Semiconductor Optoelectronic Devices

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Part II Semiconductor Lasers

Lecture 20

- Intensity modulation response

Differential rate equations

- When the laser is perturbed by a small-signal source, such as the current modulation or noises, the dynamic changes in the carrier and photon densities away from their steady-state values are small. Take differentials of the rate equations:

$$d\left[\frac{dN}{dt}\right] = \frac{\eta_i}{qV}dI - \frac{1}{\tau_{\mathrm{n}}}dN - v_g g dN_p - v_g N_p dg$$

$$d\left[\frac{dN_p}{dt}\right] = \left(\Gamma v_g g - \frac{1}{\tau_p}\right)dN_p + \Gamma v_g N_p dg + \frac{\Gamma\beta}{\tau_{sp}}dN$$

Differential rate equations

- The gain variation

$$dg = adN - a_p dN_p$$

$$a = \frac{\partial g}{\partial N}$$

$$a_p = -\frac{\partial g}{\partial N_p}$$

- The gain model

$$g = \frac{a_0}{1+\varepsilon N_p}\left(N - N_{tr}\right)$$

$$a == \frac{a_0}{1+\varepsilon N_p}$$

$$a_p = \frac{\varepsilon g}{1+\varepsilon N_p}$$

a is the differential gain

a_0 is the nominal differential gain

ε is the gain compression factor

a_p takes account the gain compression

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Differential rate equations

- Using the differential gain, the differential rate equation becomes

$$\frac{d}{dt}[dN] = \frac{\eta_i}{qV} dI - \gamma_{NN} dN - \gamma_{NP} N_p$$

$$\frac{d}{dt}\left[dN_p\right] = \gamma_{PN} dN - \gamma_{PP} dN_p$$

where

$$\gamma_{NN} = \frac{1}{\tau_{\mathrm{n}}} + v_g a N_p, \quad \gamma_{NP} = \frac{1}{\Gamma\tau_p} - \frac{\beta N}{\tau_{sp} N_p} - v_g a_p N_p$$

$$\gamma_{PN} = \frac{\Gamma}{\tau_{sp}} + \Gamma v_g a N_p, \quad \gamma_{PP} = \frac{\Gamma\beta N}{\tau_{sp} N_p} + \Gamma v_g a_p N_p$$

- Where the following relation is used

$$\frac{1}{\tau_p} - \Gamma v_g g = \frac{\Gamma\beta N}{\tau_{sp} N_p}$$

- Well above threshold,

$$\gamma_{NN} = \frac{1}{\tau_{\Delta N}} + v_g a N_p, \quad \gamma_{NP} = \frac{1}{\Gamma\tau_p} - v_g a_p N_p$$

$$\gamma_{PN} = \Gamma v_g a N_p, \quad \gamma_{PP} = \Gamma v_g a_p N_p$$

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Differential rate equations

- In the form of matrix, the differential rate equations become

$$\frac{d}{dt}\begin{bmatrix} dN \\ dN_p \end{bmatrix} = \begin{bmatrix} -\gamma_{NN} & -\gamma_{NP} \\ \gamma_{PN} & -\gamma_{PP} \end{bmatrix}\begin{bmatrix} dN \\ dN_p \end{bmatrix} + \frac{\eta_i dI}{qV}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

- In this form, the current is seen as the driving term or forcing function. When we treat noise in semiconductor lasers, the current forcing function will be replaced by noise sources. If necessary, we could choose any other parameter to be the forcing function, like the mirror loss.

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IM response

- To obtain the small-signal responses dN(t) and $dN_p(t)$ to a sinusoidal current modulation dI(t), we assume solutions of the form

$$dI(t) = I_1 e^{j\omega t}$$
$$dN(t) = N_1 e^{j\omega t}$$
$$dN_p(t) = N_{p1} e^{j\omega t}$$

- The differential rate equation becomes

$$\begin{bmatrix} \gamma_{NN} + j\omega & \gamma_{NP} \\ -\gamma_{PN} & \gamma_{PP} + j\omega \end{bmatrix} \begin{bmatrix} N_1 \\ N_{p1} \end{bmatrix} = \frac{\eta_i I_1}{qV} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Small-signal IM response

- The solutions of the carrier and photon densities are

$$\begin{bmatrix} \gamma_{NN} + j\omega & \gamma_{NP} \\ -\gamma_{PN} & \gamma_{PP} + j\omega \end{bmatrix} \begin{bmatrix} N_1 \\ N_{p1} \end{bmatrix} = \frac{\eta_i I_1}{qV} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$N_1 = \frac{\eta_i I_1}{qV} \frac{1}{D} \begin{vmatrix} 1 & \gamma_{NP} \\ 0 & \gamma_{PP} + j\omega \end{vmatrix}$$

$$N_{p1} = \frac{\eta_i I_1}{qV} \frac{1}{D} \begin{vmatrix} \gamma_{NN} + j\omega & 1 \\ -\gamma_{PN} & 0 \end{vmatrix}$$

- The IM response

$$IM = \frac{N_{p1}}{\eta_i I_1 / (qV)} = \frac{\gamma_{PN}}{D}$$

$$= \frac{\gamma_{PN}}{\omega_R^2 - \omega^2 + j\omega\gamma}$$

- Where the relaxation resonance frequency and the damping factor are

$$\omega_R^2 = \gamma_{NP}\gamma_{PN} + \gamma_{NN}\gamma_{PP}$$

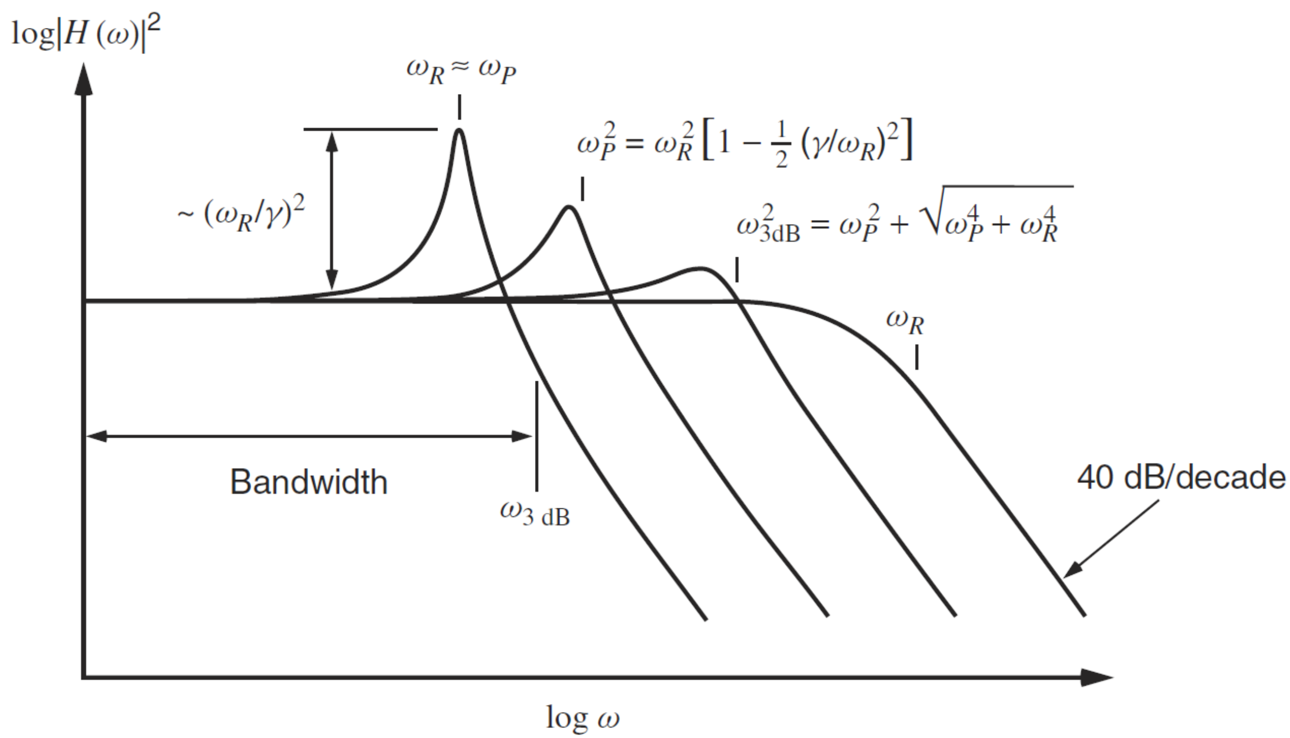
$$\gamma = \gamma_{NN} + \gamma_{PP}$$

Small-signal IM response

- The normalized IM response is described by the modulation transfer function

$$H(\omega)=\frac{IM(\omega)}{IM(0)}=\frac{\gamma_{PN}}{\omega_R^2-\omega^2+j\omega\gamma}\frac{\omega_R^2}{\gamma_{PN}}$$

$$=\frac{\omega_R^2}{\omega_R^2-\omega^2+j\omega\gamma}$$



Small-signal IM response

- The resonance frequency and the damping factor are

$$\omega_R^2 = \frac{v_g a N_p}{\tau_P} + \left[\frac{\Gamma v_g a N_P}{\tau_{\Delta N}} + \frac{\Gamma \beta R_{\mathrm{sp}}}{N_P \tau_n}\right]\left(1 - \frac{\beta \tau_n}{\tau_{\mathrm{sp}}}\right) + \frac{\beta}{\tau_{\mathrm{sp}} \tau_p}$$

$$\gamma = v_g a N_p \left[1 + \frac{\Gamma a_p}{a}\right] + \frac{1}{\tau_n} + \frac{\Gamma \beta R_{sp}}{N_P}$$

- The approximation is

$$\omega_R^2 \approx \frac{v_g a N_p}{\tau_P} \quad \text{(well above threshold)}$$

$$\gamma = K f_R^2 + \gamma_0$$

$$K = 4\pi^2 \tau_p \left[1 + \frac{\Gamma a_p}{a}\right] \text{ and } \gamma_0 = \frac{1}{\tau_{\Delta N}} + \frac{\Gamma \beta R_{\mathrm{sp}}}{N_p}$$

- The K-factor describes the damping of the response, and the offset describes the effective carrier lifetime.

Small-signal IM response

- The peak frequency is obtained at H'(w)=0, which is slightly smaller than the resonance frequency,

$$\omega_P^2 = \omega_R^2\left[1-\frac{1}{2}\left(\frac{\gamma}{\omega_R}\right)^2\right]$$

- The 3-dB frequency is obtained at $H^2(w)=1/2$,

$$\omega_{3dB}^2 = \omega_P^2 + \sqrt{\omega_P^4 + \omega_R^4}$$

- For weak damping,

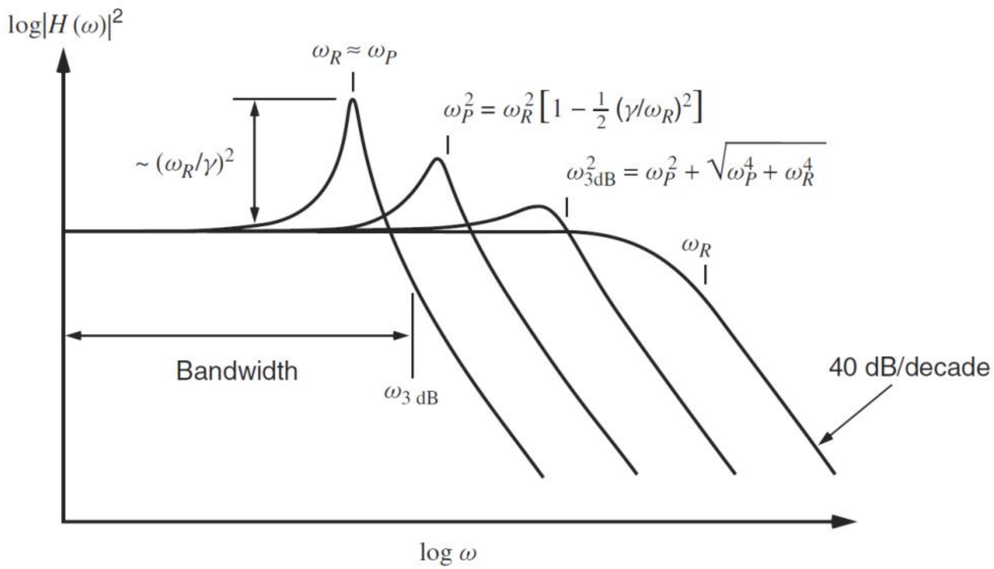
$$\omega_{3dB}^2 \approx \omega_R^2(1+\sqrt{2}), \quad (\gamma << \omega_R)$$

- The maximum 3-dB bandwidth is obtained at f_{3dB}'=0,

$$f_{3dB\_\max} = \frac{2\sqrt{2}\pi}{K}, \quad \left(\gamma / \omega_R = \sqrt{2}\right)$$

where

$$\omega_p = 0;\ \omega_R = \omega_{3dB};$$

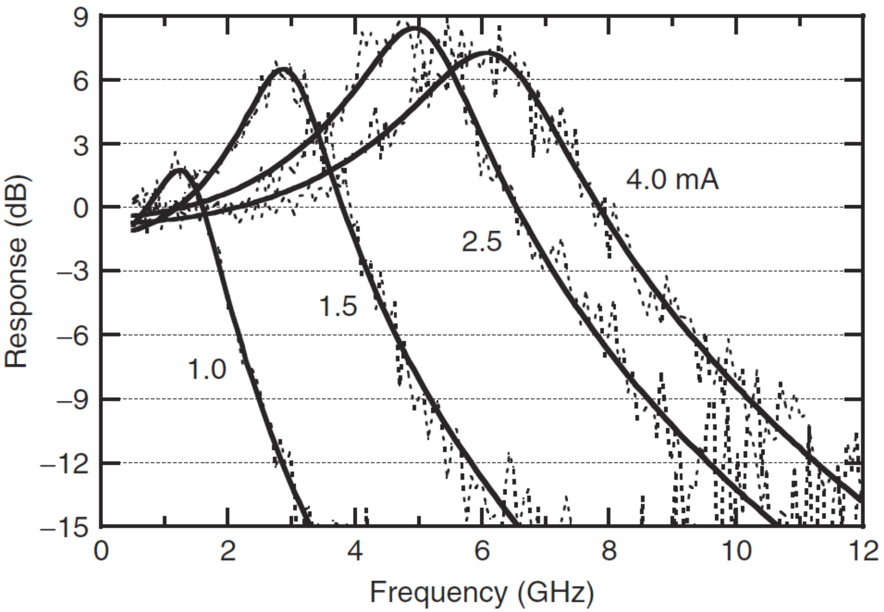


Small-signal IM response

- The power modulation response

$$\frac{P_1}{I_1} = \frac{\eta_0 h\nu N_{p1} V_p / \tau_p}{I_1}$$

$$= \eta_i \eta_0 \frac{h\nu}{q} H(\omega)$$



- The electrical power received is of more fundamental interest, thus in decibel, the normalized response is usually expressed as

$$10\log_{10}\left|H(\omega)\right|^2$$

- The phase shift of the modulation is

$$\angle H(\omega)$$

- Frequency chirp

Linewidth enhancement factor

- The linewidth enhancement factor describes the coupling between the carrier-induced variation of the real and imaginary parts of the complex susceptibility. It can be expressed using the complex refractive index:

$$\tilde{n} = n + jn_i;$$

$$\alpha = -\frac{\partial n / \partial N}{\partial n_i / \partial N} \approx -\frac{\Delta n}{\Delta n_i};$$

- The material gain (without gain compression) is linked with the imaginary part of the refractive index:

$$g = \frac{4\pi}{\lambda} n_i;$$

$$\alpha = -\frac{\partial n / \partial N}{\partial n_i / \partial N} = -\frac{4\pi}{\lambda}\frac{\partial n / \partial N}{\partial g / \partial N} \approx -\frac{4\pi}{\lambda}\frac{\Delta n}{\Delta g}$$

Linewidth enhancement factor

- The lasing frequency change is linked with the refractive index variation:

$$\frac{\Delta v}{v} = -\frac{\Delta n}{n};$$

$$\alpha = -\frac{\partial n / \partial N}{\partial n_i / \partial N} = -\frac{4\pi}{\lambda}\frac{\partial n / \partial N}{\partial g / \partial N} = \frac{4\pi n}{c}\frac{\partial v / \partial N}{\partial g / \partial N} = \frac{4\pi n}{c}\frac{\Delta v}{\Delta g}$$

- That is, the frequency will change if the gain changes:

$$\Delta v = \frac{\alpha v_g}{4\pi}\Delta g$$

- For the laser device, the lasing frequency change is described by the modal gain

$$\boxed{\Delta v = \Gamma \frac{\alpha v_g}{4\pi}\Delta g = \Gamma \frac{\alpha v_g}{4\pi} a \Delta N}$$

The rate equation of phase

- From the expression of the lasing frequency change, the phase variation of the electric field with respect to its threshold value is described by

$$\boxed{\frac{d\varphi}{dt} \equiv \Delta\omega = \Gamma \frac{\alpha v_g}{2}\left(g - g_{th}\right) = \frac{\alpha}{2}\left(\Gamma v_g g - \frac{1}{\tau_p}\right)}$$

Frequency modulation/chirp

- The variation in carrier density under intensity modulation leads to a corresponding change in the refractive index, and in turn changes the optical length of the cavity. Consequently, the lasing wavelength/frequency shifts back and forth. If this is desirable, it is called frequency modulation. However, in intensity modulation applications, this is not desirable and broadens the modulated spectrum of the laser, and then it is called frequency chirping.

Frequency chirp

- The gain variation can be obtained as:

$$\frac{dN_p}{dt} = \left(\Gamma v_g \frac{g}{1+\xi N_p} - \frac{1}{\tau_p} \right) N_p \Rightarrow$$

Under perturbation,

$$\frac{d\Delta N_p}{dt} = \left(\Gamma v_g \frac{g_0 + \Delta g}{1+\xi N_{p0}} - \frac{1}{\tau_p} \right)\left(N_{p0} + \Delta N_p \right) \approx \left(\Gamma v_g \frac{\Delta g}{1+\xi N_{p0}} \right) N_{p0} + \left(\Gamma v_g \frac{g_0}{1+\xi N_{p0}} - \frac{1}{\tau_p} \right) \Delta N_p$$

$$= \left(\Gamma v_g \frac{N_{p0}}{1+\xi N_{p0}} \right) \Delta g + \frac{1}{\tau_p} \left(\frac{1}{1+\xi N_{p0}} - 1 \right) \Delta N_p = \left(\Gamma v_g \frac{N_{p0}}{1+\xi N_{p0}} \right) \Delta g - \frac{1}{\tau_p} \left(\frac{\xi N_{p0}}{1+\xi N_{p0}} \right) \Delta N_p$$

$$\Rightarrow$$

$$\Gamma v_g \Delta g = \frac{1+\xi N_{p0}}{N_{p0}} \frac{d\Delta N_p}{dt} + \frac{1+\xi N_{p0}}{N_{p0}} \frac{1}{\tau_p} \left(\frac{\xi N_{p0}}{1+\xi N_{p0}} \right) \Delta N_p$$

$$= \frac{1+\xi N_{p0}}{N_{p0}} \frac{d\Delta N_p}{dt} + \frac{\xi}{\tau_p} \Delta N_p$$

Frequency chirp

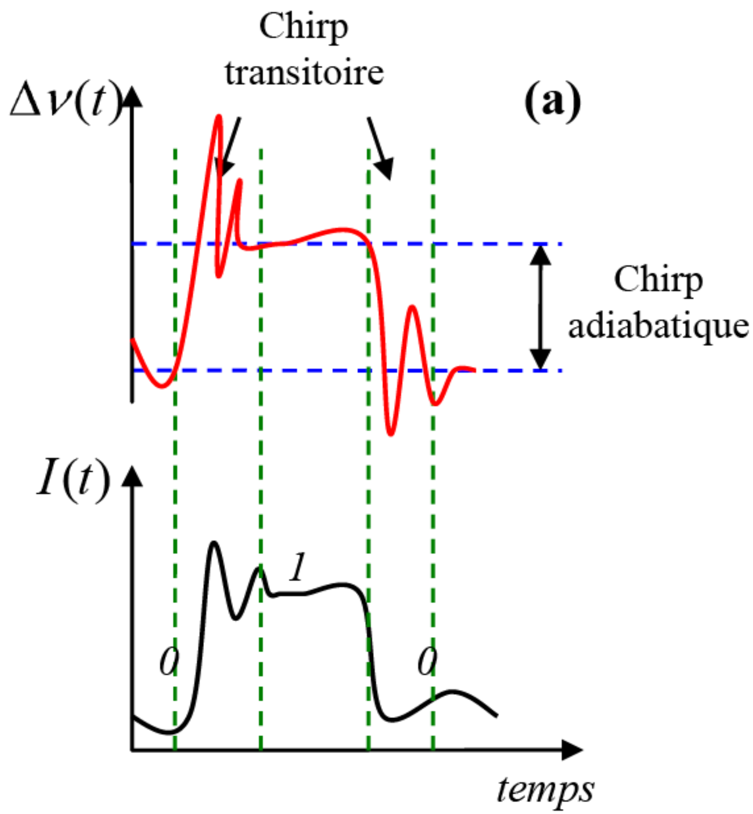
- Then, the frequency variation is

$$\Delta v = \Gamma \frac{\alpha v_g}{4\pi} \Delta g$$

$$= \frac{\alpha}{4\pi}\left(\frac{1+\xi N_{p0}}{N_{p0}} \frac{d\Delta N_p}{dt} + \frac{\xi}{\tau_p} \Delta N_p \right)$$

$$\approx \frac{\alpha}{4\pi}\left(\frac{1}{N_{p0}} \frac{d\Delta N_p}{dt} + \Gamma v_g g_0 \xi \Delta N_p \right)$$

Transient chirp　　　Adiabatic chirp



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FM response

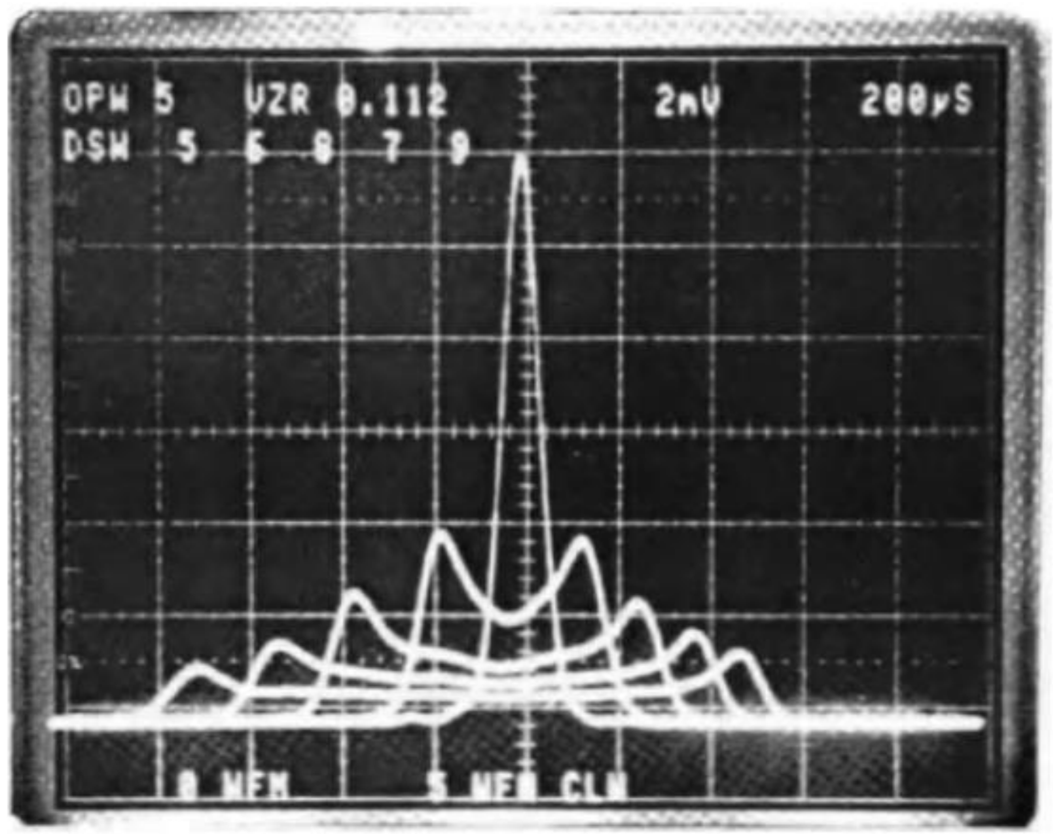
- When the laser is modulated by a small-signal sine wave:

$$\Delta v = \frac{\alpha}{4\pi}\left(\frac{1}{N_{p0}}\frac{d\Delta N_p}{dt} + \Gamma v_g g_0 \xi \Delta N_p\right) \Rightarrow$$

$$v_1 = \frac{\alpha}{4\pi}\left(\frac{j\omega}{N_{p0}} + \Gamma v_g g_0 \xi\right) N_{p1}$$

$$\approx \frac{\alpha}{4\pi}\left(j\omega + \Gamma v_g a_p N_{p0}\right)\frac{N_{p1}}{N_{p0}}$$

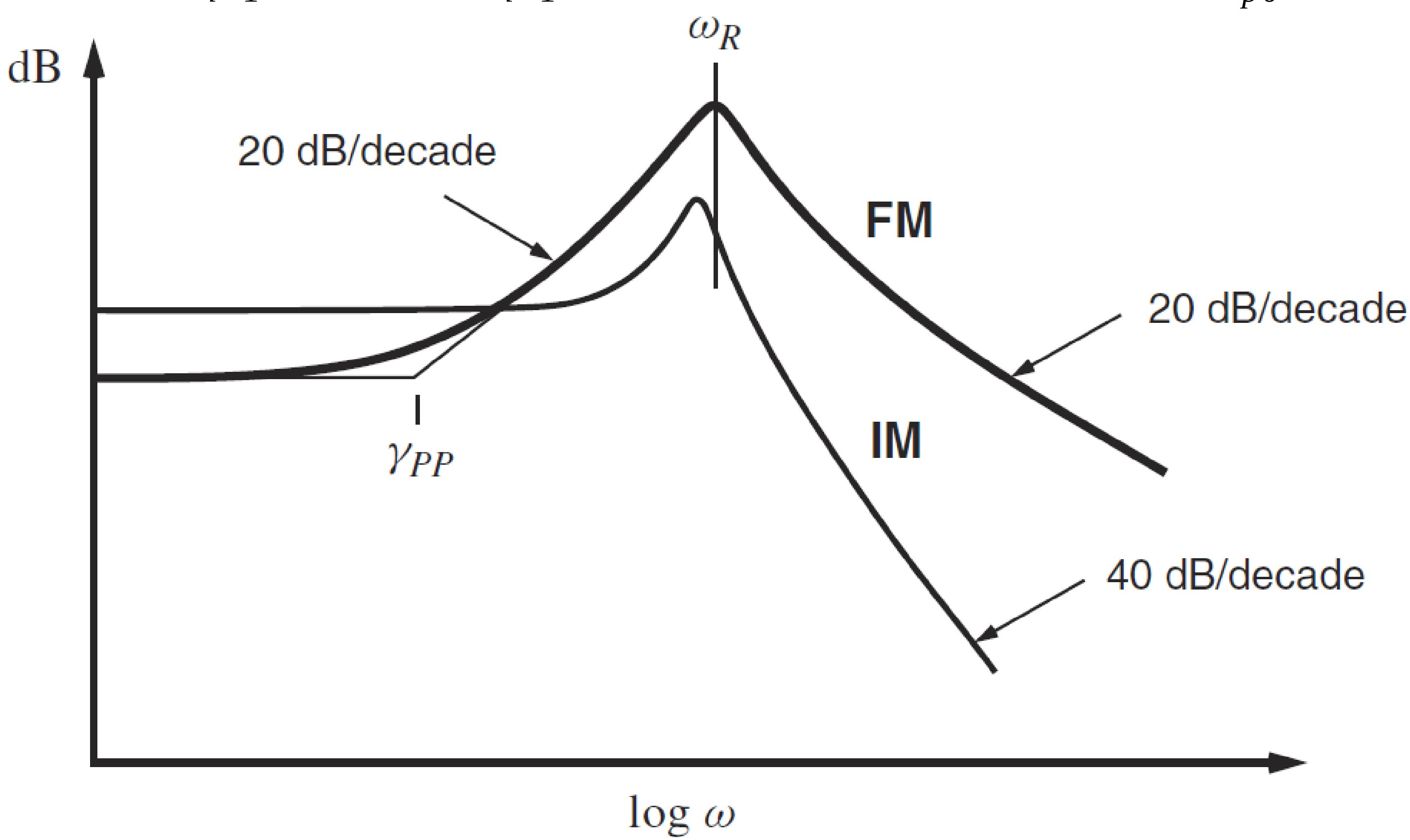
- The chirp increases linearly with the intensity modulation depth.



FM response

- Beyond the resonance, the IM decays in 40 dB/decade, while the FM decays in 20 dB/decade;

$$FM = \frac{v_1}{\eta_i I_1/(qV)} = \frac{1}{\eta_i I_1/(qV)} \frac{\alpha}{4\pi}\left(j\omega + \Gamma v_g a_p N_{p0}\right)\frac{N_{p1}}{N_{p0}}$$



Project 1 (10 scores)

- Quantum dot laser
- Quantum cascade laser
- Interband cascade laser
- InAs/GaSb quantum well laser

- Requirements:

1. Journal of Quantum Electronics template
2. No less than 15 pages
3. Reference no less than 50

Project 2 (10 scores)

- Hakki-Pauli method for the measurement of the alpha factor

- Requirements:

1. The detailed procedure of the measurements, including the formulas, figures, and the descriptions.