

Semiconductor Optoelectronic Devices

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Part I

Semiconductor electrical and optical properties

---- Electrical properties

----**Opical properties**



Macroscopic & microscopic pictures

	Macro	Micro
EM wave	E field and M field	Photons
Crystal response	Permittivity (介电常数) & permeability (磁导率)	Photon absorption & generation Carrier recombination & generation
EM wave standpoint	Wave absorption and gain	
Semiconductor standpoint	e-h generation and recombination	



□ Dielectric permittivity



- Dielectric displacement vector (C/m²) vs. electric field (V/m)

$$D(t) = \varepsilon E(t)$$

- The dielectric permittivity (F/m) of the medium is

$$\varepsilon = \varepsilon_r \varepsilon_0$$

ε_r relative permittivity

$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m vacuum permittivity

- The refractive index of the medium is

$$n_r = \sqrt{\varepsilon_r}$$



- In the frequency domain,

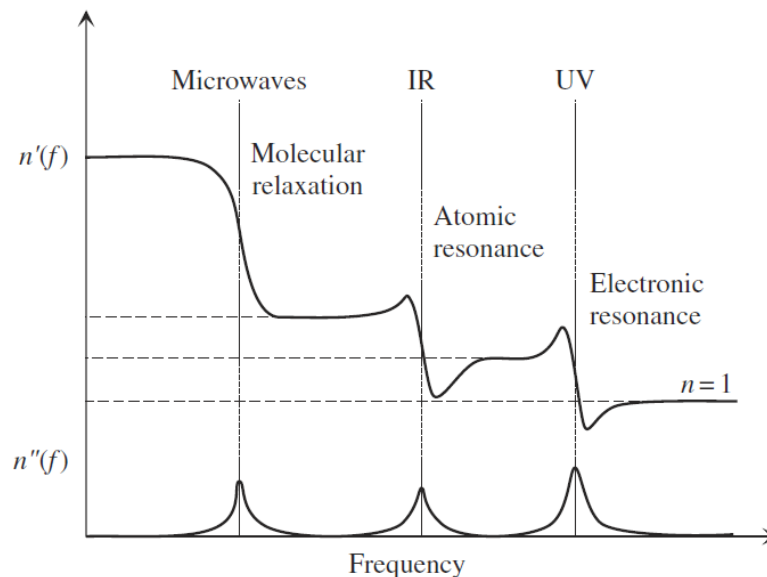
$$D(\omega) = \varepsilon(\omega)E(\omega) = (\varepsilon' - j\varepsilon'')E(\omega)$$

$$n_r(\omega) = n_r'(\omega) - jn_r''(\omega)$$

$n_r'(\omega)$ determine refractive index

$n_r''(\omega)$ determine loss

- EM waves interact with carriers of molecules, atoms, and electrons in a frequency range, respectively. Close to each cutoff frequency, the loss has a peak

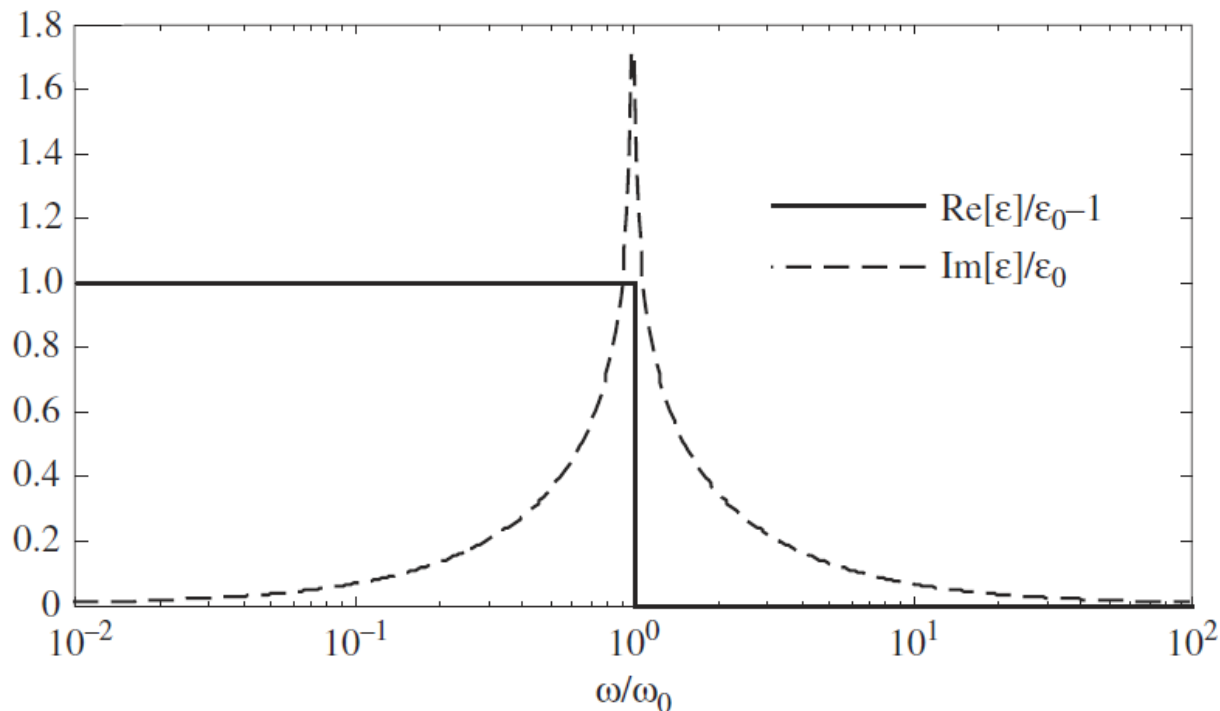


- The real and imaginary parts of the permittivity are coupled with each other, by the **Kramer-Kronig relations**:

$$\epsilon'(\omega) = \epsilon_0 + \frac{2}{\pi} \cdot \mathcal{P} \int_0^{\infty} \frac{\Omega \epsilon''(\Omega)}{\Omega^2 - \omega^2} d\Omega$$
$$\epsilon''(\omega) = -\frac{2\omega}{\pi} \cdot \mathcal{P} \int_0^{\infty} \frac{\epsilon'(\Omega) - \epsilon_0}{\Omega^2 - \omega^2} d\Omega,$$

- Once the loss spectrum (imaginary part) is measured, the refractive index spectrum of the medium can be derived.
- A variation of the real part (due, e.g., to the electrooptic effect) implies a variation of the imaginary part (i.e., of the absorption), and vice versa. This has a significant impact on the spurious frequency modulation (chirp) of electroabsorption modulators (EAMs) and of directly modulated lasers

- In general, a variation of the real part corresponds to a peak in material losses, and vice versa.



Imaginary part of permittivity in the presence of a step in the real part.

Example 2.1



- In a lossy medium the electric field propagating in the z direction is given by

$$\begin{aligned} E(z) &= E_0 e^{-jkz} = E_0 \exp\left(-j \frac{\omega}{c_0} n_r z\right) \\ &= E_0 \exp\left[-j \frac{\omega}{c_0} (n_r' - j n_r'') z\right] \\ &= E_0 \exp(-\alpha_e z) \exp(-j\beta z) \end{aligned}$$

- Here k is the **complex propagation constant**.

$$k = \beta - j\alpha_e \text{ with}$$

$$\text{The propagation constant (rad/m) } \beta = \frac{\omega}{c_0} n_r'$$

$$\text{The field attenuation (/m) } \alpha_e = \frac{\omega}{c_0} n_r''$$



- The light power/intensity is proportional to the square of the electric field magnitude.

$$|E(z)| = |E_0| \exp(-\alpha_e z)$$

$$P(z) \propto |E(z)|^2 = P(0) \exp(-2\alpha_e z)$$

- Here the **light absorption coefficient** (/m) is defined as

$$\alpha = 2\alpha_e$$



- The dielectric displacement vector are parallel with the electric field only in isotropic materials. In an anisotropic medium, they have different directions, and can be described by introducing the permittivity as a matrix.

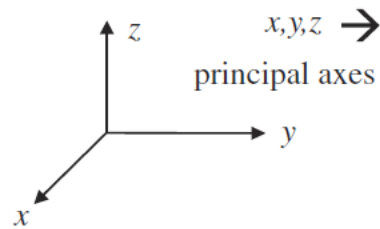
$$D = \varepsilon E \rightarrow$$

$$\begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = \begin{pmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$



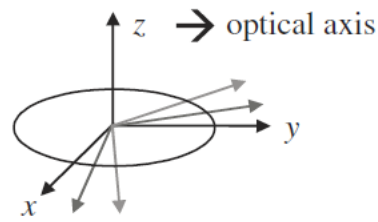
- Depending on the difference of the matrix diagonal values, the medium can be classified into biaxial, uniaxial, and isotropic medium. **All cubic semiconductors are isotropic.**

$$\begin{pmatrix} \epsilon_{xx} & 0 & 0 \\ 0 & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{pmatrix}$$



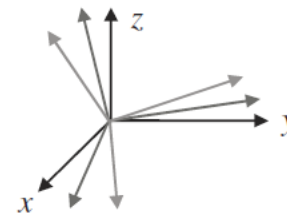
Biaxial

$$\begin{pmatrix} \epsilon_o & 0 & 0 \\ 0 & \epsilon_o & 0 \\ 0 & 0 & \epsilon_e \end{pmatrix}$$



Uniaxial

$$\begin{pmatrix} \epsilon & 0 & 0 \\ 0 & \epsilon & 0 \\ 0 & 0 & \epsilon \end{pmatrix}$$



Isotropic

Permittivity matrix in the principal reference frame (above) and optical axes (below) in a biaxial, uniaxial and isotropic medium. In the isotropic case the principal axes are arbitrary.



□ EM wave-semiconductor interaction



- ❑ At a microscopic view, photons of the EM field interacts with electrons & holes in the semiconductor medium. The interaction can be visualized as a collision or scattering process. The charged particles in motion is subject to the **Coulomb force** (EM wave electric field) and to the **Lorentz force** (EM wave magnetic field).
- ❑ The useful semiconductor response is dominated by the ability of radiation to cause band-to-band carrier transitions with corresponding *emission* or *absorption* of a photon.

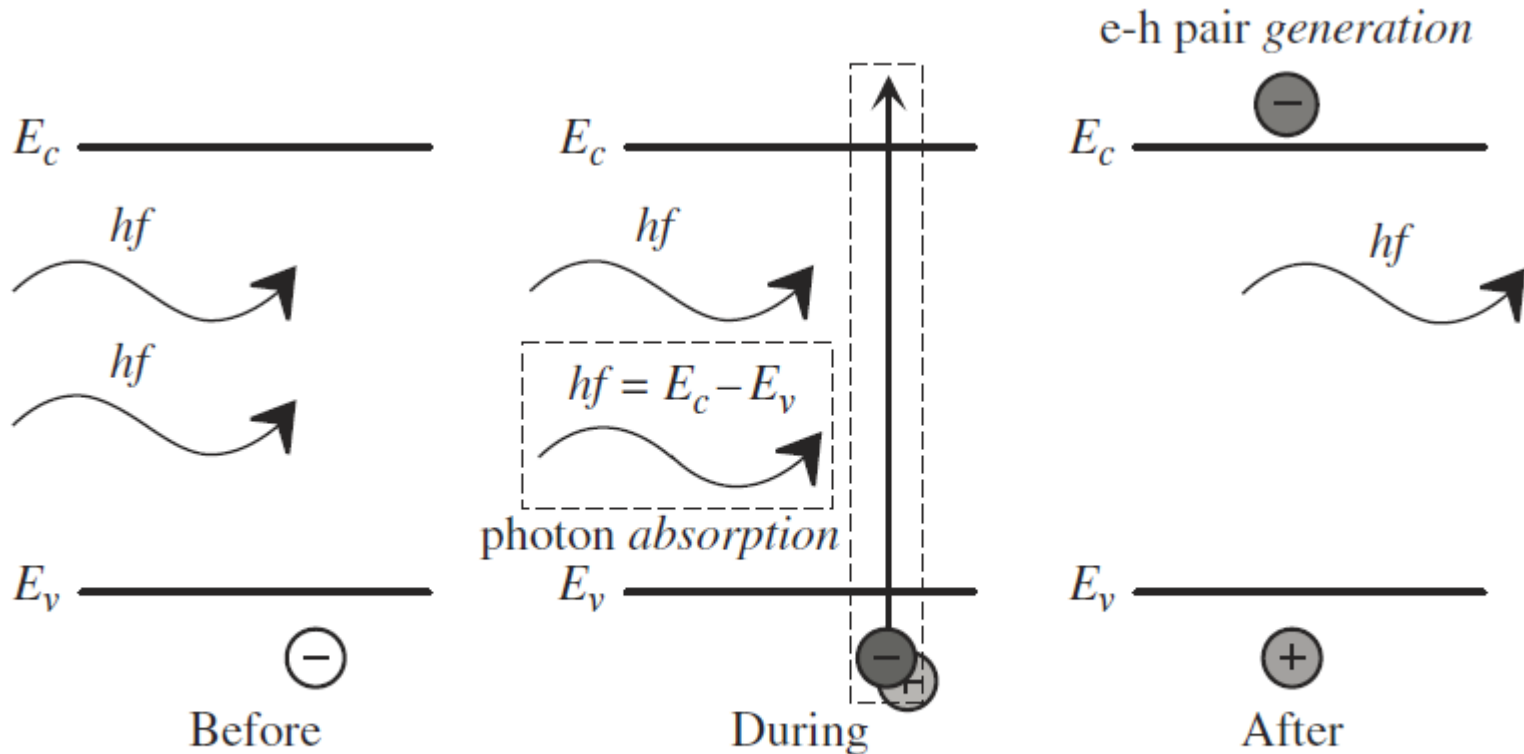


□ Depending on the difference between the photon energy and the band gap, there are three cases of the EM wave-semiconductor interaction.

1. $E_{ph} < E_g$, as in RF, microwaves, and far infrared (FIR): the interaction is weak and does not involve band-to-band processes, but only the dielectric response and interband processes (e.g., the so-called free electron/hole absorption);
2. $E_{ph} \approx E_g$ and $E_{ph} > E_g$, as in near infrared (NIR), visible light, and ultraviolet (UV): light interacts strongly through band-to-band processes leading to the *generation–recombination* of e-h pairs and, correspondingly, to the *absorption–emission* of photons;
3. $E_{ph} \gg E_g$, as for X rays: high-energy ionizing interactions take place, i.e., each photon causes the generation of a high-energy e-h pair, which generates a large number of e-h pairs through avalanche processes. This case is exploited in high-energy particle and radiation detectors, but it is outside the scope of our discussion.

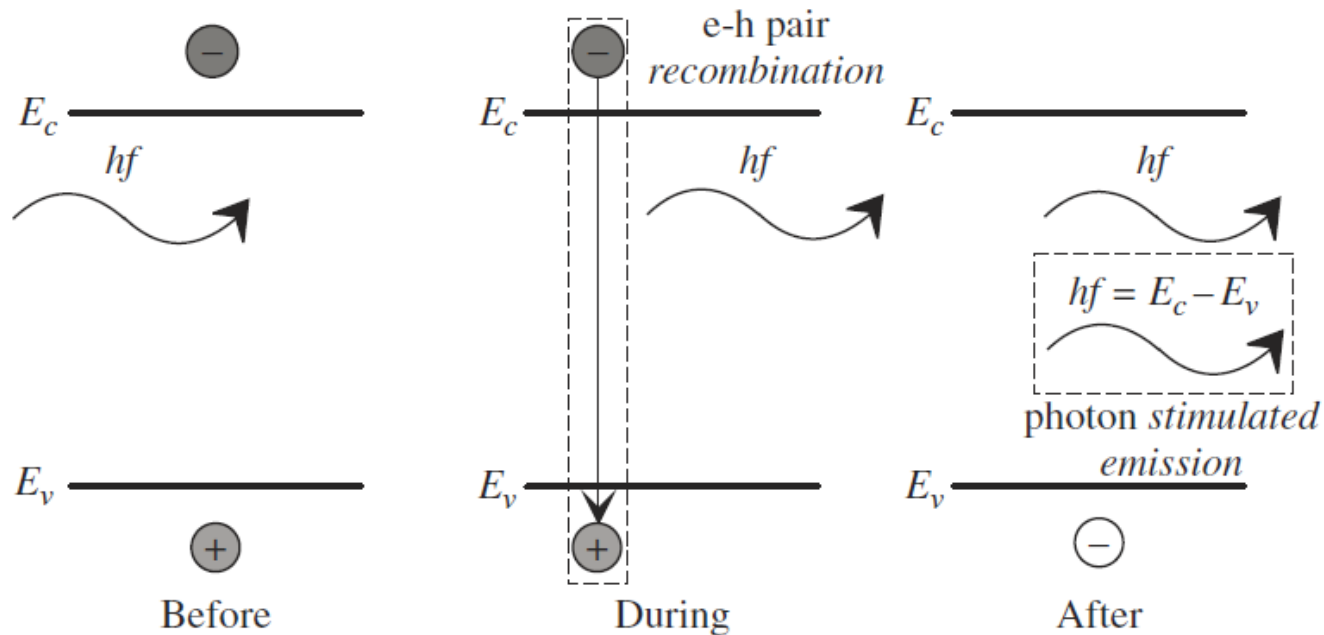


1. *Photon absorption* (and *e-h pair generation*): the photon energy (momentum conservation is discussed later) is supplied to a valence band electron, which is promoted to the conduction band, leaving a free hole in the valence band. Because of the absorption process, the EM wave decreases its amplitude and power.



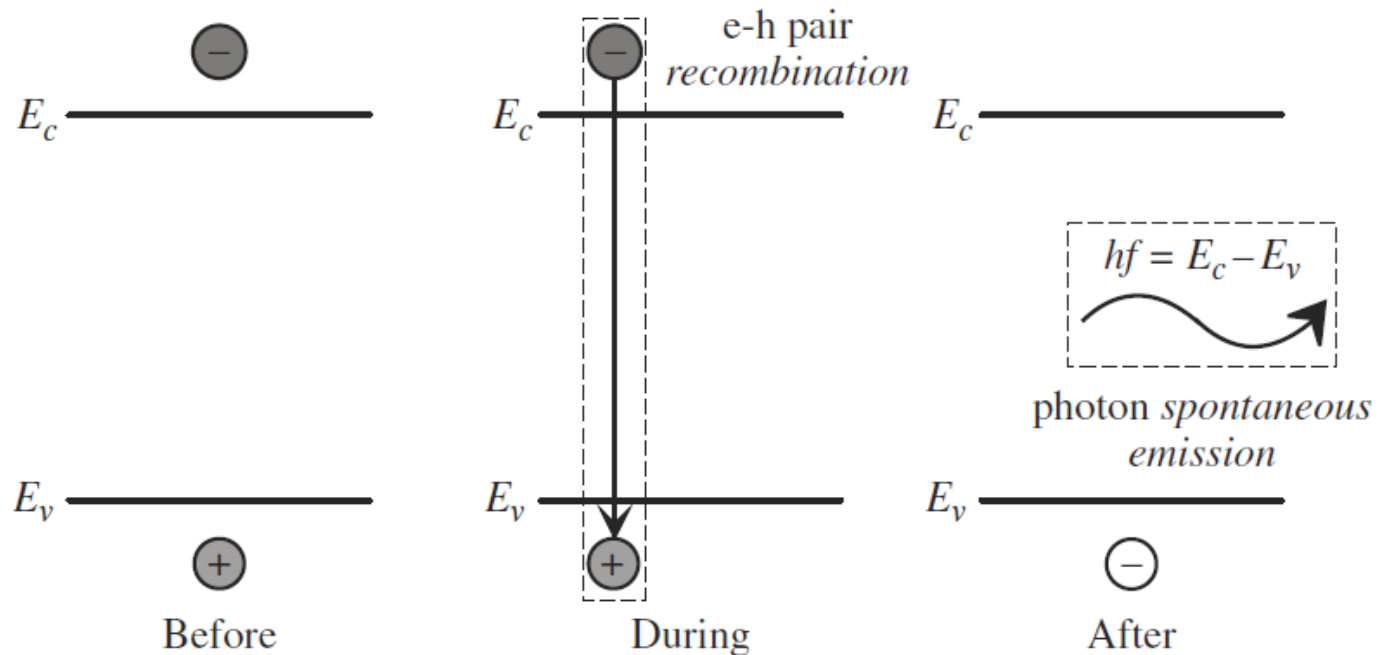
Photon absorption.

2. *Photon stimulated emission (and e-h pair recombination)*: a photon stimulates the emission of a second photon with the same frequency and wavevector; the e-h pair recombines to provide the photon energy. The emitted photon is coherent with the stimulating EM wave, i.e., it increases the amplitude of the EM field and the EM wave power through a gain process.



Photon stimulated emission.

3. *Photon spontaneous emission (and e-h pair recombination)*: a photon is emitted spontaneously; the e-h pair recombines to provide the photon energy. Since the emitted photon is incoherent, the process does not imply the amplification of an already existing wave, but rather the excitation of an EM field with a possibly broad frequency spectrum (if many photons are incoherently emitted in a specific bandwidth).



Photon spontaneous emission.

- It is obvious that the interaction between photons and carriers has to follow the energy and momentum conservation rules. For direct bandgap semiconductors,

$$\sum_i E_{i,\text{before}} = \sum_i E_{i,\text{after}}, \quad \sum_i \underline{k}_{i,\text{before}} = \sum_i \underline{k}_{i,\text{after}},$$

1. Absorption of a photon through a *direct process*, i.e., a process involving only electrons and photons. One has

$$E_f = E_i + E_{ph}, \quad \underline{k}_f = \underline{k}_i + \underline{k}_{ph},$$

where E_f is the final energy of the electron in the conduction band, E_i is the initial energy of the electron in the valence band, E_{ph} is the energy of the absorbed photon (similarly for momenta). We can equivalently write

$$\Delta E_{fi} = E_f - E_i = E_{ph} = hf, \quad \Delta \underline{k}_{fi} = \underline{k}_f - \underline{k}_i = \underline{k}_{ph} = \frac{2\pi}{\lambda} \hat{k}_{ph},$$

implying that the changes in energy and momentum of the electron are supplied by the absorbed photon.

2. Spontaneous or stimulated emission of a photon through a *direct process*, i.e., a process involving only electrons and photons. One has:

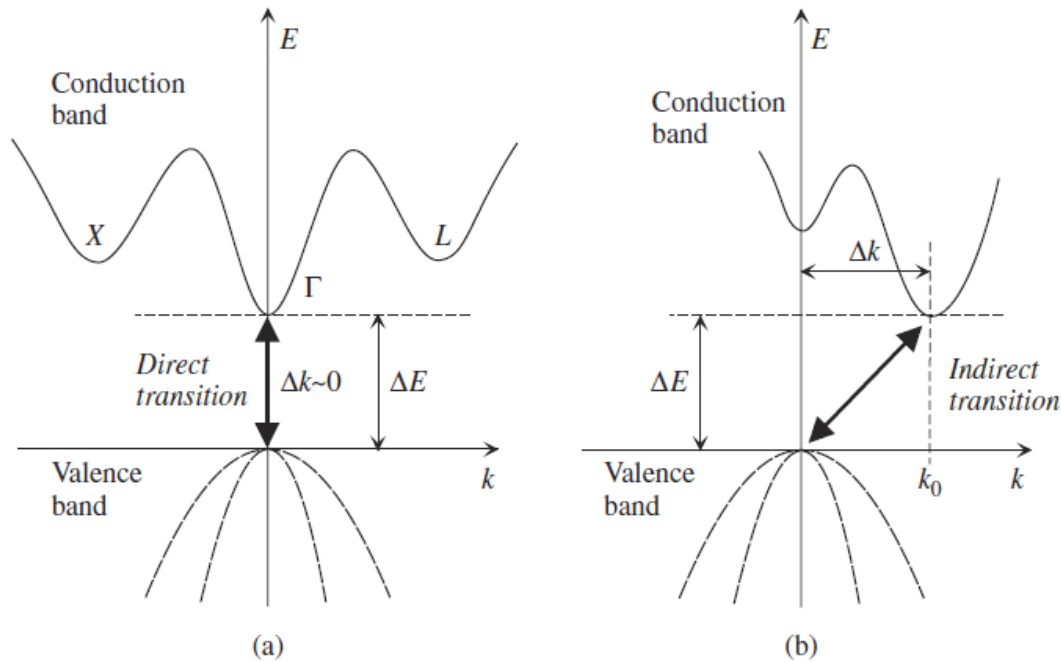
$$E_f + E_{ph} = E_i, \quad \underline{k}_f + \underline{k}_{ph} = \underline{k}_i,$$

where E_f is the final energy of the electron in the valence band, E_i is the initial energy of the electron in the conduction band, E_{ph} is the energy of the emitted photon (similarly for momenta). We can equivalently write

$$\Delta E_{if} = E_i - E_f = E_{ph} = hf, \quad \Delta \underline{k}_{if} = \underline{k}_i - \underline{k}_f = \underline{k}_{ph} = \frac{2\pi}{\lambda} \hat{k}_{ph},$$

implying that the changes in energy and momentum of the electron are supplied to the emitted photon.

- The energy of photons is larger than the bandgap, while the momentum of photons is small ($p=2\pi\hbar k$). In direct bandgap semiconductors, the momentum change is small before and after the interaction, But this change is large in indirect bandgap semiconductors.



Simplified bandstructure of (a) a direct-bandgap semiconductor (GaAs, InP, ...) showing radiative transitions involving negligible momentum difference; (b) an indirect-bandgap semiconductor (Si, SiC, [Ge]), showing radiative transitions involving a large momentum difference.

- In indirect bandgap semiconductors, the electron momentum change can be on the order

$$k_e \approx \frac{\pi}{a}$$

a is lattice constant, on the order of 0.5 nm.

say the photon wavelength is 1 micron,

the momentum difference is

$$\frac{k_e}{k_{ph}} \approx \frac{\lambda}{2a} \approx 10^3$$

- Therefore, the interaction has to be aided by involving **phonons**, which has low energy but comparatively large momentum.
- Phonons are **quantized elastic waves** carrying mechanical energy (in the form of sound or heat) through the crystal. It has its own energy and momentum

$$E_\phi = hf; \quad k_\phi = 2\pi / \lambda$$



□ A photon can be emitted or absorbed through the help of an emitted or absorbed phonon.

1. Absorption of a photon through an *indirect process*, i.e., a process involving electrons and photons but also phonons. The phonon can be absorbed or emitted.
 - a. Phonon absorption:

$$E_f = E_i + E_{ph} + E_\phi, \quad \underline{k}_f = \underline{k}_i + \underline{k}_{ph} + \underline{k}_\phi,$$

where E_f is the final energy of the electron in the conduction band, E_i is the initial energy of the electron in the valence band, E_{ph} is the energy of the absorbed photon and E_ϕ is the energy of the absorbed phonon (similarly for momenta).

- b. Phonon emission:

$$E_f + E_\phi = E_i + E_{ph}, \quad \underline{k}_f + \underline{k}_\phi = \underline{k}_i + \underline{k}_{ph},$$

where E_f is the final energy of the electron in the conduction band, E_ϕ is the energy of the emitted phonon, E_i is the initial energy of the electron in the valence band, E_{ph} is the energy of the absorbed photon (similarly for momenta).



2. Emission of a photon through an *indirect process*, i.e., a process involving electrons and photons but also phonons. The phonon can be absorbed or emitted.
- a. Phonon absorption:

$$E_f + E_{ph} = E_i + E_\phi, \quad \underline{k}_f + \underline{k}_{ph} = \underline{k}_i + \underline{k}_\phi,$$

where E_f is the final energy of the electron in the conduction band, E_{ph} is the energy of the emitted photon, E_i is the initial energy of the electron in the valence band, and E_ϕ is the energy of the absorbed phonon (similarly for momenta).

- b. Phonon emission:

$$E_f + E_{ph} + E_\phi = E_i, \quad \underline{k}_f + \underline{k}_{ph} + \underline{k}_\phi = \underline{k}_i,$$

where E_f is the final energy of the electron in the conduction band, E_{ph} is the energy of the emitted photon, E_ϕ is the energy of the emitted phonon, E_i is the initial energy of the electron in the valence band (similarly for momenta).



- Because phonon-assisted photon-carrier interactions are many-body processes, the efficiency is low. It is hardly possible to make lasers or LEDs based on stimulated or spontaneous emissions. However, photodetectors based on photo absorption process can still be realized, with inferior performance than direct-bandgap semiconductors.
- The energy and momentum difference between the final and the initial electron state is

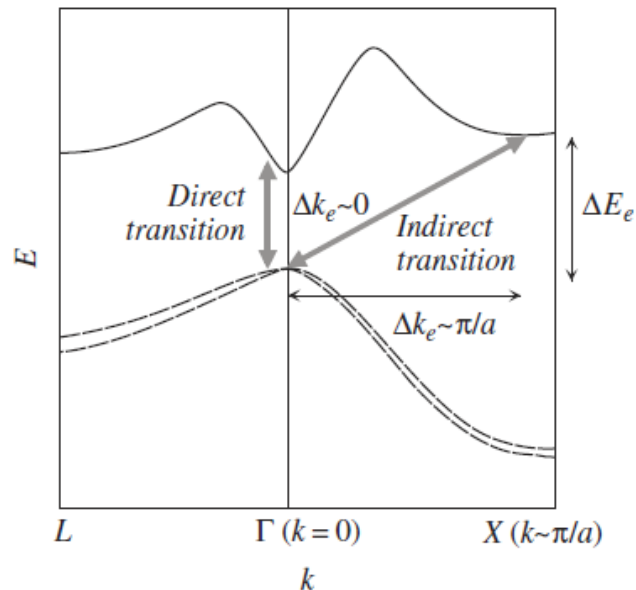
$$\Delta E_{fi} = E_{ph} \pm E_{\phi} \approx E_{ph}$$

$$\Delta k_{fi} = k_{ph} \pm k_{\phi} \approx \pm k_{\phi}$$



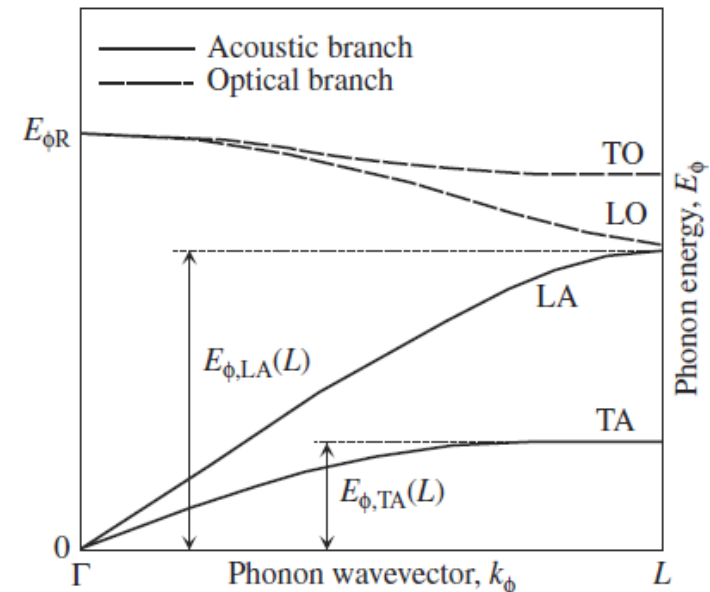
- The indirect conduction band minima are located at or near the boundary of the FBZ, so the momentum difference is

$$\Delta k_{fi} \approx \pm \frac{\pi}{a}$$



Direct and indirect optical transitions in a semiconductor: momentum and energy variations vs. the bandstructure parameters.

- ❑ The phonons are mechanical waves of **longitudinal vs. transverse**, depending on the displacement with respect to the propagation direction.
- ❑ The phonon mode is **acoustical phonon**, if the nearby atoms oscillate in phase, which has low energy. It is **optical phonon**, if the nearby atoms oscillate out of phase, which has high energy.
- ❑ Since the crystal space supporting lattice vibrations is the same periodic space in which the electron motion takes place, the phonon dispersion relation is defined in the same reciprocal space as for the electrons, i.e., the FBZ, see Fig. 2.10. The maximum value for the phonon momentum within the FBZ is therefore of the order of π/a .
- ❑ The maximum optical phonon energy is 60 meV for Si, 35 meV for GaAs; The maximum acoustic phonon energy is 20 meV for Si, 10 meV for GaAs.
- ❑ Note that the acoustical phonon energy is zero at the Gamma point.



- The phonon wavelength for the maximum phonon momentum with $k=\pi/a$ is

$$k_{\phi} = \frac{2\pi}{\lambda} = \frac{\pi}{a} \Rightarrow$$

$$\lambda = 2a$$

- The maximum phonon energy is

$$E_{\phi} = h \frac{v_s}{\lambda} = h \frac{v_s}{2a}$$

$$\text{Example: } \frac{E_{\phi}}{E_{ph}} \approx \frac{1}{30} \text{ for } 2a=1\text{nm, photon wavelength}=1 \text{ micron}$$

- Note that two or more phonons can be involved in the interaction process, rather than only one.

Example 2.2



- For direct bandgap semiconductors, the energy and momentum conservation requires that for a given photon energy, the initial and final electron states are uniquely determined.
- Assume the momentum is k , then the initial and final state energies are

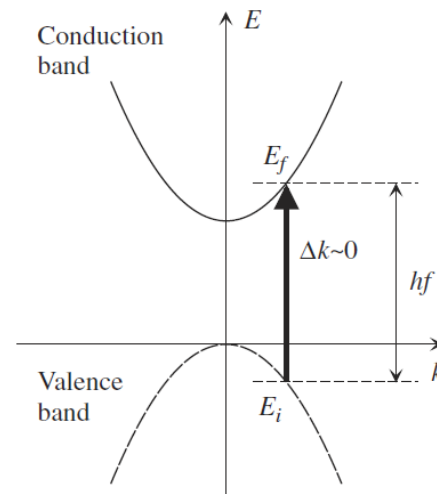
$$E_i = E_v - \frac{\hbar^2 k^2}{2m_h^*}; \quad E_f = E_c + \frac{\hbar^2 k^2}{2m_n^*}$$

- The energy conservation requires

$$E_{ph} = \hbar\omega = E_f - E_i = E_g + \frac{\hbar^2 k^2}{2m_r^*}$$

with the reduced mass

$$\frac{1}{m_r^*} = \frac{1}{m_n^*} + \frac{1}{m_h^*}$$



Energy conservation in a vertical transition (generation due to phonon absorption).



□ Therefore, the momentum and the state energies are uniquely given by

$$k = \frac{\sqrt{2m_r^*}}{\hbar} \sqrt{\hbar\omega - E_g}$$

$$E_i = E_v - \frac{m_r^*}{m_h^*} (\hbar\omega - E_g)$$

$$E_f = E_c + \frac{m_r^*}{m_n^*} (\hbar\omega - E_g)$$

□ The minimum absorbed photon energy is equal to the bandgap E_g , and

$$k = 0$$

$$E_i = E_v$$

$$E_f = E_c$$

