GAS WELLS PERFORMANCE

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The integral form of Darcy's equation for the gas flow can be approximated since the pressure function exhibits three distinct pressure application regions: low-pressure region, intermediate-pressure region, and high pressure region.

1. Radial Flow of Compressible Fluids

Let's start with the basic Darcy's Law:

$$q = \frac{kA}{\mu B}\frac{\partial p}{\partial l} \tag{1}$$

Since:

$$A = 2\pi r h \tag{2}$$

$$B = \frac{V_{g,r}}{V_{g,s}} = \frac{Z_r T_r}{p_r}\frac{p_s}{Z_s T_s} = \frac{14.696}{(1)(519.67)}\frac{ZT}{p} = 0.02828\frac{ZT}{p} \tag{3}$$

Plugging (2) and (3) into (1) yields:

$$q = \frac{(0.001127)(5.6146)(2\pi)}{0.02828}\frac{r}{\partial r}\frac{kh}{\mu ZT}p\partial p \tag{4}$$

$$q = 1.406\frac{r}{\partial r}\frac{kh}{\mu ZT}p\partial p \tag{5}$$

$$q\int_{r_w}^{r_e}\frac{\partial r}{r} = 1.406\int_{p_w}^{p_e}\frac{kh}{\mu ZT}p\partial p \tag{6}$$

$$q\int_{r_w}^{r_e}\frac{\partial r}{r} = 1.406\frac{kh}{\bar{\mu}\bar{Z}T}\int_{p_w}^{p_e}p\partial p \tag{7}$$

$$q\ln\frac{r_e}{r_w} = 0.703\frac{kh}{\bar{\mu}\bar{Z}T}(p_e^2 - p_w^2) \tag{8}$$

$$q = (703)10^{-3}\frac{kh}{\bar{\mu}\bar{Z}T}\frac{(p_e^2 - p_w^2)}{\ln\frac{r_e}{r_w}}\ SCF \tag{9}$$

$$q = (703)10^{-6}\frac{kh}{\bar{\mu}\bar{Z}T}\frac{(p_e^2 - p_w^2)}{\ln\frac{r_e}{r_w}}\ MSCF \tag{10}$$

$$q = (703)10^{-9}\frac{kh}{\bar{\mu}\bar{Z}T}\frac{(p_e^2 - p_w^2)}{\ln\frac{r_e}{r_w}}\ MMSCF \tag{11}$$

Including the skin effect, the equation is written as follows:

$$q = (703)10^{-9}\frac{kh}{\bar{\mu}\bar{Z}T}\frac{(p_e^2 - p_w^2)}{\left[\ln\frac{r_e}{r_w} - 0.75 + s\right]} \quad MMSCF \tag{12}$$

2. Pseudo-Steady State (Laminar) Flow Condition

The above formulation of (6) may be rigorously formulated as follows:

$$q\int_{r_w}^{r_e}\frac{\partial r}{r} = 1.406\int_{p_w}^{p_e}\frac{kh}{\mu ZT}p\partial p \tag{13}$$

$$q = 0.703\frac{kh}{T\ln\frac{r}{r_w}}\left(2\int_{p_w}^{p}\frac{p}{\mu Z}\partial p\right) \tag{14}$$

The term:

$$2\int_{p_w}^{p}\frac{p}{\mu Z}\partial p = 2\int_{0}^{p}\frac{p}{\mu Z}\partial p - 2\int_{0}^{p_w}\frac{p}{\mu Z}\partial p \tag{15}$$

Defining the real gas pseudo-potential as:

$$\psi = 2\int_{0}^{p}\frac{p}{\mu Z}\partial p \tag{16}$$

Similarly:

$$\psi_w = 2\int_{0}^{p_w}\frac{p}{\mu Z}\partial p \tag{17}$$

Thus:

$$2\int_{p_w}^{p}\frac{p}{\mu Z}\partial p = \psi - \psi_w \tag{18}$$

Plugging (18) into (14) yields:

$$q = 0.703\frac{kh}{T\ln\frac{r}{r_w}}(\psi - \psi_w) = \frac{kh}{1.422T\ln\frac{r}{r_w}}(\psi - \psi_w) \qquad SCF/D \tag{19}$$

Including the skin effect and writing q in MSCF yields:

$$q = \frac{kh}{1422T\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right]}(\psi - \psi_w) \qquad MSCF/D \tag{20}$$

Where:

q	Gas flow rate, MSCF/D
k	Permeability, mD
h	Reservoir thickness, ft
T	Reservoir temperature, °R
r_e	External radius, ft
r_w	Wellbore radius, ft
s	Skin factor
μ	Gas viscosity, cp
Z	Gas compressibility factor
ψ	Average (static) reservoir real gas pseudo-pressure, psi^2/cp
ψ_w	Wellbore bottom-hole flowing real gas pseudo-pressure, psi^2/cp

Equation (14) can be approximated since the pressure function exhibits three distinct pressure application regions: low-pressure region, intermediate-pressure region, and high pressure region. Including the skin effect in (14) and writing q in MSCF yields:

$$q = \frac{kh}{1422T\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right]}\left(2\int_{p_w}^{p}\frac{p}{\mu Z}\partial p\right) \tag{21}$$

A. Low Pressure Region

When both p_e and p_w are less than 2000 psi, the pressure functions $\frac{2p}{\mu Z}$ and $\frac{1}{\mu Z}$ exhibit a linear relationship with pressure; i.e. the product μZ is essentially constant when evaluated at pressures below 2000 psi. Implementing this observation into the above integral gives the following approximation:

$$q = \frac{kh}{1422T(\mu Z)\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right]}(p_e^2 - p_w^2) = J(p_e^2 - p_w^2) \tag{22}$$

Where (μZ) term is evaluated at $0.5(p_e^2 + p_w^2)$ and J is the productivity index of the gas well which is analogous to that of an oil well; i.e.:

$$J = \frac{q}{(\psi - \psi_w)} = \frac{kh}{1422T(\mu Z)\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right]} \tag{23}$$

Construct the IPR curve by assuming various values of p_w and calculating the corresponding q. This method is commonly called *the pressure-squared approximation method.*

B. Intermediate Pressure Region

When both p_e and p_w are between 2000 and 3000 psi, the pressure functions $\frac{2p}{\mu Z}$ and $\frac{1}{\mu Z}$ exhibit a distinct curvature relationship with pressure. Construct the IPR curve by assuming various values of p_w and calculating the corresponding q using:

$$q = \frac{kh(\psi - \psi_w)}{1422T\left(\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right)} = \frac{kh\left(2\int_{p_w}^{p}\frac{p}{\mu Z}\partial p\right)}{1422T\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right]} \tag{24}$$

C. High Pressure Region

When both p_e and p_w are greater than 3000 psi, the pressure functions $\frac{2p}{\mu Z}$ and $\frac{1}{\mu Z}$ are nearly constants. This suggests that the pressure term in the above equation is taken outside the integral to give the following approximation:

$$q = \frac{kh(p_r - p_w)}{141.22x10^3(\mu B)\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right]} \tag{25}$$

Where (μB) term is evaluated at $0.5(p_r + p_w)$. Construct the IPR curve by assuming various values of p_w and calculating the corresponding q. This method is commonly called *the pressure approximation method.*

3. Semi-Steady State (Turbulent) Flow Condition

To account for the additional pressure drops due to the turbulent flow, the rate-dependent skin factor Dq is included in the pseudo-steady state equations. The resulting set of equations is:

A. Pressure-Squared Approximation Form

$$q = \frac{kh}{1422T(\bar{\mu}\bar{Z})} \frac{(p_e^2 - p_w^2)}{\left[\ln\frac{r_e}{r_w} - 0.75 + s + Dq\right]} \quad MSCF \tag{26}$$

Where:

D Inertial or turbulent flow term $= F\left(\frac{kh}{1422T}\right)$

F Non-Darcy flow coefficient = $3.161x10^{-12}\left(\frac{\beta T\gamma}{\mu h^2 r_w}\right)$

β Turbulence parameter = $1.88x10^{-10}k^{-1.47}\phi^{-0.53}$

B. Pressure Approximation Form

$$q = \frac{kh(p_r - p_w)}{141.22x10^3(\mu B)\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s + Dq\right]} \tag{27}$$

C. Real Gas Potential (Pseudo-Pressure) Form

$$q = \frac{kh(\psi - \psi_w)}{1422T\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s + Dq\right]} \tag{28}$$

The above equations do not represent explicit expressions for calculating the gas flow rate. There are two separate empirical treatments that can be used to represent the turbulent flow problem in gas wells. Both treatments are directly derived from the above equations. The two treatments are called the *simplified treatment approach* and the *laminar inertial turbulent approach.*

1. The Simplified Treatment Approach

Based on the analysis of a large number of gas wells, Rawlins and Schellhardt (1936) postulated that the relationship between the gas flow rate and pressure can be expressed as:

$$q = C(p_e^2 - p_w^2)^n \tag{29}$$

Where:

q	Gas flow rate, MSCF/D
p_r	Average reservoir pressure, psi
p_w	Wellbore flowing pressure, psi
n	Deliverability exponent
C	Performance coefficient, MSCF/D/psi^2

This equation is called *Deliverability* or *Back-pressure* equation. Taking the logarithm of both sides of the equation yields:

$$\log q = \log C + n \log(p_e^2 - p_w^2) \tag{30}$$

This equation suggests that a plot of q versus $(p_e^2 - p_w^2)$ on a log-log paper should yield a straight line having a slope of n. In the industry, the plot is reversed by plotting $(p_e^2 - p_w^2)$ versus q produce a straight line with a slope of (1/n). This plot is called *Deliverability* or *Backpressure* plot. The deliverability exponent can be determined from any two points on the line. C is then calculated as follows:

$$C = \frac{q}{(p_e^2 - p_w^2)^n} \tag{31}$$

Once n and C are determined, the gas flow rate q at any pressure p_w can be calculated and the IPR curve may be constructed. There are essentially three types of deliverability tests. These are:

- Conventional Deliverability (Backpressure) Test
- Isochronal Test
- Modified isochronal Test

2. The Laminar Inertial Turbulent Approach

Based on pressure ranges, the laminar inertial turbulent flow is categorized into the following cases:

A. **Pressure-Squared Quadratic Approach** (recommended at pressures below 2000 psi)

$$q = \frac{kh}{1422T(\bar{\mu}\bar{Z})} \frac{(p_e^2 - p_w^2)}{\left[\ln\frac{r_e}{r_w} - 0.75 + s + Dq\right]} \quad MSCF \tag{32}$$

can be written in a more simplified form as follows:

$$(p_e^2 - p_w^2) = aq^2 + bq \tag{33}$$

Where:

$$a = \frac{1422T(\bar{\mu}\bar{Z})}{kh} D \tag{34}$$

$$b = \frac{1422T(\bar{\mu}\bar{Z})}{kh}\left[\ln\frac{r_e}{r_w} - 0.75 + s\right] \tag{35}$$

Equation (33) can be linearized by dividing both sides of the equation by q to yield:

$$\frac{(p_e^2 - p_w^2)}{q} = aq + b \tag{36}$$

The coefficients a and b are determined by plotting $\frac{(p_e^2 - p_w^2)}{q}$ versus q on a linear scale and should yield a straight line with a slope of a and an intercept of b. Given the values of a and b, the quadratic flow equation can be solved for q at any p_w from:

$$q = \frac{-b + \sqrt{b^2 + 4a(p_e^2 - p_w^2)}}{2a} \tag{37}$$

The IPR curve is constructed by assuming various values of p_w and calculating the corresponding q.

B. Pressure Quadratic Approach (recommended at pressures above 3000 psi)

The pressure equation:

$$q = \frac{kh(p_r - p_w)}{141.22x10^3(\mu B)\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s + Dq\right]} \tag{38}$$

can be written in a more simplified form as follows:

$$(p_r - p_w) = aq^2 + bq \tag{39}$$

Where:

a = Inertial-turbulent flow coefficient, which is given by:

$$a = \frac{141.22x10^3(\mu B)}{kh} D \tag{40}$$

b = laminar flow coefficient, which is given by:

$$b = \frac{141.22x10^3(\mu B)}{kh}\left[\ln\frac{r_e}{r_w} - 0.75 + s\right] \tag{41}$$

The above equation can be linearized by dividing both sides of the equation by q to yield:

$$\frac{(p_r - p_w)}{q} = aq + b \tag{42}$$

The coefficients a and b are determined by plotting $\frac{(p_r - p_w)}{q}$ versus q on a linear scale should yield a straight line with a slope of a and an intercept of b. Given the values of a and b, the quadratic flow equation can be solved for q any p_w from:

$$q = \frac{-b + \sqrt{b^2 + 4a(p_r - p_w)}}{2a} \tag{43}$$

The IPR curve *is* constructed by assuming various values of p_w and calculating the

Corresponding q.

C. **Pseudo-pressure Quadratic Approach** (more rigorous and applicable to all ranges of pressure)

The pseudo-pressure equation:

$$q = \frac{kh(\psi - \psi_w)}{1422T\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s + Dq\right]} \tag{44}$$

can be written in a more simplified form as follows:

$$(\psi - \psi_w) = aq^2 + bq \tag{45}$$

Where:

a = Inertial-turbulent flow coefficient, which is given by:

$$a = \frac{1422T}{kh} D \tag{46}$$

b = laminar flow coefficient, which is given by:

$$b = \frac{1422T}{kh}\left[\ln\frac{r_e}{r_w} - 0.75 + s\right] \tag{47}$$

The above equation can be linearized by dividing both sides of the equation by q to yield:

$$\frac{(\psi - \psi_w)}{q} = aq + b \tag{48}$$

The coefficients a and b are determined by plotting $\frac{(\psi - \psi_w)}{q}$ versus q on a linear scale and should yield a straight line with a slope of a and an intercept of b. Given the values of a and b, the quadratic flow equation can be solved for q any ψ_w from:

$$q = \frac{-b + \sqrt{b^2 + 4a(\psi - \psi_w)}}{2a} = \frac{-b + \sqrt{b^2 + 4a\left(2\int_{p_w}^{p} \frac{p}{\mu Z}\partial p\right)}}{2a} \tag{49}$$

The IPR curve is constructed by assuming various values of ψ_w and calculating the

Corresponding q.