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HOW TO SIZE A MOTOR

Calculations, advice and formulas for ensuring you have the right motor for your application

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QUESTION

A Control Design reader writes: The multi-station dial indexers we build are in multiple diameters and different cycle rates. We have one basic design that is customized based on the needs of the application. How can we be sure we're sizing the right type of motor correctly? The tables' motors currently can vary from induction to servo. We're also thinking of adding a gear box or direct drive. What are the calculations we can use to determine which motor to use?

ANSWERS

Load inertia

The number one calculation to make when sizing a motor for a rotary application is the calculation of the load inertia. The more accurately this can be calculated, the better. To make this calculation, it's generally best to approximate the load as a disc or cylinder. In the case of a dial indexer, this is fairly straightforward, since the dial is already essentially a disc shape and the load is typically evenly distributed around the dial. The basic load inertia calculation in SI units looks like this:

$mr^2/2$

where:

m = the total mass of the dial, with all tooling included, in units of kg

r = the radius of the dial, in units of m or cm.

In English units the calculation looks like this:

$Wr^2/2g$

where:

W = the total weigh of the dial, with all tooling included, in units of lbf

r = the radius of the dial, in units of in or ft

g = gravity constant, 386 in/sec^2.

The result of this calculation will be in units of kg-m^2 or kg-cm^2 if using SI units, or units of in-lb-sec^2 or ft-lb-sec^2 if using English units.

With this load inertia calculation complete, it is possible to begin looking for an appropriate motor to rotate the dial. The most common rules of thumb are to look for a motor whose rotor inertia is no less than 1/5 or 1/10 the load inertia. If the dial is large, this may be near impossible, and this is where a gearbox becomes important.

The advantage of a gearbox is that it reduces the effective inertia of the load by the square of the gearbox ratio. For example a 25:1 gear ratio will reduce the effective load inertia by 25^2, or 625 times. This reduction in load inertia makes it much easier to find a suitable motor.

The trade-off of a gearbox is that it reduces the output speed of the motor/gearbox combination, so it is imperative that the maximum rotational speed and cycle time requirements of the application are also calculated. In the example above, if the 25:1 gear ratio in combination with the proposed motor's rated speed indicate that the required output speed and cycle times are not achievable, a smaller gear ratio and larger motor, with larger rotor inertia, must be considered.

Another option is to go with a direct drive motor. This can often eliminate the need for a gearbox and increase overall performance by eliminating mechanical components, such as gearboxes, that can introduce backlash or other unwanted mechanical inaccuracies. However, integration of the direct drive motor is generally more complex than a standard motor and needs to be considered earlier in the design stage of the machine. It is also generally a more difficult process to retrofit an existing machine with a direct drive motor than a standard motor.

In conclusion, selecting the best motor and mechanical solution—gearbox or direct drive—is often an iterative process, where different approaches must be considered at the same time and iterations on gear ratio and motor inertia values must be made and compared to the load inertia and cycle time requirements of the application. An accurate calculation of the load inertia is essential to the process and is always the starting point for finding the best solution.

– Eric Rice, national marketing director, Applied Motion Products, **www.applied-motion.com**

DETERMINE THE TECHNOLOGY

The first thing is to determine which technology to use, induction or servo. DC motors and drives are also used in some of these applications because they are simple and have good low-end torque, but you don't mention this as an option. The technology will be driven by requirements for accuracy and performance. If the application doesn't need to index into position quickly—rapid cycle positioning—and the accuracy is not so critical that you can get away with using limit switches to sense when the indexer is in position, then you could use induction. However, if the application requires high performance—the ability to cycle rapidly and/or precisely—then I would use a servo motor. Servo motors are relatively inexpensive these days, and, once commissioned, they are very reliable. It will cost a little to have an integrator install and commission it, if you don't have someone in-house who is capable, but, once installed, along with the improved performance—faster cycles and more accurate positioning—you also benefit from the efficiency. Since the servo is typically a synchronous motor, it will be more efficient than an induction motor that would suffer from rotor losses.

Once you have determined the technology, you'll need to size and select the motor. Most motor manufacturers offer sizing software to help with this process. Enter the application data—load, technology—along with the move profile—speed, distance, accel, at speed, decel, idle time—and the software will calculate the required torque, speed and reflected inertia of the motor. Based on the results, it recommends a motor that best suits the application.

Whether or not you need a gearbox depends on the torque and speed required to index the machine. Generally the servo gearbox is used as a reducer. The output of the gearbox will increase in torque and reduce in speed by the ratio of the gearbox. Again, you can use the sizing software to put in a gearbox and change its ratios to see what that does to your torque and speed requirements for the motor. In addition to allowing you to use a smaller motor, the gearbox will also reduce the reflected inertia by the square of the ratio. For example, if you have a 10:1 gearbox, the reflected inertia to the motor will be reduced by a factor of 100. For a closed-loop servo system, having the right reflected inertia can be as critical as having the correct amount of torque. If your load-to-motor inertia ratio exceeds 10:1, you could have problems trying to tune the drive. With the introduction of high-resolution feedback devices, the inertia ratio isn't as critical as it used to be, but it still needs to be considered because it can affect the system's ability to respond quickly without overshoot. Luckily the sizing software will take the inertia into consideration, as well as the torque to help you select the proper motor for the job. I have taught a lot of fundamental servo-motor classes, and I always had the class do the hand calculations for things such as torque, inertia and friction, and then I would have them do the same exercise using sizing software just to show how simple it can be. They key is to find a good software package and get familiar with it. Your motor salesperson should be able to show you where to find a good sizing software tool.

– Bob Merrill, product manager – servo motors, Baldor, **www.baldor.com**

MOTOR FACTORIAL

Choosing the type of motor technology to drive the indexer is based on specific mechanical factors as moving load weight, dynamics and position accuracy. System cost is also a selection factor. Each motor control technology has advantages and disadvantages; finding the optimum solutions may require a compromise between engineering and economics.

AC motors are good for heavy loads, but you will need a vector drive to control it, to be able to produce positioning functionality. The ac motor/drive advantage is only ac motor cost; vector drive cost is similar to servo drive. The typical ac motor runs at 1,750 rpm—1,800 minus slip—so using a gearbox will get just over half the output torque, compared to using a servo motor with a nominal speed of 3,000 rpm for the same gearbox output shaft rpm. You will have to nearly double the torque of the ac motor to compensate for the lower gearbox input rpm. Also, if you have high dynamic indexing, the ac motor may not be able to cool itself properly, and you may have the motor overheating. Another disadvantage is in lower dynamic capabilities—lower acceleration due to higher rotor inertia.

Closed-loop stepper motors are also to be considered. Stepper motors can do position control, and closing the feedback loop—stepper motors with encoder feedback—ensures no skipping steps, so no positioning error. Using microstepping technology in modern drives will increase the resolution and create a smoother rotor movement. Stepper motors and drives have a price advantage over servo drives/motors, but with a trade in application dynamics, missing the capability to produce the peak torque for fast acceleration. Stepper technology will be the choice where cost is an issue and dynamics are not critical.

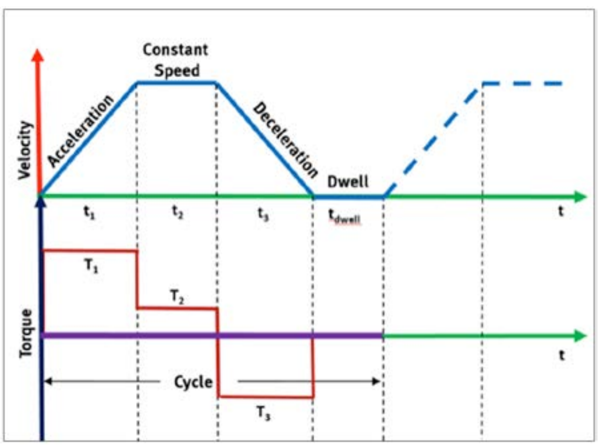
Servo motor/drive technology is the best fit for this application. Positioning accuracy, high dynamics and small format factor are just a few of the advantages. In addition, servo-drive-technology capability of pre-set indexing positions, synchronization of multiple-axis, triggering events based on position, changing torque value on the fly based on position and/or external events are just a few features that helps to simplify machine control design.

When sizing the servo motor, dynamics and mechanics associated with the application have to be considered. The most critical data you have to have is the torque rms required by the application indexing cycle and reflected load inertia mismatch. Servomotor rated torque rms should not be exceeded by the application indexing cycle required torque rms (T_{RMS}).

Calculation of the T_{RMS} required for the indexing cycle is based on the formula:

$$(T_{RMS})^2 = \frac{(T_1^2 t_1 + T_2^2 t_2 + T_3^2 t_3 + \ldots T_n^2 t_n)}{(t_1 + t_2 + t_3 + \ldots t_n)}$$

where T is torque associated with acquiring/maintaining desired velocity and t is time duration corresponding to movement. Please include in the indexing cycle the moving profile and the dwell time until the next cycle is starting.



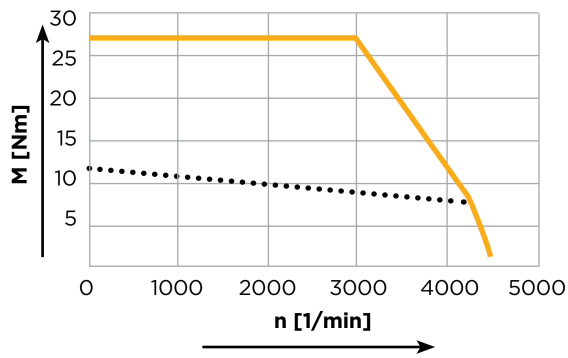
The servo motor's main feature is the capability to develop peak torque for 1-3 seconds, so for an indexing application the entire move is usually only acceleration/deceleration, with acceleration using the peak torque of the motor.

The servo drive associated with the motor has to be able to deliver the peak current needed to power the servo motor peak torque. If the servo motor peak torque requires more current than the servo drive peak current is rated, the servo drive will protect itself going overcurrent; that may either reduce the current output—current limitation—or trip in overcurrent protection. Neither of the two situations will provide sufficient current for the motor peak torque, so the motor will not be able to reach the required acceleration. Pay attention to the time associated with the servo drive peak current; it has to be at least sufficient to cover your longest acceleration time at peak torque or the drive may trip in thermal protection or go in current limitation—3 seconds if the motor peak torque is 3 seconds.

Inertia mismatch is the reflected load inertia to servo-motor rotor. The higher inertia mismatch, the slower the response time from the motor. Inertia mismatch can be anywhere between 1:1 and 100:1, or it can be even higher if acceleration/deceleration times are long and you can live with a sluggish system response. For a dial indexer where acceleration/deceleration and positioning in a 30°-60° range is close to 1 second, a 20:1 inertia mismatch should be sufficient.

Using a gearbox will decrease the reflected inertia, divided by the gearbox ratio and allow for a better usage of the servo motor. Servo-motor best torque usage is at the maximum speed before torque is starting to decrease—typically, 3,000 rpm. Using a 10:1 gearbox, for example, will multiply the motor torque 10 times, decrease inertia mismatch 10 times and allow the motor to reach higher operation speed. From the cost point of view, adding the gearbox will be compensated by reduced cost of smaller size servo motor, servo drive and probably cables.

On the servo motor torque graph, the peak torque knee is at 3,000 rpm.

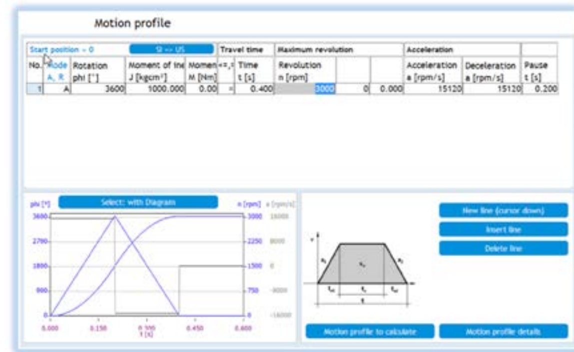


Environment vibrations and/or temperature: For high vibrations and temperature variations, a resolver feedback on the servo motor will be better than an encoder with glass disk.

Backlash associated with the gearbox: It will affect positioning accuracy. Using a low-backlash planetary will create a typical positioning error of 3 arcmin (0.05°).

The easiest way to size your servo motor is using the sizing software of the servo motor/drive manufacturer. The majority of servo manufacturers have sizing software that does all the calculations for you and provides a servo motor/drive selection. You just have to enter the application data.

In addition to sizing the servo motor, the sizing software can also select additional mechanical components, such as gearbox and linear actuator.



Critical data, such as precision accuracy at the end of travel, can be calculated and documented. One advantage of the servo software is that it is looking at application vs. capability of the entire servo system verifying for example that the servo-drive selection has sufficient thermal capability to handle the peak current of the indexing cycle, offering in the final selection different packages with different system loading capability. This load capacity can be very important in designing different load/cycle variations of the same machine, especially when some input data can change in time, such as friction due to poor machine maintenance.

The technical report on the system selection that the sizing software provides is a complex document with application-sizing data and motor, drive, gearbox and actuator technical data, as well as device loadings for the specific application. The report eliminates the errors in auxiliary components needed, such as connector size on system cables or adapting flange for gearbox/servo motor.

The bill of material (BOM) with all of the correct part numbers and quantities can be also provided, making the PO documentation really easy to process and error-free.

– Paul Plavicheanu, EA regional product manager, Region Americas, Festo Americas, www.festo.com/us

CALCULATE, CALCULATE, CALCULATE

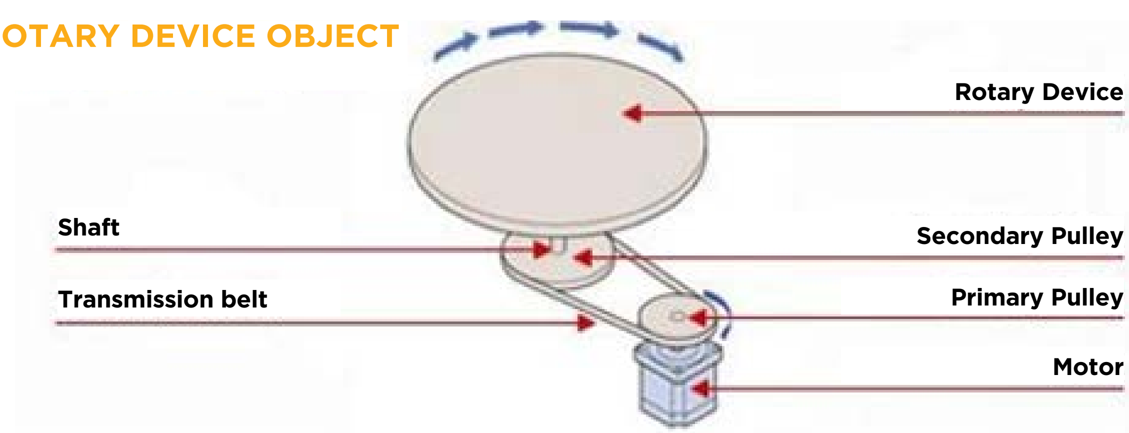
There are three main calculations needed to determine the proper product for an application. These calculations are for the load inertia of the system, the required torque and the speed in which the motor needs to rotate. Since there is one design that is customized based on the various dial indexers and different cycle rates, these calculations will need to be done for each unique application. However, in a situation where a single motor is required and the system will be varied, select the worst-case scenario—largest diameter with fastest cycle rate—when sizing.

From the mechanical setup, gather the following information for a sizing.

First, calculate for the inertia of all components being used. The inertia should be calculated first as the inertia value will be used later to calculate for the torque. To calculate inertia of the dial, use the following equation: $J_D= (1/8)mD^2$, where m is the weight of the dial in ounces and D is the diameter of the dial in inches. If the weight is unknown, then the inertia can be calculated by using the dial thickness and material density. The equation to use is: $J_D= (\pi/32)\rho LD^4$, where ρ= material density in oz-in³, L= dial thickness in inches and D= dial diameter in inches. Either equation can be used to size for the inertia of the dial, J_D.

Next, if the load is not being directly driven, but instead the system includes a shaft, then the inertia of the drive shaft, Js will

ROTARY DEVICE OBJECT



also need to be calculated. To calculate the inertia of the drive shaft use the following equation: $J_S = (1/8)\, m_2 D_2^2$, where m_2 is the weight of the shaft in ounces and D_2 is the diameter of the shaft in inches.

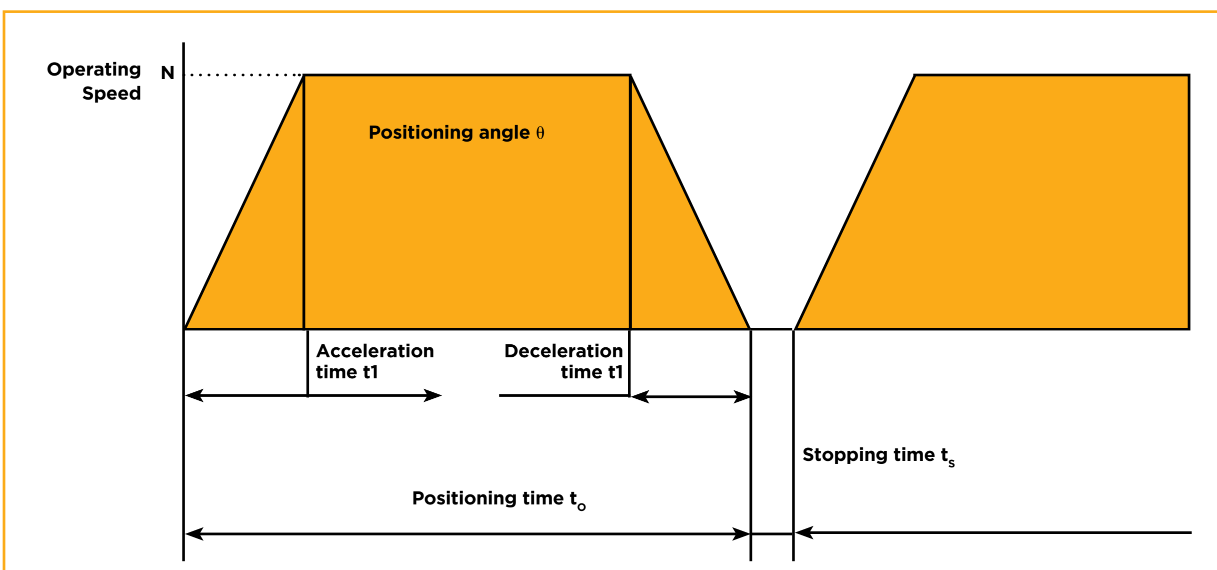
If a gearing system—pulleys and belts—is used to gear the system, then the inertia of the gearing or transmission will need to be calculated, as well. To calculate for the inertia of the transmission pulleys, the following equations are used: $J_{DP1} = (1/8) m_{p1} D_{p1}^2$ and $J_{DP2} = (1/8) m_{p2} D_{p2}^2$. The J_{DP1} is for the inertia of the primary pulley and J_{DP2} is for the inertia of the secondary pulley where m_{p1} is the weight of the primary pulley in ounces, m_{p2} is the weight of the secondary pulley in ounces, D_{p1} is the diameter for the primary pulley in inches and D_{p2} is the diameter for the secondary pulley in inches. If there is no external gearing, then the inertia calculations for J_{DP1} and J_{DP2} can be skipped.

To calculate the total system inertia, J_L, use the following equation: $J_L = (J_D + J_S + J_{DP2}) * (D_{p1}/D_{p2}) + J_{DP1}$.

If no external gearing is being used, then the equation for the total system inertia, J_L, is: $J_L = J_D + J_S$.

Now that the total system inertia, J_L, has been calculated, size for the speed of the motor.

To size for the speed of the motor V_m in rpm, use the following equation: $V_m = (\theta/360)(60/(t_0 - t_1))(D_{p2}/D_{p1})$ where θ is the indexing distance in degrees, t_0 is the total time for positioning in seconds, t_1 is the acceleration/deceleration time in seconds, D_{p1} is the diameter for the primary pulley in inches and D_{p2} is the diameter for the secondary pulley in inches. If there is no external gearing, then leave (D_{p2}/D_{p1}) out of the equation.



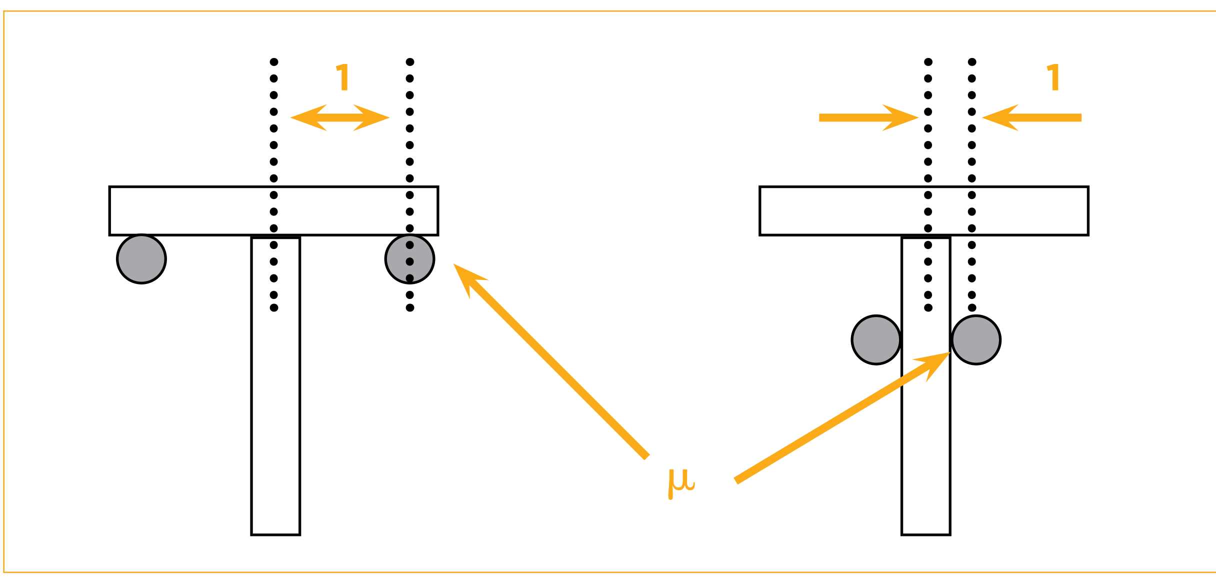
Last, calculate for the required torque, T, in lb-in. The required torque is made up of the combination of the acceleration torque, T_a, and the load torque, T_L. First calculate the acceleration torque, T_a. T_a= $((1.2J_L)/386)(V_m/(9.55t_1))(1/16)$, where J_L is the total system inertia, V_m is speed of the motor, and t_1 is the acceleration /deceleration time.

To size for load torque, T_L, use the following equation: $T_L = W\mu l(1/(\eta 0.01))(D_{p1}/D_{p2})$ where W is the weight of the dial in lb, μ is the friction coefficient between the dial and the supporting mechanism, l is the distance from the dial center to the supporting mechanism in inches and η is the system efficiency left as a percentage value such as 80%.

Once both the acceleration torque, T_a, and the load torque, T_L, are sized, then determine the total torque required, T. To calculate for the required torque, use the following equation: $T = (T_a + T_L)$(safety factor). Generally, when sizing, "2 times safety factor" is used.

Now that the sizing is completed, make note of the final values for J_L, V_m and T. Select a product based on those values. When matching a stepper motor please take into account the system load inertia, J_L, and the motors rotor inertia, J_o. For an open-loop system this ratio J_L/J_o should be less than 10 while for closed-loop systems it should be less than 30.

– Lizbeth Lopez, technical support engineering supervisor, Oriental Motor, **www.orientalmotor.com**



HOW MUCH TORQUE?

Determine torque needed, how long it's needed and the speeds it will need. To get an idea of where these come into the motor selection process, it would be good to understand the terms used in motor datasheets, starting with continuous and peak torque values.

Direct drive motors have a continuous torque value, which is the amount of torque it can hold indefinitely before overheating. A motor can operate below or above—up to the peak torque value—freely but needs to average out to its continuous value to keep from overheating. It's important to know not only how much torque the motor needs to deliver but for how long. Speed is important to know also because depending on the motor type there will be a drop-off of achievable torque at higher speeds due to back EMF Voltage. On low-pole motors such as servo motors, the overall force is comparatively low, but the achievable speeds for the torque ranges are a lot higher. Oppositely, a torque motor has a high pole count, which greatly increases the achievable torque, but can only maintain the torque at relatively low speeds. If a higher speed is required, then the drop-off in torque must be considered in the motor selection.

The exact measurements to get the torque values would be the ones used to measure the inertia of the payload, and the formula would depend on the shape of the object being rotated. Once that's done, then you determine the speeds you wish to get to and how fast you'd like to get there, which gives the acceleration. Using inertia and angular acceleration gives you the required torque. This is how you would select the motor. From there, you can adjust the acceleration depending on the motor you'd select for it.

In a best-case scenario, you'd be able to put together a full duty cycle charting distance and time, which would ultimately give you an average torque value, but, if you are anticipating unexpected movements, then you'd have to at least gauge what would be the ideal amount of power you'd like your machine to deliver and average from there.

– Brian Zlotorzycki, Heidenhain product specialist, ETEL, www.etel.ch

INDEXING APPLICATIONS

Indexing tables are easier to calculate than many other applications but still require a few input values and preliminary calculations to size properly. Base application values include orientation of move (horizontal, vertical or angled), inertia of table, inertia of payload, friction torque of table, move time/distance and then finally the accuracy required.

Here is a basic power calculation example assuming the following application information:

- move is horizontal
- 20-in radius table weighing 10 lb
- 10-lb load positioned on a 10-in radius
- 2-second 90° move.

For ease of calculation and to yield a less extensive response, we'll pre-determine table inertia, velocity and acceleration, and then we'll convert to metric values.

Application information

Description	Value
Mass of load	(4.54 kg)
Radius of load	(0.254 m)
Table inertia	0.146 kgm^2
Friction torque (table)	1 Nm
Acceleration	1.57 rad/s^2
Velocity	(1.57 rad/sec)

First we calculate total inertia. The following formula includes an inertia calculation for the mass in motion: ($m_L r_L^2$). We have pre-determined 0.146 kgm^2 for the table inertia, (J_{add}).

$J_{sum} = J_{add} + m_L r_L^2$

0.146 kgm^2 + (4.54 kg * 0.254 m^2) = 0.438 kgm^2 (J_{sum})

Then we calculate the dynamic torque of the application:

$M_{dyn} = J_{sum}\alpha$

0.438 kgm^2 * 1.57 rad/s^2 = 0.69 Nm (M_{dyn})

Then we calculate the overall torque of the application. Most often, we can simply add friction and dynamic torque values:

$M_{App} = M_{dyn} + M\mu{,}L$

0.69 Nm + 1 Nm= 1.69 Nm (M_{App})

Finally, we calculate power:

$P_{App} = M_{App} n(2\pi/60)$

1.69 Nm * 15 rpm * 0.1047 = 2.65 W

Selecting base motor technology and whether or not a gearbox is suitable depends on the following key factors: dynamic response, inertia mismatch, spatial constraints, cost, energy efficiency and accuracy.

SYMBOLS USED

Symbol	Description	Dimension unit
J_{add}	Moment of inertia of the rotary table	kgm^2
J_{sum}	Total moment of inertia	kgm^2
m_L	Mass of payload	kg
M_{dyn}	Dynamic torque	Nm
M_{App}	Required torque of the application	Nm
$M\mu{,}L$	Friction torque of the load	Nm
n	Speed of application	rpm
P_{App}	Power of the application	w
r_L	Radius of movement of the payload	m
α	Angular acceleration	rad/s^2
ω	Angular velocity	rad/s

Here are some general insights regarding each contributing element.

Dynamic response: With applications that require acceleration below 0.5 seconds, a general rule of thumb is to move toward synchronous servo motors. Their permanent magnet design enables a lower rotor inertia, which contributes to quicker dynamic response. Conversely, if the application doesn't require such high dynamics, asynchronous or induction motor technology may be suitable.

Inertia mismatch: As a general rule of thumb, the inertia mismatch between the application and motor rotor should be 20 or less. If the inertia mismatch is higher, there are a few ways to address. Two examples are a larger motor with a heavier rotor and the introduction of a gearbox.

Spatial constraints: If space is at a minimum, permanent magnet motors may be worth the additional cost.

Cost: In many cases we can utilize induction motor technology with traditionally touted servo applications, saving the customer significant initial outlay. This initial cost benefit may be counter-effective if energy consumption and spatial concerns have greater value, or if the application duty cycle is too demanding.

Energy efficiency: Permanent magnet motors offer the greatest efficiency, but this value may be counterproductive, depending on the duty cycle and whether or not regenerative power can be produced.

Accuracy: In the past it was automatically assumed servo technology was the best way to approach high accuracy demands. With feedback devices and capable drives, induction motors are also capable of handling very high-accuracy applications. Depending on the accuracy required, introduction of a gearbox may work against accuracy requirements. Knowing the final accuracy requirements is the best approach.

All of the aforementioned factors interplay when deciding on a final solution, meaning there is no short answer.

As noted, many applications can employ either servo or induction technology to handle the same task. The best approach is to work with an experienced supplier, capable of providing multiple engineering solutions that are supported by application specific software, and a wide range of technology. If you found the right supplier, they should be able to articulate advantages and disadvantages among various solutions with ease.

– Alby King, product manager, electromechanical, Lenze Americas, www.lenze.com/en-us

MORE TORQUE

In sizing a motor for a rotational application, the available torque of the motor needs to be the torque required to perform the desired operation. For rotational movements, you will first need to calculate the moment of inertia of all rotating components and constant forces working against the rotation, in order to calculate the torque (T) required to accelerate the table to the required rotational speed. The moment of inertia can be thought of as the resistance of the item to a change in its rotation. Relevant equations for components in your system are:

In the direct drive example below, the moment of inertia of the motor is neglected for simplicity, but in general it can contribute to the required torque if the motor is large enough, and the same goes for a gear head if utilized.

Direct drive example: Motor connects directly to a disk and spins it to 1,000 rpm in 5 seconds. The disk is made of aluminum 6061 and is 6 mm thick with a diameter of 100 mm.

Since this is the torque required to accelerate the disk to top speed, with no other forces operating to slow the disk down, such as friction, this is the value that should be used to size the motor.

To more closely resemble one of your options, we will include the use of a gear head to drive a disk along with a friction load during rotation of the disk in our next example. In the example below, the gear head moment of inertia is utilized, as is the motor's moment of inertia. The required torque will include the additional torque required to overcome the friction forces present. The friction force is representative of any constant force that needs to be overcome by the motor to maintain rotation. These include gravity, preloads or other push-pull forces.

Gearbox example: A motor is connected to a titanium disk via a gear head. Disk diameter is 0.5 m, and it is 10 mm thick. Peak rotation of the disk is 100 rpm in 5 seconds. There is a constant frictional interaction on the disk due to a contact sensor pushing on the outside diameter of disk with a force

Item	Equation	Variable units
Moment of inertia of a disk, pulley, gears, and shafts (J)	$J=\frac{1}{2}Mr^2$	M=mass (kg); r=radius (m)
Angular acceleration (α)	$\alpha=\frac{\Delta\omega}{\Delta t}$	ω=angular velocity (radians/sec); t=time (sec)
Torque required	$T=J\alpha$	radians/sec^2
Torque at the motor using a gearhead	$T_{@m} = G_r\frac{T_L}{e}$	T_L= Load torque; G_r=gear ratio; e=gear efficiency

Disk Mass: disk volume x aluminum density = $(\pi * 0.050\ m^2 * 0.006\ m) \times 2720 \frac{Kg}{(m^2)}$

$$= 4.71 \times 10^{-5}\ m^3 \times 2720 \frac{Kg}{(m^2)} = 0.128\ Kg$$

Disk moment of inertia of disk = $\frac{1}{2} Mr^2 = \frac{1}{2} 0.128\ Kg * 0.050\ m^2 = 0.00016\ Kg*m^2$

Angular acceleration: $1000\ RPM = 1000 \frac{revolution}{Minute} \times \frac{1\ minute}{60\ seconds} \times 2\pi \frac{radians}{Revolution} \times 104.7 \frac{radians}{sec}$

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_{final} - \omega_{initial}}{5\ seconds} = \frac{(104.72) \frac{radians}{second}}{5\ seconds} = 20.9 \frac{radians}{s^2}$$

Torque required = $T = J\alpha = 0.00016\ Kg * m^2 \times 20.9 \frac{radians}{s^2} = 0.003 \frac{Kg * m^2}{s^2} = 3\ mN * m$

of 2 N. The gear head in this example has a gear ratio (G_r) of 1/50 with an efficiency of 81% and a moment of inertia of $3.7 * 10^{-5}$ Kg * m². The motor has a moment of inertia value of $57 * 10^{-7}$ Kg * m². As can be seen in the Method 1 table, the motor moment of inertia is significantly less than the load moment of inertia in this example.

As you can see, Method 1 gives a slightly larger value for required torque than Method 2, as it takes into account other rotating components in the system. In this example, the extra attention of Method 1 is not warranted, but in your case it might be. With the use of a gearing mechanism, attention should also be paid to the speed of the motor. In this example, the motor turns 50 times for each time the load turns, so for a load to turn at an rpm of 100, the motor has to be able to operate at 5,000 rpm. This may require reducing the gear ratio to bring the speed of the motor down while increasing the torque requirement on the motor. Things to consider in calculating the required torque include parasitic forces such as viscosity of lubricants in the gearing, drag or frictional forces, preloads, and stepped or segmented motion profiles, among others.

– Brian Scott, Ph.D., application engineer, Nippon Pulse America, www.nipponpulse.com

DEVIL IN THE DETAILS

The calculations for sizing the appropriate motor for an application are complex. The basic formulas for calculating motor torque are readily available, but the devil lives in the details. Are all of the factors accounted for? Is there a friction component that was not calculated? How much power is lost in the gearbox? Are there other requirements, such as IP rating, certification or dimensions? What drive is compatible with the selected motor? All of these factors must be accounted for correctly.

– Tom Tichy, system design engineer for Motion Analyzer Software, Rockwell Automation, www.rockwellautomation.com

METHOD 1: MOTOR INERTIA AND GEAR MOMENTS OF INERTIA

Quantity	Calculated values	Equation
Mass of disk	8.7 Kg	See above
Load angular velocity ($\Delta\omega$)	$10.5 \frac{\text{radians}}{\text{s}}$	See above
Load angular acceleration (α)	$2.1 \frac{\text{radians}}{\text{s}^2}$	$\frac{\Delta\omega}{\Delta t}$
Load moment of Inertia (J_L)	0.2718 $\text{Kg}\cdot\text{m}^2$	$\frac{1}{2} Mr^2$
Gear box moment of inertia (J_g)	3.7 x 10-5 $\text{Kg}\cdot\text{m}^2$	Provided
Motor moment of inertia (J_M)	57 x 10-7 $\text{Kg}\cdot\text{m}^2$	Estimated
Moment of inertia reflected to motor (J_{LM})	0.0067 $\text{Kg}\cdot\text{m}^2$	$G_r * \frac{J_L}{e}$
Total of moments of inertia (J_t)	0.0002 $\text{Kg}\cdot\text{m}^2$	$J_M + J_g + J_{LM}$
Acceleration torque require (T_a)	14.2 mN·m	$= J_t * \alpha$
Constant force torque (T_c)	12.3 mN·m	$G_r * \frac{(F*r)}{e}$ r=radius at point of force
Torque required	26.5 mN·m	$T_a + T_c$

METHOD 2: QUICK AND DIRTY

Quantity	Calculated values	Equation
Mass of disk	8.7 Kg	See above
Load angular velocity ($\Delta\omega$)	$10.5 \frac{\text{radians}}{\text{s}}$	See above
Load angular acceleration(α)	$2.1 \frac{\text{radians}}{\text{s}^2}$	$\frac{\Delta\omega}{\Delta t}$
Load moment of Inertia (J_L)	0.2718 $\text{Kg}\cdot\text{m}^2$	$\frac{1}{2} Mr^2$
Acceleration torque require at load	569 mN·m	$= J_L * \alpha$
Constant force torque (T_c)	500 mN·m	$= F * r$; r=radius force application (0.25 m here)
Torque required at load	1069 mN·m	$T_L + T_c$
Torque required at motor using gearbox	26.4 mN·m	$G_r * \frac{T_L}{e}$

SYSTEM PARAMETERS

The first mistake made when sizing motors is to immediately jump to selecting the motors. Before sizing a system the parameters for the system should be well-established. These parameters would include cycle rate, bandwidth, positional accuracy and environment.

Cycle rate will tell us how much time we have to index each part and must include the dwell time and settling time. For example, if a machine is to produce 10 parts per minute, then the total time allowed for each part, or cycle time, is 6 seconds. If there is a process that takes 3 seconds to complete once the part is in place, the index time plus settling time is 3 seconds. The dwell time, or the time for the process to take place, will be 3 seconds.

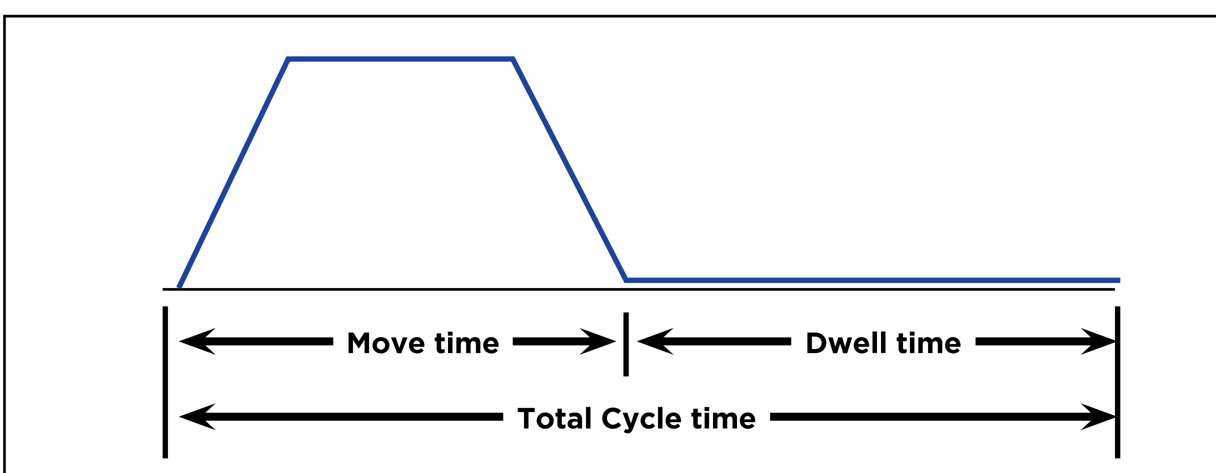
When the move time and dwell are equal we call this a 50% duty cycle. One mistake is to assume that the motor will not need to produce torque during dwell. If a force is applied during the dwell in the direction of rotation, then the motor will have to produce toque to hold position. Vertical forces and forces that act straight into the bearing require no holding torque.

The move time can be used to calculate the velocity maximum (Vmax) and the acceleration. If our particular machine has six stations, each station will be 60° apart, and each move will be 0.167 revolutions of the motor. For a 1/3-1/3-1/3 trapezoidal move, we can calculate Vmax as follows:

V_{max} = (d/t)1.5 = (0.167 revs/3 seconds) 1.5 = 0.0833 rps

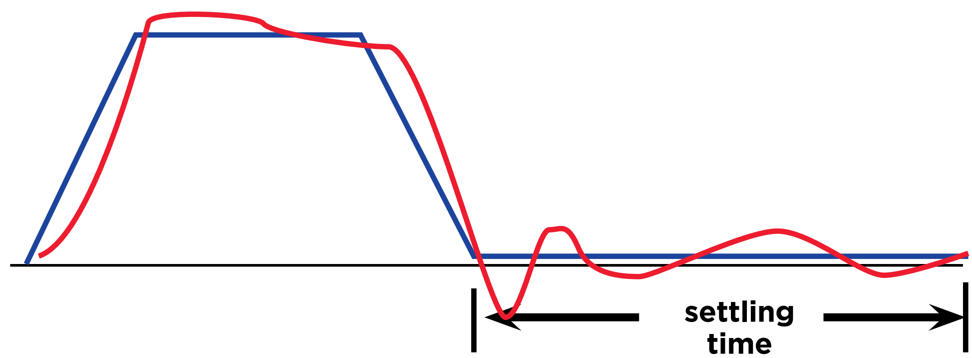
The motor's maximum velocity is 0.0833 rps, or 5 rpm. The 1/3-1/3-1/3 trapezoidal move allows 1/3 of the time for acceleration (t_a), 1/3 of the time for constant velocity (t_c), and 1/3 of the time deceleration (t_d). Acceleration would be calculated as follows:

Accel = V_{max}/t_a = 0.0833 rps/1s = 0.0833 rps/s



α = Accel * 2π = 0.0833 rps/s * 2απ = 0.5234 Rad/s²

- **The angular acceleration will be used later to calculate the peak torque required (T_{pk}).**
- **Bandwidth of the system is basically the performance of the system. The Bandwidth can be calculated by using the settling time of the system. The settling time is that which is required for the system to stop moving.**



The angular acceleration is shown above.

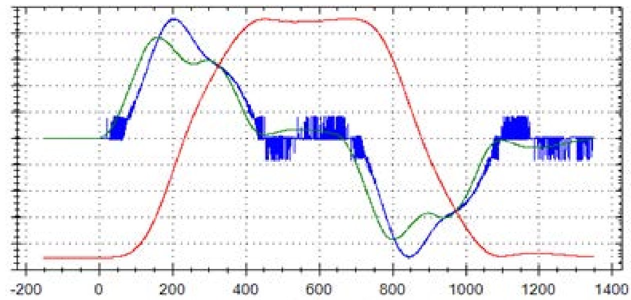
The bandwidth is the inverse of settling time, or 1/settling time. If after a move has been completed the motion required 0.05 seconds for all things to stop moving, then the bandwidth of the system is:
BW = 1/0.05 = 20.

The settling time must be included as part of the original move. For very compliant systems, those with a lot of mechanical components, the settling time can be as high as a few seconds. This means the bandwidth of the system will be very low. Positional accuracy requires a lot of thought about the feedback type to be used. When sizing motors for an index table, high-resolution feedback should be the standard. The high-resolution feedback is not only for position accuracy but for tuning the system. Index tables are usually very high inertial loads. To control the high inertias the system's gains will need to be high; moreover, to ensure the gains can be high and the system remain stable, there needs to be a lot of feedback.

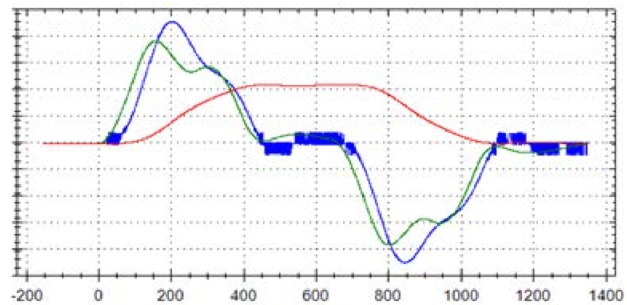
Two systems with the same inertia ratio, J_L/J_M, but different feedback resolutions will perform very differently. The motors will produce the same amount of torque, but the information is coming back at different rates.

A lot of times the environment is overlooked when beginning to apply motors into an application. In addition to the moisture, dust and chemicals, the ambient

Because this system has a lower feedback resolution, 4096/rev, the changes are much greater as the point corrections are being made. The actual velocity, in blue, is allowed to get out of position much more than the higher resolution feedback.



Because the higher resolution feedback is providing twice as much information, 8196/rev, corrections are made much sooner and require less adjustment. In diagram we can see the peaks for the velocity feedback are much lower.



temperature of the environment must be considered.

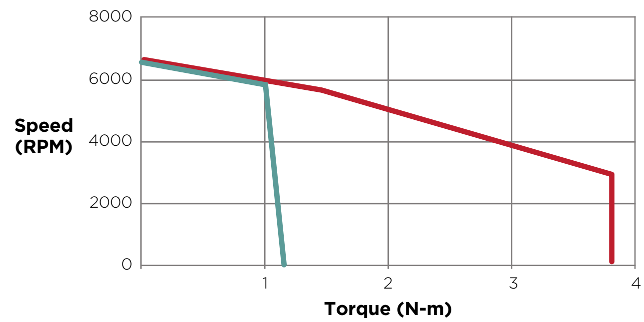
To protect against moisture and dust, the motor's IP rating will need to be taken into consideration. A motor with an IP rating of 54 provides little protection against moisture or dust. The first number, in this example, 5, will only protect against dust particle larger than 1 mm in diameter. The second number, 4, will only protect against low pressure water such as a drip.

To protect against chemicals, seals and cables will need to be selected for their resistance to the particular chemicals that will be encountered. In many cases a synthetic rubber, such as Viton, will be used for O-rings or gaskets.

Temperature comes in play when calculating the continuous torque of the motor. Since the continuous torque of a motor, T_C, is based on the thermal properties of the motor, ambient temperature affects the available torque. All motors and drives have a speed/torque curve, or torque/speed curve. This is the expected performance of the system based on a beginning ambient temperature.

If the ambient temperature changes from this standard, so will the available continuous torque. If the ambient is less than 40 °C, the continuous torque will improve. If the ambient is more than 40 °C, the continuous torque will decrease. We can calculate the change with the following equation:

The chart at the right shows the speed and torque for a selected motor and drive combination. From the chart we can see the expected torque at a given speed. This particular chart for created for an ambient temperature of 40 °C.



$$T_{Cnew} = T_{Crated} \frac{\sqrt{(T_{Max} - T_{ambient})}}{T_{max} - 40°C}$$

T_{max} is the rated temperature of the windings; $T_{ambient}$ is the temperature surrounding the motor; T_{Crated} is the rated continuous torque; T_{Cnew} is the continuous torque adjusted for the ambient temperature.

The inertia of the load will affect the torque required to accelerate to the maximum velocity. Newton's First Law describes inertia as the resistance to change. The more inertia a system has, the more resistance to change, whether this resistance is to acceleration or deceleration.

To calculate the inertia of an index table we use the basic equation:

$$J = \frac{1}{2} M r^2$$

J is the inertia, M is the mass, and r is the radius of the table. It is unusual for the mass of system to be specified. It is much more common to have the specification in weight or in dimension and material.

We more commonly use the following equation, which is based on the weight of the load:

$$J = \frac{W}{2g} r^2$$

Since weight divided by the gravitational constant equals mass, we are returning to the original equation.

When the dimensions and materials of the table are given, we use the equation below:

$$J = \frac{\pi L \rho r^4}{2g}$$

This equation will calculate the volume of the disc ($\pi L r^2$) and then calculate the weight using the material density (ρ). When divided by the gravitation constant, we are back to the original equation:

$$J = \frac{W}{2g} r^2$$

Once the inertia of the table is found, we can calculate the inertia of the load. The most common equation for calculating the load inertia is by weight. The load weight includes the part, fixture or any other items.

$$J = \frac{w}{2g}\left(r_o^2 + r_i^2\right)$$

The location of the load is defined by the inner and outer radius. Depending on the application, the load could be located outside the radius of the table.

With the inertia of the table and load known, the total inertia can be calculated. The total system inertia will include the motor inertia.

$$J_T = J_L + J_{table} + J_M$$

Using the total inertia of the system and angular acceleration we can begin to calculate the peak torque of the system. Hand calculations can always be tricky. We don't know which motor we are going to use, so how do we add in the motor inertia? One trick is to estimate the inertia you will see. If we were trying to keep the inertia ratio 10:1 or less, we can divide the sum of $J_L + J_{table}$ by a factor of 10.

$$J_T = J_L + J_{table} + ((J_L + J_{table})/10)$$

This will give us a good estimate of the system's total inertia. To find the peak torque, we use the total inertia and the angular acceleration.

$$T_{PK} = J_T\alpha + T_F$$

The frictional torque (T_F) is the torque required to overcome coefficients of friction that may result from bearings, sliding surfaces or other items that want to slow the system down. This torque usually is found during the beginning of acceleration, but can be maintained all the way up to peak speed. We plot it on the motor's speed torque curve at the peak speed to ensure we can hit the speed and torque requirements.

To calculate the continuous torque is a little more complex. The continuous torque, or rms torque, is based on all the torques found in the system. The continuous torque (T_c), or rms (root mean square) can be calculated using the T_{rms} equations. It should be noted that T_c is the same thing as T_{rms}. Adding the dwell time into the equation gives the motor more time to cool down. This in turn reduces the continuous torque of the system.

$$T_{rms} = \sqrt{\frac{Ta^2ta + Tc^2tc \quad Td^2td + Tn^2tn}{ta + tc + td + tn + t_{dwell}}}$$

The equation is showing the torques and times for calculating the continuous torque. T_a is the torque during acceleration, and t_a is the time during acceleration; T_c is the torque at constant velocity, and t_c is the time during constant velocity; T_d is the torque during deceleration, and td is the time during deceleration; T_n are other torques that might be found in the system, and t_n is the time during that torque. We can see that the T_{rms} equation can grow or shrink, depending on the type of profile used and the torques generated.

Plot the continuous torque on the motor's speed/torque curve, and check that it has about 20% safety margin. It is always a good idea to have a safety margin to prepare for the unexpected.

Adding a gearbox to the system will reduce the reflected inertia but requires more motor speed in the end. There are two terms that are used interchangeably, although there are some differences. A gearbox is usually a stand-alone device with input and output shafts. A gear head is designed to be mounted to the motor. Otherwise, they are doing the exact same thing, adding a mechanical advantage to the system.

Depending on the positional accuracy required, a gearbox might be prohibited. If you require a positional accuracy of +/- 15 arcseconds, then a gearbox will be out of the question. The best of planetary gearboxes will have a backlash of 4 arcminutes, or 240 arcseconds.

A gearbox will reduce the load's reflected inertia, the inertia to which the motor thinks it is connected, by the square of the ratio. This means that a 5:1 gearbox will reduce the inertia by a factor of 25.

$$J_{ref} = ((J_L + J_{table})/n^2) + J_M$$

The speed of the motor will increase by the ratio. With a 5:1 gearbox, the motor will need to rotate faster, by a factor of 5, than the load. With gearboxes, generally look at the input speed vs. the output speed, as well as the input torque vs. the output torque.

There are a lot of things that need to be taken into consideration when sizing any servo system. Changing one parameter can change the motor and the overall system performance. It is always good to bring in a motion control expert at the beginning of the application design phase. Many major errors can be avoided, and performance can be optimized.

Before sizing a system take the time to establish the parameters. Figure out what you really need to the system do. Also, keep in mind that sizing software is available that is free to download, and many companies offer classes on motion control and sizing. A hands-on class from a qualified instructor can pay for itself many times over.

There are a number of sizing tools available in the market, such as Motioneering, which is a full-featured application sizing and selection tool covering a variety of mechanism types, including rotary indexing tables. A good sizing tool will remind you to take all of these factors into consideration, capture inputs and assumptions, and shield you from the tedious math associated with the inputs, and ultimately, recommend a system for you.

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