

n_{r0} is at g=0



❑ For homo. broadening medium, the refractive change is given by

$$\Delta n_r(v) = \frac{c(v - v_0)}{2\pi v \Delta v_H} g_H(v, I_v)$$

Δv_H is broadening linewidth

$g_H(v, I_v)$ is saturated gain

Frequency pulling effect

- In the passive cavity, the resonance frequency is as follows, and the adjacent modes have equal FSR.

$$\boxed{\mathrm{v}_{\mathrm{q0}} = \mathrm{q}\frac{c}{2n_{r0}L}}$$

- In the active cavity with gain medium, the resonance frequency with dispersion becomes

$$\boxed{\mathrm{v}_{\mathrm{q}} = \mathrm{q}\frac{c}{2n_{r}(v_q)L}}$$

- The resonance difference is

$$\boxed{\begin{aligned} v_q - v_{q0} &= q\frac{c}{2L}\left(\frac{1}{n_{r0}+\Delta n_r(v_q)} - \frac{1}{n_{r0}}\right) \\ &= q\frac{c}{2n_{r0}L}\left(\frac{-\Delta n_r(v_q)}{n_{r0}+\Delta n_r(v_q)}\right) \\ &= \frac{-\Delta n_r(v_q)}{n_{r0}+\Delta n_r(v_q)} v_{q0} \approx -\frac{\Delta n_r(v_q)}{n_{r0}} v_{q0} \end{aligned}}$$

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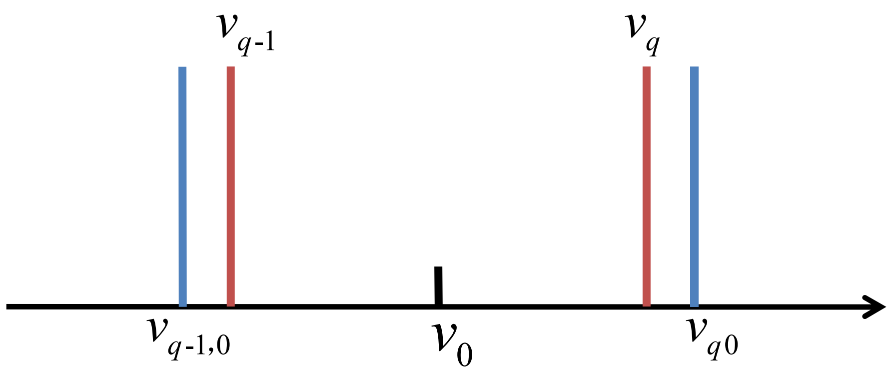
Frequency pulling effect

❑ **Frequency pulling effect:** The resonance frequency of active cavity is more close to the central frequency of the gain medium than that of passive cavity, due to the dispersion effect.

$$\boxed{\begin{array}{l}\text{For } \nu_{q0} > \nu_0, \Delta n_r(\nu_q) > 0, \nu_q < \nu_{q0} \\ \text{For } \nu_{q0} < \nu_0, \Delta n_r(\nu_q) < 0, \nu_q > \nu_{q0}\end{array}}$$



Frequency pulling effect

- Above threshold, the gain is clamped at threshold g_{th}=alpha,

$$\begin{aligned} \nu_q - \nu_{q0} &= -\frac{c(\nu_q - \nu_0)}{2\pi \nu_q \Delta\nu_H n_{r0}} \nu_{q0}\alpha \\ &\approx -\frac{(\nu_q - \nu_0)}{\Delta\nu_H}\left(\frac{c\alpha}{2\pi n_{r0}}\right) \\ &= -\frac{(\nu_q - \nu_0)}{\Delta\nu_H}\left(\frac{1}{2\pi \tau_{\mathrm{p}}}\right) \\ &= -(\nu_q - \nu_0)\frac{\Delta\nu_C}{\Delta\nu_H} \end{aligned}$$

$$\boxed{\nu_q = \frac{\nu_{q0}\Delta\nu_H + \nu_0\Delta\nu_C}{\Delta\nu_H + \Delta\nu_C}}$$

$\Delta\nu_C$ is the width of the resonance peak in passive cavity

$\Delta\nu_{\mathrm{H}}$ is the gain broadening linewidth

- Note that the frequency pulling is usually very small, because the gain broadening linewidh (say ~100 GHz) is much larger than the resonance peak linewidth (say ~1MHz).

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Chapter 7_L20

- Frequency pulling effect
- Laser linewidth and noise

Laser linewidth

- In the passive cavity, the spectral linewidth of the resonance peak (Lorentzian shape) is

$$\boxed{\Delta v_c = \frac{1}{2\pi\tau_p} = \frac{1}{2\pi} v_g \alpha_T}$$

- In the active cavity, the gain reduces the net loss coefficient,

$$\boxed{\alpha_{Ta} = \alpha_T - g}$$

- Above threshold, the gain equals to the loss, and hence the net loss coefficient is zero, so the spectral linewidth of the laser mode seems to become zero.

- The physical image is that the stimulated emission energy compensated the loss energy in the cavity, and because the stimulated emitted photons are coherent with the same phase, so the output wave is continuous and infinite, leading to a linewidth of zero.

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Laser linewidth

- However, in practice the laser linewidth can never be zero, due to the spontaneous emission, which generates incoherent photons. The spontaneous emission leads to the random fluctuation of the phase (dominate) and of the amplitude (small).
- Considering the spontaneous emission, the steady state of the photon rate equation gives

$$\boxed{\begin{aligned} &\mathrm{v_g} g N_P - \frac{N_P}{\tau_p} + \beta \frac{N_2}{\tau_{sp}} = 0 \Rightarrow \\ &\mathrm{v_g}\left(\alpha_T - g\right) = \beta \frac{N_2}{\tau_{sp} N_P} \Rightarrow \\ &\mathrm{v_g} \alpha_{T\mathrm{a}} = \beta \frac{N_2}{\tau_{sp} N_P} \end{aligned}}$$

- That is, the gain at and above threshold is actually smaller than the cavity loss, owing to the spontaneous emisison. Therefore, the net loss coefficient is not zero.

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Laser linewidth

❑ The laser linewidth determined by the spontaneous emission is given by

$$\boxed{\Delta \nu_L = \beta \frac{N_2}{2\pi N_P \tau_{sp}}}$$

❑ Or equivalently given by the following formula, which is the famous Schowlow-Townes linewidth,

$$\Delta \nu_L = \frac{N_2}{\Delta N} \frac{2\pi h \nu_L}{P} \left(\Delta \nu_c\right)^2$$

P is the output power

❑ The Schowlow-Townes linewidth gives the linewidth limit of lasers, in practice the laser linewidth is much larger than this value due to other noise sources.

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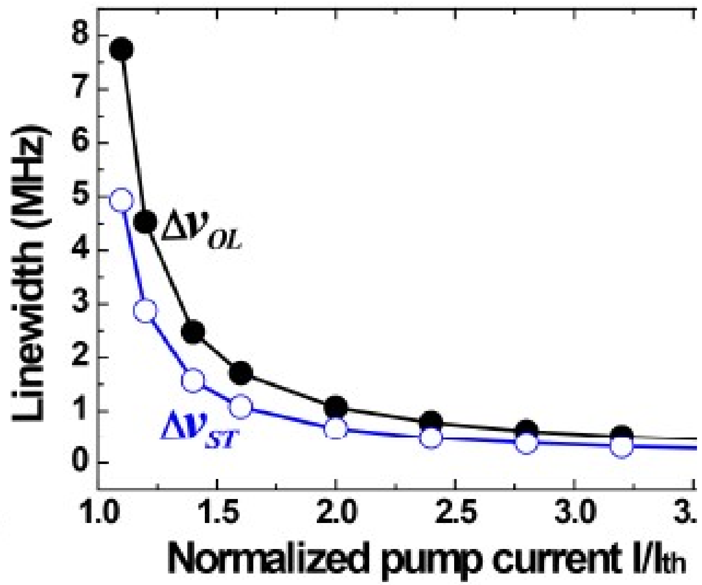
Laser linewidth

- The laser linewidth narrows with increasing output power. It is understandable because higher power means more coherent photons, stimulated emission becomes stronger than spontaneous emission.
- On the other hand, the laser linewidth can be also reduced by increasing the cavity length or reducing the cavity loss.



Examples 7.9

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Phase noise

- In practice, the laser frequency is not constant, but fluctuates randomly due to the laser noise. Short-term noise (high frequency) is due to the quantum noise (spontaneous emission, carrier noise). Long-term noise (low frequency) is due to technical noise (temperature fluctuation, driven current noise, mirror vibration…). Phase noise is also frequency nosie,

- The laser electric field with random phase:

$$\boxed{E(t) = A_0 \sin\left[2\pi v_L t + \varphi_{ran}(t)\right]}$$

- Phase variation and frequency variation relation:

$$\boxed{\Delta v_L(t) = \frac{d\varphi_{ran}(t)}{dt}}$$

- The instantaneous laser frequency:

$$\boxed{v_L(t) = v_L + \Delta v_L(t)}$$

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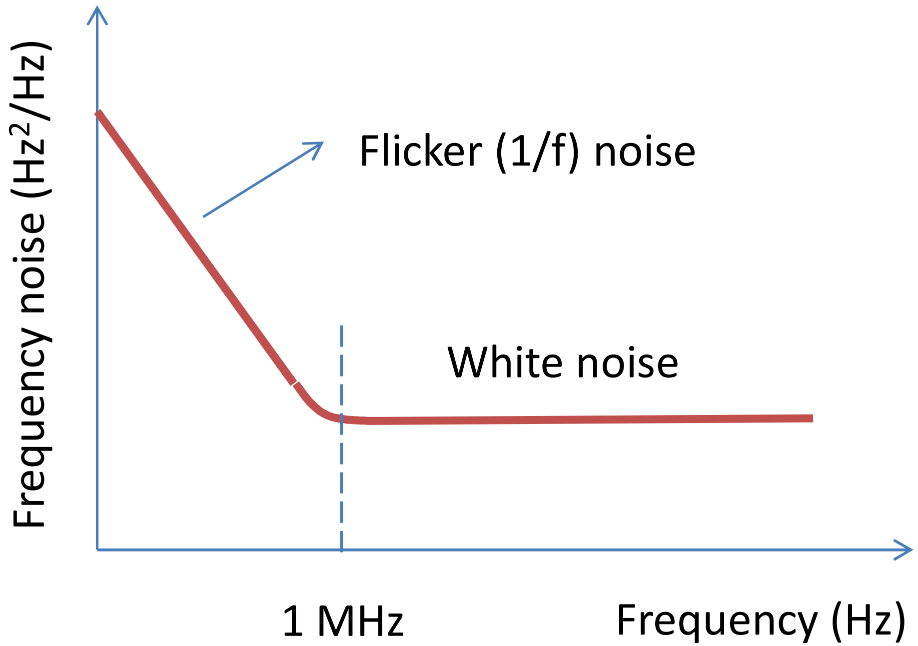
Phase noise

- Spontanenous emission noise is a kind of white noise ($S_v(f)$=const.) in the frequency (electrical) domain, which leads to a Lorentzian-shape spectral (optical) linewidth. The relation between the laser linewidth and the white frequency noise is

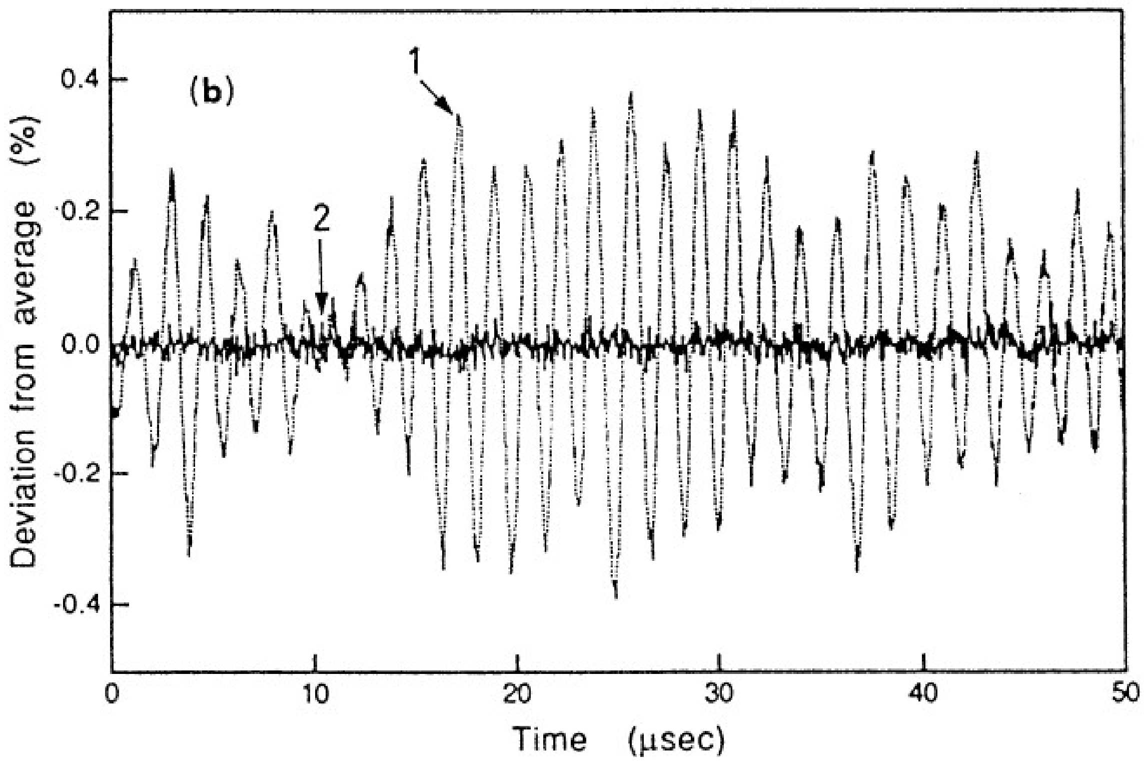
$$\boxed{\Delta v_L = \pi S_v(f)}$$



- Besides the white noise due to quantum noise, there is flicker noise at low frequency due to technical noises.

Intensity noise

❑ The sponaneous emission not only perturbs the phase of the electric field, but also alters the field amplitude. This induces the intensity noise, and the thus the laser output power varies instanenously on the time scale.



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Chapter 7_L21

- Mode selection
- Frequency fluctuation and stablization

Transverse mode selection

- TEM_{00} mode is usually desirable.
- In case the diffraction loss dominates the total loss. Cavities can be designed to select the fundamental mode, because the diffraction loss increases with the transverse mode index.
- The mode selection requirement is

$$g^{00} > \alpha_T{}^{00} \text{ and } g^{01} < \alpha_T{}^{01},\ g^{10} < \alpha_T{}^{10}$$

- The larger the diffraction loss difference between the high-order mode and the fundamental mode, the easier the mode selection.

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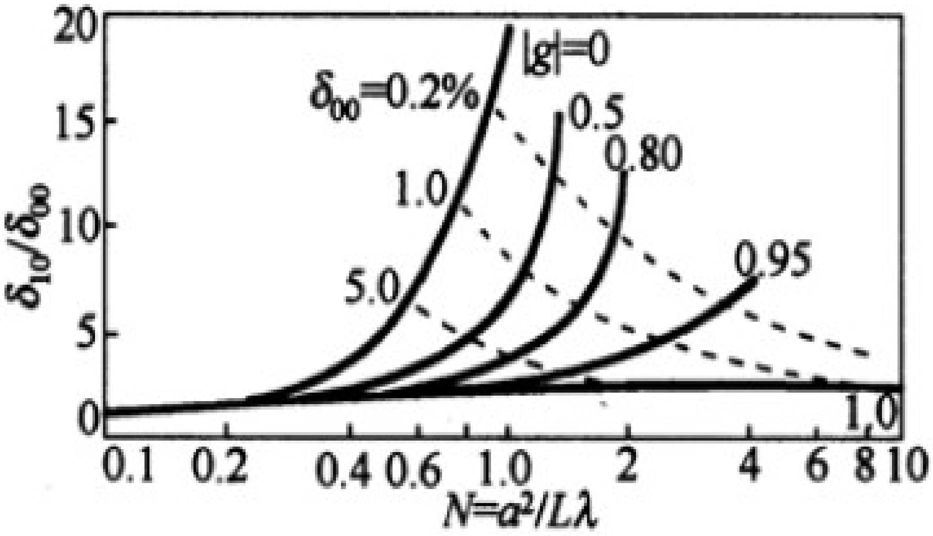
Transverse mode selection

- The diffraction loss decreases with Fresnel number, while the mode discrimination increases.
- The confocal cavity has the lowest loss, and the largest discrimination ability. The concentric cavity and the plane cavity have the highest loss, but the smallest dscrimination ability.
- However, the absolute value of diffraction loss in confocal cavity is small, while it is large in concentric cavity and plane cavity. Therefore, it is easier to select mode in the latter case.



Mode discrimination versus Fresnel number

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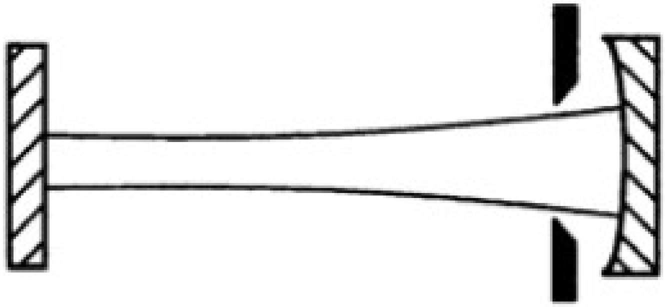
Transverse mode selection

❑ Several methods can be used to select the transverse mode

✓Design a proper aperture inside the cavity



✓Design a proper parameter g and Fresnel number N of the cavity, to make the loss of higher-order modes higher than the corresponding gain.

✓Design a proper unstable cavity, since the mode discrimination is large.

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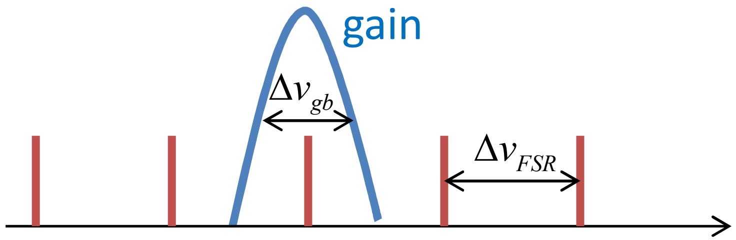
Longitudinal mode selection

- The principle is to reduce the loss of the selected longitudinal mode, while increase the loss of other modes.
- Short cavity method
  - ✓ This is appliable when the gain broadening linewidth is not large, like in gas (He-Ne) lasers (say 1.7 GHz). Piezo-electric transducer is needed to adjust the cavity length carefully, with accuracy on the order of wavelength.

$$\boxed{\begin{aligned}&\Delta\nu_{FSR} \geq \Delta\nu_g / 2 \Rightarrow \\ &L \leq \frac{c}{\Delta\nu_g}\end{aligned}}$$

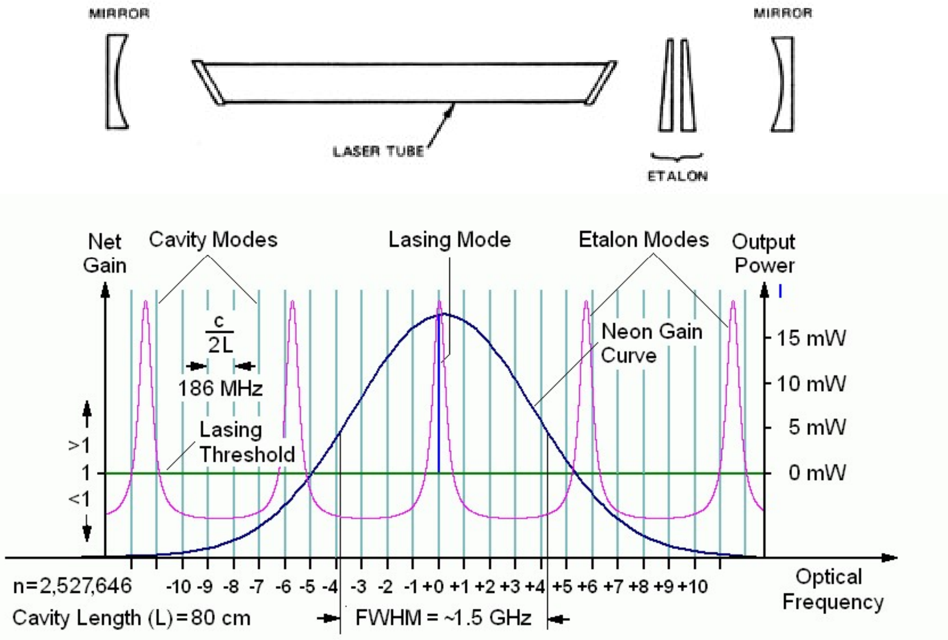


- Ring cavity with isolator
  - ✓ The traveling wave together with homogeneous broadening medium wont form any hole burning effects.

Longitudinal mode selection

- F-P inteferometer method

  ✓ Align one transmission peak of the F-P etalon with the selected longitudinal mode in the laser cavity, while others are not aligned.



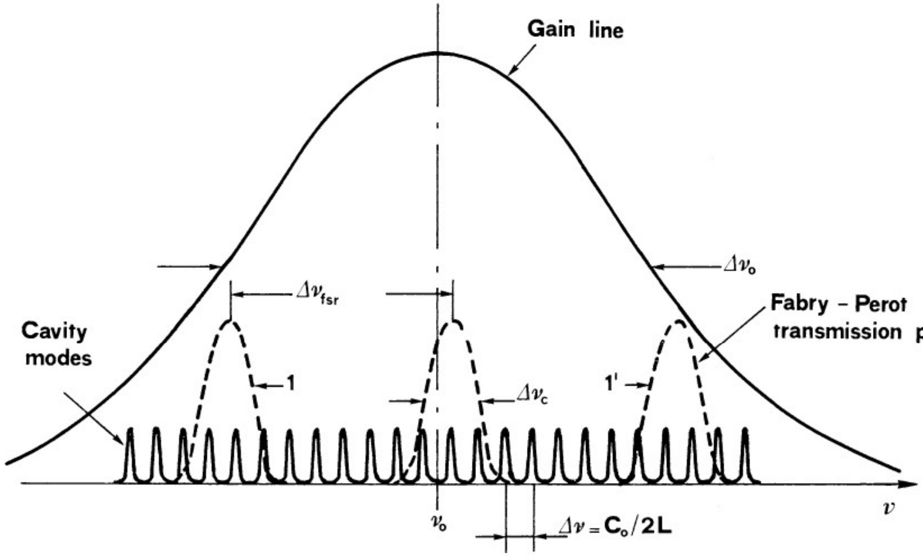
Intracavity Etalon for Line Selection in a Single Mode HeNe Laser

- Other methods include using grating, external cavity to select the mode.

Longitudinal mode selection

- The etalon half peak linewidth must be smaller the mode spacing of the laser cavity.

$$\frac{1}{2}\Delta v_{et} \leq FSR_{ls}$$



- The etalon FSR must be larger than half gain linewidth

$$\mathrm{FSR}_{\mathrm{et}} \geq \frac{1}{2}\Delta v_g$$

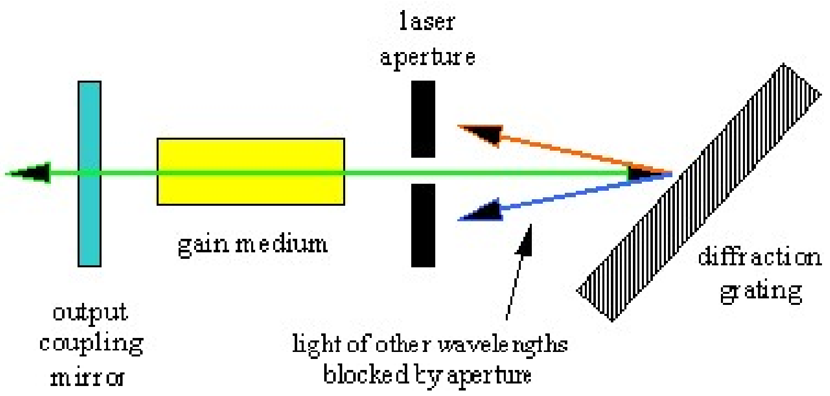
- Consequently, the requirement

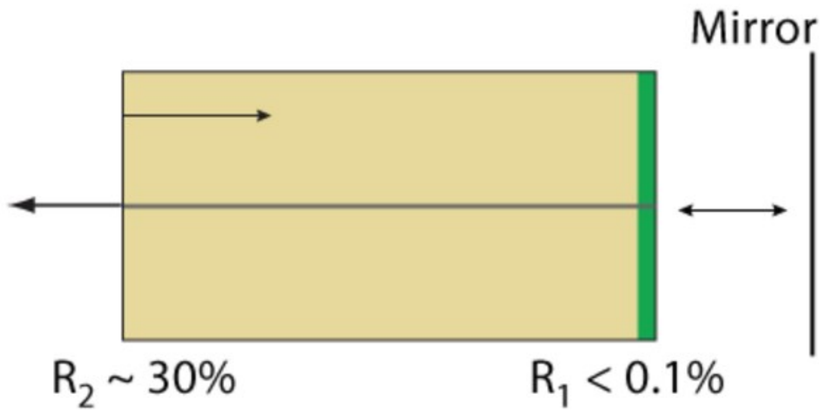
$$\frac{1}{2}\Delta v_g \leq FSR_{et} \leq 2F \times FSR_{ls}$$

$$L \leq 2F\frac{c}{n_r \Delta v_g}$$

Longitudinal mode selection

- Other methods include using grating, external cavity to select the mode.





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Chapter 7_L21

- Mode selection
- Frequency fluctuation and stablization

Frequency fluctuation

- In practice, the laser frequency is not constant, but fluctuates due to cavity length change and to refractive index change, arising from environmental changes including thermal fluctuation, acoustic vibration etc.

$$\boxed{\begin{aligned} \nu_m &= m\frac{c}{2n_r L} \\ \Delta\nu_m &= -\nu_m\left(\frac{1}{n_r}\Delta n_r + \frac{1}{L}\Delta L\right) \end{aligned}}$$

- The frequency stability is usually characterized by

$$\boxed{\left|\frac{\Delta\nu_m}{\bar{\nu}_m}\right|}$$

Examples 7.10

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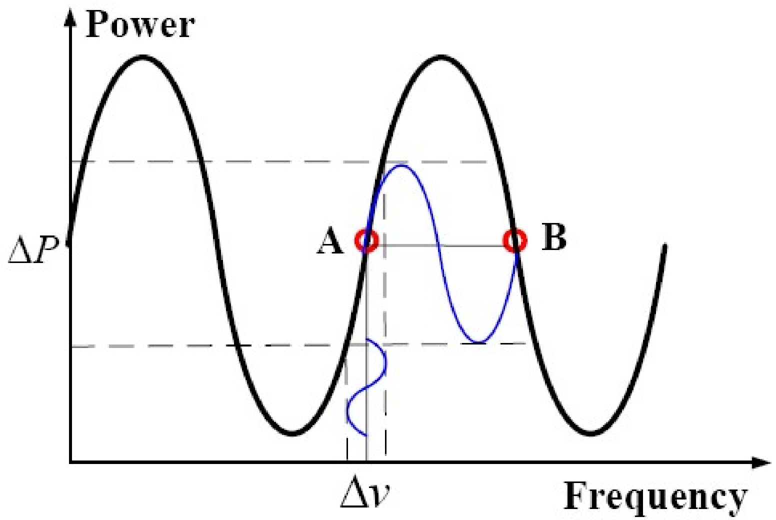
Frequency stabilization

- In order to stablize the laser frequency, the feedback method is usually used, which includes a frequency discriminator.



- It is usually hard to precisely track the laser frequency change, here the frequency discriminator can convert the frequency variation into power variation, and then is detected.

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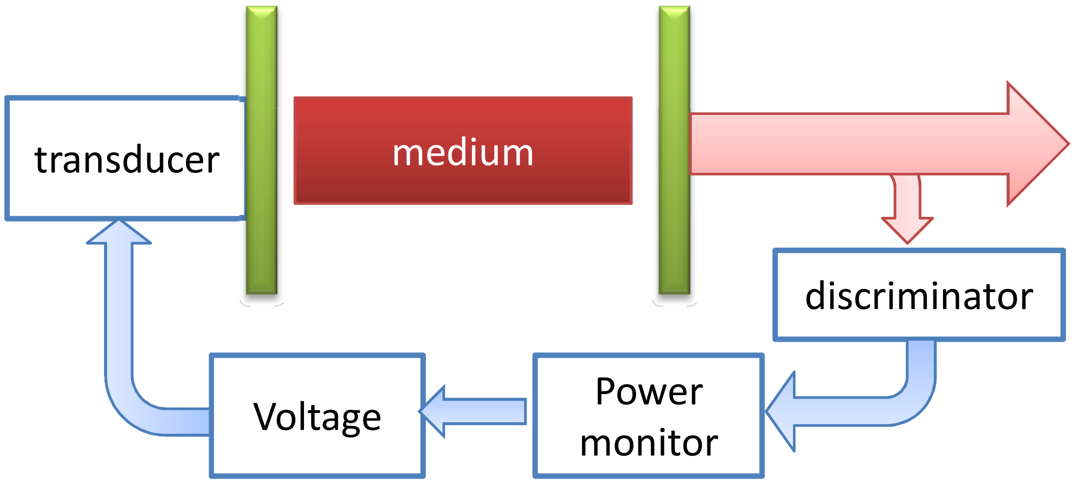
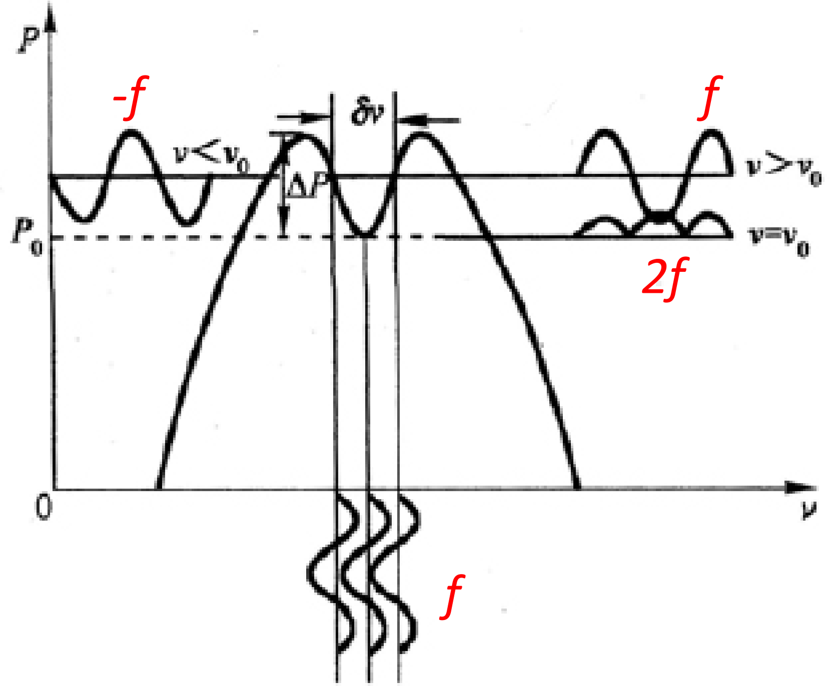
Frequency stabilization

- The basic feedback system includes a frequency discriminator and a piezoelectric transducer, which can change the cavity mirror position by controlling the voltage on it.



Frequency stabilization

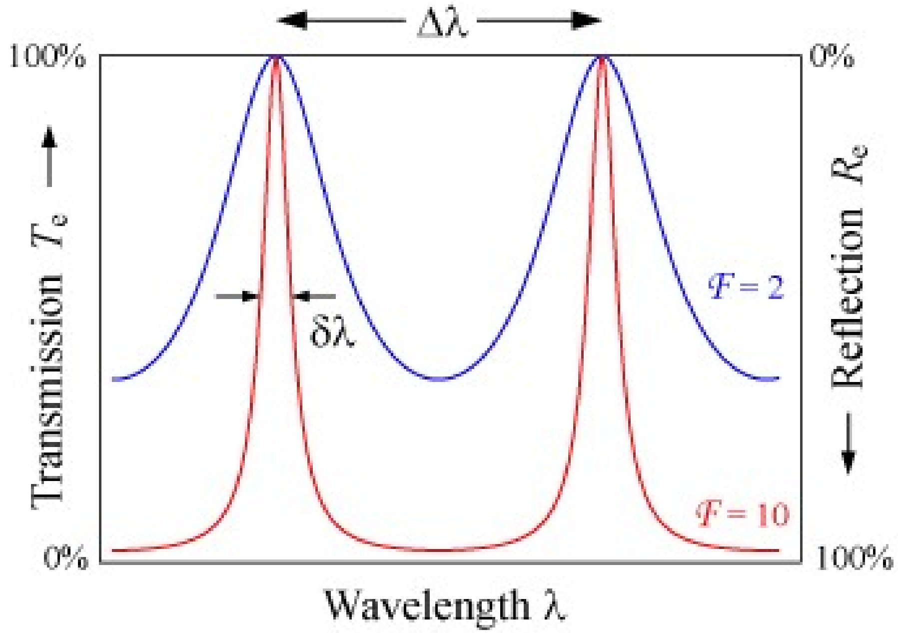
- One method is to use the Lamp dip as the frequency discriminator.



- The laser is modulated at a low acoustic frequency f.
- If the lasing frequency $v=v_0$, the lasing power modulation frequency is 2f.
- If the lasing frequency $v>v_0$, the lasing power modulation frequency is f, and it is in phase with the voltage modulation. Then, the transducer will enlarge the cavity length to move the lasing frequency to v_0.
- If the lasing frequency $v<v_0$, the lasing power modulation frequency is f, and it is out of phase with the voltage modulation. Then, the transducer will reduce the cavity length to move the lasing frequency to v_0.

Frequency stabilization

❑ Another method is to use the Fabry-Perot inteferometer as the frequency discriminator.



❑ The laser frequency is operated around the half maximum position of one transmission peak.

Homework

7.3

7.4

7.5

7.6

7.8

7.16

7.17

7.18