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**Background:**

*What are your technical skills, education, experience, etc. Especially make sure to explain with what level of mathematics you are comfortable with and on what level you would like to program.*

I received my B.A. in Mathematics at Carthage College in Kenosha, WI. I am currently attending Saint Louis University as a Ph.D. graduate student.

I have experience executing research in mathematics not only this past semester under Professor Ben Hutz, researching Moduli Spaces, as well as reviewing and executing code in Sage. I have also participated in the Student Undergraduate Research Experience, at Carthage College, with Dr. Aaron Trautwein, studying Knot Theory. This study translated into my senior thesis where I wrote a program to find the harmonic equations of knots.

Before my research in mathematics I was executing research in radiochemistry. I worked for over two years at DuraColor, a UV-ink and screen-printing company in Racine, WI. I was the research and development lab assistant. While working at DuraColor, I had to be self-motivated as there were multiple areas of testing and research going on at any time. This taught me to be very organized, and keep accurate documentation. I translated these skills into my mathematics research and studies.

As a first year graduate student I feel comfortable with mid-level graduate mathematics.

*Who are you? What makes you the best person to work on this particular project? Your personal motivation?*

I am both a left and right-brained person. Aside from my love of mathematics, I have a deep passion for art and theatre. My left brained interests allow me to approach problems from many different facets.

I am the best person for working on this project because I already have experience programming in Sage in this area of mathematics. I have a rapport with Dr. Hutz and will be continuing to work with him and Sage for the duration of my Ph.D. experience.

Over the summer I would like to continue working in my field of mathematics, instead of doing it in my spare time over the summer while working full-time elsewhere.

*What platform and operating-system are you using on your computer?*

I am using a Mac with OS X version 10.9.4.

*Are you or have you been engaged in other open-source projects?*

This is my first experience with open-source code.

*Do you code on your own pet projects?*

Yes, I do. A long-standing problem in topology has been to find differentiable parameterizations of knots. One such representation uses trigonometric polynomials, known as harmonic parameterizations. I made finding differentiable representations of knots simpler by creating a computer program to generate these harmonic parameterizations. The program was written in C++. This program takes polygonal knots in  $\mathbb{R}^3$ , determines their edge lengths, and then converts their edges into segments from  $-\pi$  to  $\pi$ . This allows us to then calculate the parametric equations for each of these edges piecewise in terms of  $\pi$ . A truncated Fourier series is then applied to approximate the linear functions representing the paths of these polygonal knots, giving us harmonic parameterizations. All of the knots found by this program are represented as 15<sup>th</sup> degree polynomials, while a smaller degree could possibly maintain their equivalence class. For further research my hope is to get the knots into their lowest degree form. Then get the knots in their lowest energy form, i.e. evenly spread out in  $\mathbb{R}^3$ . I would also like to apply known invariants to check for knot equivalences.

*Are you a Sage user, how long do you know Sage?*

I am a Sage user, my username is rlmiller and I have been using Sage since January 2016. I've been doing research in dynamical systems with Dr. Ben Hutz.

*Current Tickets I've worked on:*

Reviewer: 20168, 20067, 20059, 19979.

Author:

19891: Fixed style of projective products file.

19889: Fixed style of morphism folder.

20262: Adds the point transformation matrix for projective space. This function takes two sets of  $n+2$  points, one being our source and the other being our target. The function then determines that no  $n+1$  subsets are linearly dependent, and then finds the unique transformation matrix that takes our source to our target.

*Title, Project Synopsis: a short description and summary of its aim and scope.*

Moduli Space of Dynamical Systems.

There is functionality for working with dynamical systems over projective space in Sage. However, one of the areas lacking in functionality is the moduli space. We say two self-maps of

projective space are equivalent if there is an element of the Projective Linear Group (PGL) that conjugates one to the other. The following two algorithms should be implemented, in order to create more functionality in moduli space.

If there is time remaining I would examine some of the applications of these algorithms. Such as the concept of potential good or bad reduction; or an iterator function for reduced minimal moduli elements of bounded height, similar to `QQ.range_by_height()`.

*What is your personal involvement or relationship with your proposed project?*

I am currently working on some of the lead up programming to this project with Dr. Ben Hutz. I will be his research assistant until the completion of my Ph.D. in 2020 and will be continuing to work with him in arithmetic of dynamical systems and in Sage.

*Details: describe all the details and explain modules or parts of your whole project. Break down the whole project into individual tasks - as good as possible - and describe deliverable and quantifiable results for each of them. It also helps if you have already discussed this with a possible mentor.*

This project involves implementing two algorithms, see project synopsis.

*Schedule: A timetable, including special circumstances like exams or holidays, for the individual tasks.*

During our community bonding period I will read through the two papers containing the algorithms, to ensure a general knowledge and build towards a deeper understanding.

May 23-29th: I would spend gaining a deeper understanding of the paper *Computing Conjugating Sets and Automorphism Groups of Rational Functions*, and sketching some of the code for the algorithm.

May 30th- June 13th: I would spend writing the code.

June 14th- 20th: I would be testing and implementing the code.

June 21st-27th: I would spend gaining a deeper understanding of the paper *On the Reduction of Binary Forms* and making a sketch of the code.

June 28th- July 11th: I would spend writing the code.

July 12th-18th: I would spend testing and implementing the code.

July-19th-August 23rd: Learn about the applications of these algorithms. Code, test and implement as many of these applications as time allows.

No holidays or days off needed, will be working from home.

*Risk Management: Try to anticipate potential problems and explain, how to mitigate them. Propose alternative scenarios, if a particular milestone isn't reached, to still successfully complete the project.*

My proposed schedule leaves time to overcome problems. Some potential problems could be gaining a deeper understanding of the papers containing the algorithms. To overcome this problem I would read additional sources and consult with my mentor. This could add a week to each project, which my schedule leaves room for.

Another problem could be implementing the  $P^n$  case. If I find myself behind I could implement the  $P^1$  case and return to the  $P^n$  case. The  $P^1$  case is still very beneficial to the study of dynamical systems.