Liquidity Regulation in a Monetary Economy*

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Abstract

Although it is commonly argued that the prevention of bank runs is the main reason to regulate banks' asset portfolios, I show that a market failure that justifies such regulation lies on the incompleteness of financial markets when there is risk about the *aggregate distribution of transaction types*. I develop a framework in which *outside* (fiat, government-provided) and *inside* (plastic, bank-created) money co-exist as means of payment under either complete or incomplete financial markets for aggregate risk. The welfare analysis is reduced to comparing only two parameters: the currency-to-liability ratio δ which is set by the government and the fraction ρ of banks' depositors engaged in *cash-only transactions* (inside money cannot be accepted). In equilibrium, when $\delta < \rho$ fiat currency is relatively scarce in the inter-bank market and then government bonds (which are transformed into liquid liabilities by banks) are less valuable than cash. This forces banks to offer higher consumption with plastic money to induce self-selection among depositors. Welfare is lower under incomplete markets: depositors exert a higher labor effort (precautionary motive) to accumulate more assets as perfect risk-sharing is unattainable (unlike the case of complete markets). Also, a higher cash requirement on banks is equivalent to an implicit increase in the policy parameter δ which makes bonds scarcer and more valuable in the inter-bank market. Therefore, a liquidity requirement is not welfare-improving because it reduces the likelihood of bank runs but because it increases the inter-bank market price of bonds which in turn improves risk-sharing. Finally, when the government sets $\delta = \rho$ the welfare measures under complete and incomplete markets coincide as the *Friedman rule* holds.

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1 Introduction

Liquidity is a class of assets that are used as means of payment (Hicks, 1962). In the United States a liquidity measure is given by M1 (funds that are readily accessible for spending) which, in addition to government-issued (fiat) currency, includes demand deposits in commercial banks, traveler's checks and other checkable deposits. Figure 1 displays the composition of US M1 from January of 1963 to August of 2017 and allows to conclude that, excepting for the years associated to the last financial crisis, the means of payment were, to a high extent, provided by depository institutions.[1] This observation is relevant to understand the new type of regulation introduced within the Third Basel Accord (Bank for International Settlements, 2011, 2013) which focuses on banks' asset holdings in terms of their liquidity.

In this paper I analyze the welfare implications of regulating the amount of liquid assets held by banks. This is done for an economy in which government-issued currency is *essential* (i.e. welfare improving) as emphasized by the *money search* literature (Lagos and Wright, 2005) and banks are also *essential* as they transform illiquid assets into means of payment (Diamond and Dybvig, 1983). As usual, such regulation is justified as long as there exist a market failure.

The **novel feature** of my approach lies on introducing the incompleteness of financial markets for aggregate risk as a market failure that, *ceteris paribus*, leads to a lower welfare. It is worth to clarify the notion of aggregate risk employed here by means of an example. Consider a situation in which only two banks exist, A and B, and there exist two states of nature. In state 1 bank A demands a low amount of cash which equals 1 and bank B demands a high amount of cash which equals 2. Conversely, in state 2 the roles are reversed: bank A demands 2 and bank B demands 1. Therefore, under both states of nature the total demand for cash equals 3 and there is a difference between the total demand for cash (which always equals 3) and the risky distribution of banks' demand for cash which is the notion of aggregate risk that I rely on. On the other hand, the liquidity risk I consider distinguishes between cash-only transactions and transactions in which a broader class of asset can be used (e.g. checks and/or debit cards).

One of the advantages of my approach relies on its ability to reduce the discussion on banks' liquidity to an economic analysis in standard terms as follows. Risk-averse individuals exert labor effort and earn a wage that is used to make a deposit in a bank in order to insure themselves against liquidity risk. This so happens because depositors cannot access to a certain type of financial markets (e.g. the inter-bank market). In addition to fiat currency, banks also use these deposits to accumulate government bonds whose gross return is converted into *liquid claims on bank accounts* in the inter-bank market.

In equilibrium all markets clear and, in particular, the relative price of bonds is determined in the inter-bank market where banks re-balance their asset holdings according to their depositors' needs of means of payment. Such price depends on the fraction δ of total

[1] Introducing alternative liquidity measures like M2 and M3 reinforces this statement.

government liabilities that is constituted by fiat currency and the fraction ρ of total bank depositors that can only use cash in their transactions. When $\delta < \rho$ the available amount of currency is relatively "low" compared to the needs of depositors and then bonds are less valued than cash in the interbank-market. Given this, banks induce self-selection among depositors by offering more consumption if claims are reported to be needed as means of payment.

When banks have access to a complete set of Arrow securities, these intermediaries can perfectly smooth depositor's consumption across states of nature for each transaction type. Also, under this financial market structure the inter-bank (spot) market becomes redundant. These properties constitute a benchmark to be compared with and economy where (*ceteris paribus*) the financial market structure is different. Indeed, under a total lack of Arrow securities, the spot inter-bank does matter. However, since banks face different liquidity needs, perfect consumption-smoothing is impossible to attain. This induces a higher accumulation of assets by banks which corresponds to a higher labor effort by depositors. Therefore, under incomplete markets the economy accumulates a higher amount of assets but exhibits a lower welfare when compared to the case of complete markets.

On the other hand, when $\delta = \rho$ fiat currency is provided in the same proportion as needed by depositors and the *Friedman rule* holds as bonds are as valuable as cash. Since the currency-to-liability ratio δ is a policy variable, a direct recommendation consists on setting $\delta = \rho$ which, *ceteris paribus*, makes both welfare measures under complete and incomplete markets to coincide.[2] Finally, I also analyze the welfare effects of a policy that requires banks to hold an infinitesimally higher percentage of cash in their portfolios than in a *laissez faire* situation. By construction, implementing this policy requires the government to marginally increase δ (to provide banks with the necessary resources) and to make holdings of fiat currency less attractive (to provide banks with the necessary incentives).

The remaining sections of this paper are organized as follows. Section 2 elaborates on the way my approach relates to prior contributions. In section 3, I formally describe the unaltered elements of my economy such as timing, preferences, production technology, transaction technology, uncertainty structure, among others. Section 4 closes the model by specifying complete financial markets for aggregate uncertainty. Section 5 defines, characterizes and examines the properties of a competitive equilibrium with Arrow securities. Section 6 departs from complete financial markets assumption and specifies financial markets as incomplete in order to close the model. Section 7 defines, characterizes and analyzes the properties of a competitive equilibrium under incomplete markets and compares it against the case of complete markets. Section 8 formally defines a regulated equilibrium for this framework and discusses its implementation through regulation of banks' behavior. Section 9 summarizes the properties through a series of numerical examples. Section 10 concludes.

[2] I rule out the case in which $\delta > \rho$ since it delivers counter-intuitive results (e.g. a negative nominal interest rate) which cannot support a meaningful equilibrium concept.

2 Related literature

I study a monetary economy in which deposit contracts offered by one-period-lived banks are accepted by infinitely-lived agents who have no access to financial markets. A distinctive feature is that, in equilibrium, claims on bank accounts themselves are also used as a medium of exchange or inside money as originally proposed by Williamson (2012). Moreover, the new element here is the introduction of a tractable structure of risk about the aggregate distribution of agents' liquidity needs as originally proposed by Allen and Gale (2004) but for production economies.

In this Diamond-Dybvig framework, such extra layer of uncertainty allows me to formally show that, even when deposit contracts are state-contingent and then bank runs do not occur, the incompleteness of financial markets for aggregate risk leads to a lower welfare. Also, it allows me to take the argument by Geanakoplos and Polemarchakis (1986) one step further and study the inefficiency of the competitive equilibrium with incomplete financial markets for the case in which banks' liquidity is endogenous and provide a welfare-based foundation for its regulation. It is worth to mention that, by construction, this result is related to relevant policy variables.

Given the lower welfare, two types of policies have been usually proposed. The first type consists on introducing more contingent markets, although Hart (1975) has demonstrated that this does not even weakly increase welfare. The second way consists on taking the incompleteness of financial markets as given and support a welfare-improving allocation, which is the approach I adopt here.

The way in which my study is related to the existing literature can be better understood across four dimensions. First, the contribution by Diamond and Dybvig (1983) was the first to make explicit the role of banks in transforming illiquid assets into liquid liabilities. Moreover, under certainty at the aggregate level the full-information optimum can be achieved by implementing a suspension of convertibility. In a similar fashion. I study the extent to which several notions of optimality can be achieved by regulating banks' behavior under aggregate uncertainty. It is worth to mention that there has been a common perception that relaxing the assumption of aggregate certainty makes the modeling intractable. To overcome this problem, I briefly describe in more detail the logics behind the probabilistic structure proposed by Allen and Gale (2004). Consider a situation in which only two banks exist, A and B, and there are only two states of nature. In state 1, bank A experiences a low demand for fiat currency and equal to 1 whereas bank B experiences a high need for fiat currency which equals 2 (see equation 2.1). In state 2, the former situation is reversed 2 (see equation 2.2).

$$\text{Aggregate state 1}: \underbrace{\text{Type-A banks' demand}}_{=1} + \underbrace{\text{Type-B banks' demand}}_{=2} = 3 \tag{2.1}$$

$$\text{Aggregate state 2}: \underbrace{\text{Type-A banks' demand}}_{=2} + \underbrace{\text{Type-B banks' demand}}_{=1} = 3 \tag{2.2}$$

The key observation is that under both scenarios, the aggregate demand for fiat currency equals 3, which in turn reflects that there is a difference between aggregate uncertainty in quantities (which I do not assess) and uncertainty on the aggregate distribution of liquidity needs (the one I focus on), and that the class of aggregate distributions I consider is mean-preserving. This property will allow me to construct several stationary equilibria and analyze their properties. Second, a lot of attention has been devoted to the non-contingent nature of deposit contracts and specially to its connection with the possibility of bank runs. One of the purposes of this paper is to show that even when deposit contracts are state-contingent, the key market failure (incomplete financial markets) remains.

Third, and related to the previous point, the role for welfare-improving regulation arises from the government's ability to induce a consumption transfer across states of nature. To illustrate this, consider a representative depositor's expected utility E(U) in 2.3 with utility arising from two consumption levels: U(good state) and U(bad state) with probabilities Prob(good state) and Prob(bad state), respectively.

$$\mathrm{E(U)} = \mathrm{Prob(good\ state)} \times \mathrm{U(good\ state)} + \mathrm{Prob(bad\ state)} \times \mathrm{U(bad\ state)} \tag{2.3}$$

My analysis focuses on the intensive margin of risk diversification or, equivalently, the ability to transfer consumption across states of nature, U(good state) and U(bad state), as opposed to the extensive margin that focuses on changing the probability of certain events (such as bank runs in other contexts). In my framework, the distribution of such probabilities is exogenous.

Fourth and finally, I acknowledge that there is a role for credit in the liquidity creation process. Although authors such as Bianchi and Bigio (2014) showed that bank credit is a sufficient condition for generating endogenous liquidity, I show that it is not necessary as the key feature is the acceptability of claims on bank accounts as medium of exchange. Also, although the analysis of economic environments with credit and default is relevant for its real world implications (and in order to keep the mechanism as clean as possible), I adopt a rather minimalist approach and rule out these two elements.

In this regard, my approach differs from Berentsen, Camera, and Waller (2007) who argue that (outside) money and credit co-exist because banks realistically take cash deposits and make cash loans, while I show that banks not only operate with outside money but create their own private (inside) money.

The **main motivation** for my study relies on the fact that the last financial crisis in the United States has represented a challenge both for policy makers and scholars. First, the collapse of the financial system and the transmission mechanisms of conventional monetary policy lead to the Federal Reserve System to implement three quantitative easing programs (QE1, QE2, and QE3) in order to stimulate the economy. Second, fiscal policy aimed at increasing the debt ceiling in order to promote economic growth. Third, by that time it became evident that the prevailing financial regulation policy had little (if any) success at ameliorating the impact of the crisis on the financial sector. That is, the solvency requirements of the Basel II regulatory framework were not effective as the banks' stress scenario mainly consisted of a lack of liquidity. In recent years, this situation lead to a new set of liquidity requirements within a new regulatory framework known as Basel III (Bank for International Settlements, 2011, 2013) (see table 1). Given this situation, policy makers are nowadays not exclusively interested in the effects of monetary/fiscal policies but also in a deeper understanding of the way the financial sector operates. Specifically, the concerns on how to reduce systemic risk through the banking system and the macroeconomic implications of such policies are collected into the field of macro-prudential regulation. In particular, one of the goals of Basel III consists of improving the banking sector's ability to absorb shocks arising from financial and economic stress which, for example, can be achieved by imposing liquidity requirements on commercial bank in order to avoid mismatches.

Even though the above situation has motivated a vast amount of literature on the optimality of monetary and fiscal policies during a period of crisis, relatively few attention has been placed on the rationale behind the new regulation of banks' and with those few studies in the field focusing either on solvency risk and capital requirements (see Park, 2016, chap. 1) or the probability of bank runs (Gertler and Kiyotaki, 2015). Moreover, there seems to be a disconnection between real-world concerns and the economic analysis of liquidity regulation as there is no reference to the market failure that justifies such intervention.

For this task, and keeping in mind that in recent years there has been an increasing interest in the general notion of liquidity *per se* within a theoretical branch of the literature, I **borrow** some elements from the New Monetarist literature (Williamson and Wright, 2010b,a; Lagos, Rocheteau, and Wright, 2017). It is worth to mention that a distinctive feature of this literature relies on its emphasis on the explicit modeling of the exchange process as a double coincidence problem in which agents trade with each other (see Lagos and Wright, 2005) and the presence of frictions (such as limited commitment and imperfect record keeping) that incentive fiat currency holding and make money essential (that is, its introduction leads to a higher welfare and/or a larger incentive-feasible allocation set). In this sense, I follow Lagos (2006) and treat money as an asset that serves as a medium of exchange since it improves the allocation in the economy as it constitutes an imperfect form of memory (see Kocherlakota, 1998). In particular, I exploit the search nature of transactions and the quasi-linearity of the preferences involved.

It is also worth to mention that for the New Monetarist literature money is not the only institution that facilitates the exchange process. Attention has also been placed on the micro-foundation of the frictions that make banking essential. This constitutes a relevant feature because recent issues associated to the financial crisis have their root on the way the banking system facilitates the exchange process. Particularly, the contribution by Diamond and Dybvig (1983) constitutes a workhorse for studying one of the reasons why financial intermediaries exist: banks provide depositors with an insurance against liquidity needs by diversifying their portfolio across liquid and illiquid assets and by transforming illiquid assets into liquid liabilities.

On the other hand, and by preserving the same spirit as in the Diamond-Dybvig model, Williamson (2012) develops a model of banking under exchange frictions in which the depositors can also use bank liabilities in transactions with third parties as inside money or, equivalently, claims on bank accounts are employed as a medium of exchange even when no financial intermediary can issue private notes. Among the features that make this novel (and realistic) approach appealing there is its ability to conform a bridge between theoretical models and the evaluation of their empirical performance. For instance, Lagos and Wright (2005, pg. 476) aimed to explain why agents hold positive amounts of fiat currency but rather use U.S. M1 (fiat currency and bank accounts) to calibrate their model whereas fiat currency has not been the largest component of this conventional measure of liquidity (see figure 1). A similar argument applies to Aruoba and Schorfheide (2011). Another reason for adopting this approach is that, despite recent theoretical advances in the analysis of Diamond-Dybvig economies, little effort has been made to integrate this framework into mainstream macroeconomics. That is, unlike conventional infinite-horizon economies with production, most of the analysis is based on three-period endowment economies and without policy variables.

To summarize, in order conduct a formal welfare analysis of the liquidity risk on banks' behavior under the presence policy variables, I develop a framework in the spirit of Williamson (2012) in which I incorporate aggregate uncertainty à la Allen and Gale (2004) in a tractable way. Another advantage of this approach is the ability to model the way in which the central bank operates as a financial intermediary as it explicitly describes how central bank's liabilities (outside money) and claims on bank deposits (inside money) are used in the exchange process. Therefore, the interaction between monetary and fiscal policies and financial regulation is examined. The main modification with respect to Williamson (2012) is that the fraction of depositors needing fiat currency is also random and the implied ex-post heterogeneity allows for the introduction of an inter-bank market in which banks adjust their portfolios according to their needs. On the other hand, the main modification with respect to Allen and Gale (2004) is that the aggregate amount of banks' deposits and the real return on assets are endogenous. It is in this sense that my model lies within the class of New Monetarist models of liquidity provision by banks under aggregate uncertainty.

3 Model setup

For the sake of simplicity, two annotations are in order. First, henceforth any individual agent will be referred to as "she" or "her". Second, for any time-indexed variable Z_t with $t = 1, 2, \dots$ represented simply by Z, let Z_- (Z') denote its corresponding lag Z_{t-1} (lead Z_{t+1}). The key elements of the model, which remain unaltered through the entire exposition, are described in this section as follows. Time is discrete with infinite horizon $(t = 0, 1, 2, \dots)$ and there are two sub-periods within each period called day and night. There is a perfectly divisible homogeneous consumption good that can be produced one-for-one with labor input during the night and stored until the next day. Also, there is a unit mass of infinitely-lived individuals[3] who consume the good during the day and exert labor effort during the night. At the beginning of each night, each individual forms a profit-maximizing firm that lasts for two consecutive sub-periods and demands labor for a given real wage w_t, measured in units of the consumption good per unit of labor input. During the night individuals and firms meet up and trade labor in a walrasian market which will be referred to as the labor market. During the day, there is two-sided search between individuals (who demand the consumption good) and firms (which sell the stored consumption good). As a result, each individual is randomly matched with a firm. Individuals' preferences are represented by

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[u\left(x_t\right) - L_t \right] \right\} \tag{3.1}$$

where $\beta \in (0,1)$ is the subjective discount factor, x_t denotes the consumption during the day and L_t denotes the extent of labor effort during the night.

The instantaneous utility function $u : \mathbb{R}_+ \to \mathbb{R}$ is assumed to be strictly increasing, strictly concave and twice continuously differentiable with $u(0) = 0$, $\lim_{x \downarrow 0} u\prime(x) = +\infty$, $\lim_{x \uparrow +\infty} u\prime(x) = +\infty$ and with the property that there exists some $\hat{x} > 0$ such that $u(\hat{x}) = \hat{x}$ (see Figure 2).

The unit mass of individuals is such that they are arranged into two equally sized groups and each of these will be referred to as an ex-ante type i with $i \in 1, 2$ (see Figure 3, part a). At the beginning of the night, each individual knows her ex-ante type and this is public information.

During the night, each individual is uncertain about the nature of her transaction during the next day. Specifically, her transaction can be either *anonymous* or *monitored*. On the one hand, if the transaction is ***anonymous*** then individuals and firms cannot agree on a credit arrangement since they know they will never meet again. However, there exists an intrinsically useless piece of paper (which is perfectly divisible and storable in any positive quantity) that can be widely used as a claim to be exchanged for goods. It is assumed that this ***fiat currency*** (also called cash) can only be issued by the government since it is difficult or impossible for private agents to produce it. Therefore, any individual that wants to acquire goods from a firm must necessarily hold it. On the other hand, if the transaction is ***monitored*** then credit between the individual and the firm is not feasible either but, in addition to fiat currency, a communication technology is <u>**also**</u> available for free and allows the individual to transfer either ***interest-bearing assets*** (which in this model consist only of government bonds) or the ownership of a ***claim on a financial intermediary*** (also called a claim on a bank account) to the firm.

[3] Let $j \in I \equiv [0,1]$ denote the j-th atomistic individual lying in the unit interval I. The mass of any semi-open interval $(\underline{\alpha}, \overline{\alpha}] \in I$ with $0 \leq \underline{\alpha} < \overline{\alpha} \leq 1$ is quantified by the Lebesgue measure $\mathcal{L}$, defined over $\mathcal{B}(I)$ the Borel σ-field consisting of all the semi-open intervals in I, and equals $\mathcal{L}(\underline{\alpha}, \overline{\alpha}] \equiv \overline{\alpha} - \underline{\alpha}$. Since $\mathcal{L}(I) = 1$, the measure space $(I, \mathcal{B}(I), \mathcal{L})$ can be interpreted as a probability space. Specifically, if an individual is randomly picked from the interval I with uniform distribution then $\mathcal{L}(\underline{\alpha}, \overline{\alpha}]$ measures the probability of such individual lying on the interval $(\underline{\alpha}, \overline{\alpha}]$. In this latter sense, the terms *mass* and *probability* will be indistinctively used.

Although individuals of a given ex-ante type are identical at the beginning of the night, each one of them receives a private, idiosyncratic, shock at the beginning of the next day that specifies whether her transaction is anonymous or monitored. For $i \in 1, 2$, the idiosyncratic shock corresponding to an ex-ante type i individual is denoted by $\theta_i \in \Theta_i \equiv 0, 1$ where

$$\theta_i = \begin{cases} 0 & \text{if the transaction is } \textit{anonymous}, \\ 1 & \text{if the transaction is } \textit{monitored}. \end{cases}$$

Henceforth, the idiosyncratic shock θ_i will be referred to as the individual's ex-post **type** and since this is private information no contract can be made explicitly contingent on it.

This economy is also subject to an aggregate shock that affects the cross-sectional distribution of needs for *assets used as means of payment* (i.e. liquidity). Since at the beginning of each day the uncertainty is resolved and each individual knows her transaction type in the goods market, for each *ex-ante type* the proportion of individuals in anonymous (and monitored) transactions is also determined. It is in this latter sense that, for $i \in \{1, 2\}$, I will refer to the *ex-post type* of a whole group of ex-ante type i individuals.

Let the aggregate shock be denoted by the random vector $\eta \equiv (\rho_1, \rho_2) \in [0, 1] \times [0, 1]$ where ρ_1 (ρ_2) denotes the proportion of ex-ante type 1 (2) individuals engaged in anonymous transactions.[4] For a sake of tractability, I assume that $\eta \in H \equiv \{\eta_1, \eta_2\}$ with $\eta_1 \equiv (\rho_L, \rho_H)$, $\eta_2 \equiv (\rho_H, \rho_L)$ and $0 < \rho_L < \rho_H < 1$. This assumption corresponds to a narrowed-down version of the aggregate risk structure in Allen and Gale (2004) and allows me to relax the lack of aggregate uncertainty in Diamond and Dybvig (1983) in the simplest way possible by not only allowing for one but two aggregate states of nature. Furthermore, the analysis under complete (incomplete) markets can be easily performed by only including two (one) financial assets (asset).

Uncertainty is resolved at the beginning of the day when the aggregate state η is drawn and each individual discovers her idiosyncratic shock. Given the assumptions on the distribution of individuals, the probability of being an individual of type (i, θ_i) conditional on state η is denoted by $\lambda_i(\theta_i, \eta) > 0$ for $\eta \in H$ and $i = 1, 2$. Also, the ex-ante probability of being an individual of type i is $1/2$. Therefore, consistency requires that (see figure 3, part b)

$$\sum\nolimits_{\theta_i \in \Theta_i} \lambda_i(\theta_i, \eta) = \lambda_i(0, \eta) + \lambda_i(1, \eta) = 1/2 \tag{3.2}$$

for $\eta \in H$ and $i = 1, 2$. To summarize, the cross-sectional distribution of ex-ante types is assumed to be the same as the probability distribution λ and then, by the **law of large numbers** convention, $\lambda_i(\theta_i, \eta) > 0$ is interpreted as the "mass" of individuals of type (i, θ_i) in state η.

[4] Conversely, $1 - \rho_i$ denotes the proportion of ex-ante type i individuals engaged in monitored transactions for $i \in \{1, 2\}$.

4 The case of complete financial markets

In this section I complement the aforementioned environment with specific financial markets, provide a liquidity-insurance role for banks as in Diamond and Dybvig (1983) and introduce a public sector in a consistent way. Namely, I first assume the existence of complete financial markets as there are as many Arrow securities as aggregate states of nature or, equivalently,

$$\text{number of Arrow securities} = 2 = \text{number of aggregate states of nature.}$$

The only interest-bearing asset in this economy is a government bond which is defined as an account balance held with the government. This bond is sold during the night of period t for one unit of fiat currency and pays off $g'(\eta')$ units of fiat currency at the beginning of the night of period $t+1$ if $\eta' \in H$ is drawn. Also, let ϕ denote the price of fiat currency in terms of consumption goods[5] during the night of period t and let $r'(\eta') \equiv \frac{\phi'}{\phi} g'(\eta')$ denote the gross real interest rate on government debt if $\eta' \in H$ is drawn during the day of period $t+1$.

4.1 Banks

Given the *aggregate risk* structure, one-period lived banks form at the beginning of each night and each bank can be run by any individual (before each one knows whether her transaction will be anonymous or monitored during the subsequent day). These banks dissolve at the beginning of the next night and are replaced by new one-period lived banks. During the night each individual exerts labor effort in order to make a deposit with a bank in exchange for a risk sharing contract.[6]

4.2 Asset markets

4.2.1 Arrow security markets (night)

Only banks have access to a complete set of Arrow security markets during the night: for each aggregate state $\eta' \in H$ there is a security traded during the night that promises one unit of the consumption good during the next day if the state η' is drawn and nothing otherwise. Let $q(\eta') > 0$ denote the price of one unit of the Arrow security corresponding to the aggregate state η' or, equivalently, the number of units of the consumption good during the night needed to buy one unit of that good in state η' during the next day.

[5] Let P denote the price of a unit of the consumption good in terms of fiat currency. Therefore, $\phi = 1/P$ is interpreted as the price of a unit of fiat currency in terms of consumption goods.

[6] Without loss of generality, I assume that each bank is run by only one individual and that no individual can offer herself a deposit contract.

4.2.2 Spot markets (day)

Since all uncertainty is resolved at the beginning of the day there is no need to trade contingent securities during this sub-period. I instead assume that there exist spot markets both for fiat currency (cash) and interest-bearing assets (government bonds) during the day. Let the fiat currency be the numeraire so its real price will be denoted by $p_{\text{cash}}(\eta') = 1$ for all $\eta' \in H$. Then, the price of the government bond $p_{\text{bond}}(\eta') > 0$ is the number of units of the consumption good during the day needed to purchase one unit of the bond in state η'. Therefore, let $(p_{\text{cash}}(\eta'), p_{\text{bond}}(\eta')) = (1, p(\eta'))$ denote the vector of asset prices during the day in state η'. It is also assumed that only banks have access to this spot (inter-bank) market and that the government bonds cannot be liquidated during the day. That is, a bank facing a relatively high need for cash during the day sells bonds to other bank.

4.3 Deposit contract and mechanism design

Even though individuals are able to self-insure during the night by acquiring fiat currency and/or interest-bearing assets on their own, they also have the option to participate in the rest of the financial markets indirectly, though financial intermediaries.

A bank, understood as a financial intermediary, is a risk-sharing institution that invests in financial assets (Arrow securities, fiat currency and government bonds) on behalf of its depositors (individuals) and provides them with assets that allow them to consume during the next day. These banks use financial markets to hedge the risks that they manage for depositors.

A bank servicing an ex-ante type i individual offers a contingent deposit contract[7] that promises to provide a certain amount of units of fiat currency (in real terms) if an anonymous transaction is reported and other certain amount of units of claims on bank deposits (also in real terms) if a monitored transaction is reported.

It is worth to mention that for each ex-ante type i individual, the realization of her ex-post type during the next day is private information which in turn implies that if such individual is engaged in an anonymous transaction she will report it since claims on bank accounts are useless for exchange. However, it also implies that if the same individual is engaged in a monitored transaction but the benefit from (mis)reporting an anonymous transaction is greater than those arising from telling the truth, then that depositor has the incentive to lie (i.e. to misrepresent herself). In order to induce truth-telling, each bank imposes a condition on the design of its deposit contract which will be referred to as the *incentive constraint* and is stated as follows: for an ex-ante type i individual (depositor) in a monitored transaction ($\theta_i = 1$) to tell the truth, it has to be the case that the benefit from reporting $\hat{\theta}_i = 1$ is not lower than those from reporting $\hat{\theta}_i = 0$. A formal version of the latter condition is provided below.

[7]Equivalently, this deposit contract is assumed to be efficient.

Given a real wage w, each ex-ante type i individual exerts a certain amount of labor effort and the corresponding firm opens a bank account on her behalf. Such bank account (which is backed by the final goods stored by the firm) is worth enough units of consumption for the servicing bank to purchase $m_i^{night} \geq 0$ units of fiat currency and $b_i^{night} \geq 0$ units of government bonds (both in real terms) during the night. In exchange she receives a bundle of assets

$$x_i(\theta_i, \eta') \equiv (m_i^{day}(\theta_i, \eta'), a_i^{day}(\theta_i, \eta')) \in \mathbb{R}_+^2,$$

where the first (second) component denotes the amount of fiat currency (claims on her bank account) in real terms, such that

$$x_i(\theta_i, \eta') = \begin{cases} (m_i^{day}(\eta'), 0) & \text{if } \theta_i = 0 \text{ and} \\ (0, a_i^{day}(\eta')) & \text{if } \theta_i = 1 \end{cases}$$

for all $\eta' \in H$. The function $x_i : \Theta \times H \to \mathbb{R}_+^2$ is a *direct mechanism* that maps depositors' (individuals') reports into asset allocations. The previous description states that each depositor receives only one type of asset based on her **reported (ex-post) type**: she will retrieve $m_i^{day}(\eta') \geq 0$ ($a_i^{day}(\eta') \geq 0$) real units of fiat currency (claims on his bank account) if she reports an anonymous (a monitored) transaction.

For each aggregate state $\eta' \in H$, let $U_i(x_i(\cdot, \eta'), \cdot) : \Theta_i \times \Theta_i \to \mathbb{R}_+$ be the function such that $U_i(x_i(\hat{\theta}_i, \eta'), \theta_i)$ represents the instantaneous payoff during the day of an individual that reports the ex-post type $\hat{\theta}_i \in \Theta_i$ and whose (private) ex-post type is $\theta_i \in \Theta_i$. Specifically,

$$U_i(x_i(0, \eta'), 0) = u(m_i^{day}(\eta')), \tag{4.1}$$

$$U_i(x_i(1, \eta'), 0) = u(0), \tag{4.2}$$

$$U_i(x_i(0, \eta'), 1) = u(m_i^{day}(\eta')) \text{ and} \tag{4.3}$$

$$U_i(x_i(1, \eta'), 1) = u(a_i^{day}(\eta')). \tag{4.4}$$

Some comments are in order. In (4.1), a truth-telling ex-ante type i individual in an anonymous transaction retrieves $m_i^{day}(\eta')$ units of fiat currency (in real terms) and consumes $m_i^{day}(\eta')$ during the day. If this individual misreports her ex-post type, the right-hand side term in (4.2) reflects the fact that claims on bank accounts are not accepted as a medium of exchange. On the other hand, the right-hand side of (4.3) states that if an individual in a monitored transaction misreports his type she can still get to consume during the day as fiat currency is also accepted as a medium of exchange. Finally, in (4.4) a truth-telling ex-ante type i individual in an monitored transaction is handed $a_i^{day}(\eta')$ units of claims on bank accounts (in real terms) and consumes $a_i^{day}(\eta')$. Therefore, within this framework the **incentive constraint (IC)** is stated as

$$U_i(x_i(\theta, \eta'), \theta_i) \geq U_i(x_i(\hat{\theta}, \eta'), \theta_i), \forall \theta_i, \hat{\theta}_i \in \Theta, \forall \eta' \in H$$

which, in particular, implies that any notion of truth-telling equilibrium with banks must necessarily satisfy

$$u(m_i^{day}(\eta')) \geq u(0) \text{ and} \tag{4.5}$$

$$u(a_i^{day}(\eta')) \geq u(m_i^{day}(\eta')), \forall \eta' \in H. \tag{4.6}$$

Given the properties of the instantaneous utility function u, it is easy to show that (4.5) and (4.6) are equivalent to $m_i^{day}(\eta') \geq 0$ and $a_i^{day}(\eta') \geq m_i^{day}(\eta')$, $\forall \eta \in H$, respectively.

4.4 Banks' behavior in equilibrium

Banks are profit-maximizing and each one offers a deposit contract. As a consequence, it is easy to show that perfect competition and free entry (which are assumed in this paper) force banks to undermine each other by making more attractive offers to individuals. As a result, each of the two prevailing banks' offers maximizes the expected utility of the corresponding ex-ante depositor and earns zero profits. Formally, given a lump-sum tax $\tau(\eta)$ corresponding to the already known aggregate shock η during the night of period t and given the collection $(w, \phi\prime/\phi, p(\eta_1), p(\eta_2), q(\eta_1), q(\eta_2), r(\eta_1), r(\eta_2)) \in \mathbb{R}^8_{++}$, for $i = 1, 2$ the deposit contract offered by the bank servicing the ex-ante type i buyers solves

$$\max_{(L_i, m_i^{night}, b_i^{night}, m_i^{day}(\eta_1), a_i^{day}(\eta_1), m_i^{day}(\eta_2), a_i^{day}(\eta_2)) \in \mathbb{R}^7_+} \Big\{ -L_i \\ + \beta \underbrace{\left[\lambda_i(0,\eta_1) u\left(m_i^{day}(\eta_1)\right) + \lambda_i(1,\eta_1) u\left(a_i^{day}(\eta_1)\right) \right]}_{\text{includes conditional expected utility, given } \eta=\eta_1} \\ + \beta \underbrace{\left[\lambda_i(0,\eta_2) u\left(m_i^{day}(\eta_2)\right) + \lambda_i(1,\eta_2) u\left(a_i^{day}(\eta_2)\right) \right]}_{\text{includes conditional expected utility, given } \eta=\eta_2} \Big\} \quad (4.7)$$

subject to

$$m_i^{night} + b_i^{night} \leq wL_i - \tau(\eta), \quad (4.8)$$

$$a_i^{day}(\eta_1) \geq m_i^{day}(\eta_1), \quad (4.9)$$

$$a_i^{day}(\eta_2) \geq m_i^{day}(\eta_2) \quad (4.10)$$

and

$$q(\eta_1) \underbrace{\left[\lambda_i(0,\eta_1) m_i^{day}(\eta_1) + \lambda_i(1,\eta_1) p(\eta_1) a_i^{day}(\eta_1) \right]}_{\text{cost of assets when } \eta=\eta_1} \\ + q(\eta_2) \underbrace{\left[\lambda_i(0,\eta_2) m_i^{day}(\eta_2) + \lambda_i(1,\eta_2) p(\eta_2) a_i^{day}(\eta_2) \right]}_{\text{cost of assets when } \eta=\eta_2} \\ \leq q(\eta_1) \underbrace{\left[\tfrac{1}{2} \tfrac{\phi'}{\phi} m_i^{night} + \tfrac{1}{2} p(\eta_1) r'(\eta_1) b_i^{night} \right]}_{\text{value of investments when } \eta=\eta_1} + q(\eta_2) \underbrace{\left[\tfrac{1}{2} \tfrac{\phi'}{\phi} m_i^{night} + \tfrac{1}{2} p(\eta_2) r'(\eta_2) b_i^{night} \right]}_{\text{value of investments when } \eta=\eta_2}. \quad (4.11)$$

In 4.7, each expression in square brackets involves the expected utility of an ex-ante type i depositor, given a realization of the aggregate shock. This so happens because there exist two possible aggregate distributions for liquidity needs during the next day. Also, the previous adoption of the law of large numbers convention allows to reinterpret the bank's objective function as one corresponding to a social planner focused on maximizing the welfare of only a certain ex-ante type. In this setting, the contingent deposit contract involves not only the asset holdings during the night $(m_i^{night}, b_i^{night})$ but also the labor effort required to acquire it. This can be seen in 4.8 where $wL_i - \tau(\eta)$ denotes the depositor's net income. On the other hand, 4.9 and 4.10 represent the incentive constraints: in any truth-telling equilibrium, each bank provides its depositors in monitored transactions with at least as much consumption as those provided to its depositors in anonymous transactions.

In state $\eta' \in H$, the cost of the assets for depositors who report $\theta_i = 0$ equals $m_i^{day}(\eta')$, the cost for depositors who report $\theta_i = 1$ equals $p(\eta')a_i^{day}(\eta')$ and there are $\lambda_i(0,\eta')$ and $\lambda_i(1,\eta')$ such depositors, respectively. The total cost therefore equals $\lambda_i\left(0,\eta'\right) m_i^{day}\left(\eta'\right) + \lambda_i(1,\eta')p(\eta')a_i^{day}(\eta')$. Multiplying by the cost of one unit of consumption during the day in state η' and summing across aggregate states of nature η' delivers the total cost of the direct mechanism, in terms of units of consumption during the night, as the left-hand side of 4.11. The right-hand side is the total value of investments by the bank. In state $\eta' \in H$ fiat currency yields $\frac{\phi'}{\phi} m_i^{night}$ units of the consumption good during the day and the government bond yields $r'(\eta')b_i^{night}$ units of the good at the beginning of the next night so the total value of the portfolio during the day is $\frac{\phi'}{\phi} m_i^{night} + p(\eta')r'(\eta')b_i^{night}$ times the mass of ex-ante type i buyers which equals 1/2. Multiplying by the price of a unit of the good during the day in state η' and summing across these states gives the total value of the bank's portfolio in terms of units of the consumption good during the night. Finally, it is worth to emphasize that the existence of a unique budget constraint for each bank reflects the assumption of complete financial markets as each intermediary is able to transfer consumption across aggregate states of nature.

4.5 Public sector: monetary and (passive) fiscal policies

It is assumed that the government levies lump-sum taxes on individuals during the beginning of each night before the corresponding goods market opens. For later use let $\tau(\eta)$, with $\eta \in H$, denote the real (i.e. in units of consumption goods) tax per individual, let M denote the units of government's outstanding currency during the night of period t and let B denote the stock of bonds of the consolidated government (all of which are held by the private sector). The consolidated government budget constraint is therefore given by

$$\phi\left(M + B\right) + \tau(\eta) = \phi(M_{-} + g(\eta)B_{-}), \text{ for } t = 1, 2, \ldots \tag{4.12}$$

or, equivalently, the total value of government's net outstanding liabilities (in real terms) at the end of period t, plus tax revenues, equals government's net outstanding liabilities at the beginning of the same period (also in real terms). Assume further that private agents are endowed with no outstanding liabilities at the beginning of the first period:

$$\phi_0\left(M_0 + B_0\right) + \tau_0 = 0. \tag{4.13}$$

The class of rules here considered assumes that the monetary authority commits to a policy such that the total stock of nominal currency M grows at a constant gross rate $\mu > 0$,

$$M' = \mu M, \tag{4.14}$$

and the ratio of currency to total nominal government debt remains fixed at $\delta \in (0, 1)$,

$$\frac{M}{M + B} = \delta. \tag{4.15}$$

I will consider the case in which monetary policy leads and the path of lump-sum taxes changes passively to support it. Given this assumption, (4.12)-(4.15) imply that the lump-sum taxes are determined according to (see appendix A)

$$\tau(\eta) = -\frac{\phi M}{\delta}\left(1 - \frac{1}{\mu}\right) + \frac{\phi}{\phi_-}\phi_- M_-\left(\frac{1}{\delta} - 1\right)\left[\frac{\phi_-}{\phi}r(\eta) - 1\right],\ t = 1, 2, \ldots, \text{ and} \tag{4.16}$$

$$\tau_0 = -\frac{\phi_0 M_0}{\delta}. \tag{4.17}$$

4.6 Labor and asset market-clearing conditions

To close the model, certain conditions are required for the goods, labor and financial markets to clear up. On the one hand, given the real wage w, firms are assumed to choose the product-labor combination (Y, N) to solve

$$\max_{(y,n)\in\mathbb{R}^2_+} \{y - wn\} \tag{4.18}$$

subject to the technological constraint

$$y = n. \tag{4.19}$$

The solution to the problem above leads to the demand for labor in this economy. On the other hand, the solution to the banks' problem already described leads, among others, to their ex-ante demand for fiat currency and government bonds. During the night, the perfectly inelastic supply for both assets is provided by the government. During the next day, all the uncertainty is resolved which implies that each bank experiences a specific liquidity need. It is assumed that, to re-balance their portfolios, banks meet up in an inter-bank (Walrasian) market where the relative price of government bonds $p(\eta')$ is determined for each $\eta' \in H$. It is also assumed that no individual depositor has access to this inter-bank market.

Equations (4.20)-(4.26) summarize the market-clearing conditions for both assets during the night and the next day. For a sake of exposition, let each left-hand (right-hand) side term denote the supply (demand) in the corresponding market:

$$\begin{aligned}
\sum\nolimits_{i=1}^{2}\tfrac{1}{2}L_i &= N, && (4.20)\\
\phi M &= \sum\nolimits_{i=1}^{2}\tfrac{1}{2}m_i^{night}, && (4.21)\\
\phi B &= \sum\nolimits_{i=1}^{2}\tfrac{1}{2}b_i^{night}, && (4.22)\\
\tfrac{\phi'}{\phi}\sum\nolimits_{i=1}^{2}\tfrac{1}{2}m_i^{night} &= \sum\nolimits_{i=1}^{2}\lambda_i(0,\eta')m_i^{day}(\eta') \text{ for } \eta'\in H, && (4.23)\\
r'(\eta')\sum\nolimits_{i=1}^{2}\tfrac{1}{2}b_i^{night} &= \sum\nolimits_{i=1}^{2}\lambda_i(0,\eta')a_i^{day}(\eta') \text{ for } \eta'\in H, && (4.24)
\end{aligned}$$

$$\begin{aligned}
&\left[\lambda_1(0,\eta_1)m_1^{day}(\eta_1)+\lambda_2(0,\eta_1)m_2^{day}(\eta_1)\right]+p(\eta_1)\left\{\lambda_1(1,\eta_1)a_1^{day}(\eta_1)+\lambda_2(1,\eta_1)a_2^{day}(\eta_1)\right\}\\
&\quad=\tfrac{\phi'}{\phi}\left[\tfrac{1}{2}m_1^{night}+\tfrac{1}{2}m_2^{night}\right]+p(\eta_1)r'(\eta_1)\left\{\tfrac{1}{2}b_1^{night}+\tfrac{1}{2}b_2^{night}\right\} \text{ and}
\end{aligned} \tag{4.25}$$

$$\begin{aligned}
&\left[\lambda_1(0,\eta_2)m_1^{day}(\eta_2)+\lambda_2(0,\eta_2)m_2^{day}(\eta_2)\right]+p(\eta_2)\left\{\lambda_1(1,\eta_2)a_1^{day}(\eta_2)+\lambda_2(1,\eta_2)a_2^{day}(\eta_2)\right\}\\
&\quad=\tfrac{\phi'}{\phi}\left[\tfrac{1}{2}m_1^{night}+\tfrac{1}{2}m_2^{night}\right]+p(\eta_2)r'(\eta_2)\left\{\tfrac{1}{2}b_1^{night}+\tfrac{1}{2}b_2^{night}\right\}.
\end{aligned} \tag{4.26}$$

In 4.20, it is assumed that all firms demand the same extent of labor effort during the night and that the supplied labor effort is the same within ex-ante types. Equation 4.21 states that during the night the real supply of fiat currency must equal the demand for real money balances across banks working for ex-ante individuals. A similar description applies to equation 4.22 where the real supply of bonds must equal the demand for these assets by banks. The real demand for assets during the night determines its real supply during the next day, when banks trade with each other in the inter-bank market, which is reflected in 4.23 for the real demand for fiat currency. Equation 4.24 applies the same logic to the real demand for interest-bearing assets during the day. Finally, 4.25 and 4.26 represent the market-clearing conditions for Arrow securities under the aggregate states η_1 and η_2, respectively.

5 Competitive equilibrium with complete markets

5.1 Definition

Equipped with the above structure, in this section I define the corresponding competitive equilibrium and use the upper-bar notation $\bar{x}$ to represent the equilibrium value of x. It is also worth to mention that, provided with the initial conditions ϕ_0 and M_0, the policy mix (μ, δ) determines the path for the nominal variables $(\phi_t, M_t, B_t)_{t=0}$. Finally, the assumption of quasi-linear preferences implies both $\beta/\mu \leq 1$ and $\beta r'(\eta') \leq 1$ for $\eta' = \eta_1, \eta_2$ and the non-negativity of the nominal interest rates requires $\mu r'(\eta') \geq 1$ for $\eta' = \eta_1, \eta_2$. These conditions establish upper and lower bounds for the real return on interest-bearing assets.

Definition 1. *Given the initial condition (ϕ_0, M_0) and a monetary policy (μ, δ), a stationary **competitive equilibrium with complete financial markets** consists of a price system $(\bar{p}(\eta_1), \bar{p}(\eta_2), \bar{q}(\eta_1), \bar{q}(\eta_2))$, price dynamics $\bar{\phi}'/\bar{\phi}$, a real wage $\bar{w}$, gross real returns $(\bar{r}(\eta_1), \bar{r}(\eta_2))$, a feasible allocation $(\bar{Y}, \bar{N})$, complete contingent deposit contracts for individuals $\left\{(\bar{L}_i, \bar{m}_i^{night}, \bar{b}_i^{night}, \bar{m}_i^{day}(\eta_1), \bar{a}_i^{day}(\eta_1), \bar{m}_i^{day}(\eta_2), \bar{a}_i^{day}(\eta_2))\right\}_{i=1,2}$, taxes $(\bar{\tau}(\eta_1), \bar{\tau}(\eta_2))$ for periods $t = 1, 2, \ldots$ and an initial tax $\bar{\tau}_0$ such that:*

1. *The gross real returns are bounded: $\bar{\phi}'/\bar{\phi} \leq \bar{r}(\eta) \leq 1/\beta$ for $\eta = \eta_1, \eta_2$.*

2. *Given the equilibrium wage $\bar{w}$, the feasible allocation $(\bar{Y}, \bar{N})$ solves the profit maximization problem 4.18 subject to the technological constraint 4.19.*

3. *For $i = 1, 2$, given the equilibrium values $\bar{\tau}(\eta)$, $\bar{w}$, ϕ'/ϕ, $(\bar{p}(\eta_1), \bar{p}(\eta_2), \bar{q}(\eta_1), \bar{q}(\eta_2))$ and $(\bar{r}(\eta_1), \bar{r}(\eta_2))$, the complete contingent deposit contract offered to ex-ante type i individuals $\left(\bar{L}_i, \bar{m}_i^{night}, \bar{b}_i^{night}, \bar{m}_i^{day}(\eta_1), \bar{a}_i^{day}(\eta_1), \bar{m}_i^{day}(\eta_2), \bar{a}_i^{day}(\eta_2)\right)$ solves the problem of maximizing their expected utility in 4.7 subject to 4.8-4.11.*

4. *The monetary policy rules 4.14 and 4.15 hold.*

5. *For $\eta \in H$, $\bar{\tau}(\eta)$ satisfies 4.16 for $r(\eta) = \bar{r}(\eta)$ and $\bar{\tau}_0 = -\phi_0 M_0/\delta$ satisfies 4.17.*

6. *Labor and asset markets clear: 4.20-4.26 hold.*

Therefore, in a competitive equilibrium with complete financial markets: 1) the real return on interest-bearing assets is bounded 2) firms take the real wage as given and maximize profits, 3) banks take prices and returns as given and provide a deposit contract that maximizes depositors' utility, 4) monetary policy rules hold, 5) the government's budget constraint hold and 6) the labor market clears, the government supply of fiat currency and bonds meets the banks' ex-ante demand for these assets during the night, and the banks' holdings of fiat currency and government bonds meet their ex-post demand for these assets during the day.

5.2 Characterization and some properties

The equilibrium previously defined extends the analysis of Allen and Gale (2004) in two directions. First, it extends to the class of production economies where both the aggregate amount of assets and their (real) return are endogenously determined. Second, money is essential as in Lagos and Wright (2005) and banking is essential as in Diamond and Dybvig (1983) within a framework that resembles Williamson (2012). However, the main departure from Williamson (2012) consists on the inclusion of only one group of individuals as the so-called New-Monetarist "sellers" are replaced by firms that behave as retailers during each day. An appealing property of this approach relies on the way to perform welfare analysis as the expected utility banks seek to maximize becomes the welfare measure as well. I this regard, banks behave as social planners that take prices as given.[8]

The characterization to the above equilibrium is shown in Appendix B. Since the instantaneous utility function u is strictly increasing, B.1-B.4 represent the bank's binding constraints in 4.8 during the night for $i = 1, 2$ and $\eta = \eta_1, \eta_2$, whereas B.5 and B.6 denote the binding budget constraints during the day 4.11 for banks servicing ex-ante depositors $i = 1$ and $i = 2$. At the individual level, in B.7, if the right-hand side is greater (lower) than ϕ'/ϕ, then government bonds are (fiat currency is) better at transferring consumption to the next day and banks have the incentive to accumulate an unlimited amount of bonds (cash) and no cash (bond) at all. In equilibrium, for both assets to co-exist in positive quantities it must be the case that banks are indifferent among these two financial assets. The associated condition formalizes the argument provided in Allen and Gale (2004). Conditions B.8 and B.9 are the cash-related Euler equations for ex-ante types $i = 1$ and $i = 2$, respectively. Also, conditions B.10 and B.11 are the bond-related Euler equations for ex-ante types $i = 1$ and $i = 2$, respectively. On the other hand, B.12 and B.13 are the pricing equations for bonds in the inter-bank market and, as usual, the equilibrium price of bonds reflects the marginal rate of substitution of fiat currency for claims on bank accounts in real terms. Condition B.14 is the pricing equation (in relative terms) for Arrow securities. Conditions B.15 and B.16 summarize the monetary policy rules and B.17, B.18 and B.19 reflect the fiscal policy consistent with this monetary policy. The labor market-clearing condition is given by B.20. Also, B.21 and B.22 denote the cash and bond market-clearing conditions during the night. On the other hand, equations B.23, B.24, B.25 and B.26 represent the market-clearing conditions during the day under η_1 and η_2 for cash and bonds, respectively. Finally, B.27 and B.28 are the market-clearing conditions for both aggregate states of nature.

For a sake of exposition, I assume that the probabilities in $\{\{\{\lambda_i(\theta_i, \eta)\}_{\theta_i \in \Theta_i}\}_{\eta \in H}\}_{i=1,2}$ hereafter satisfy

$$\begin{aligned}
\lambda_1(0, \eta_1) &= \lambda_2(0, \eta_2) = \tfrac{1}{2}\rho_L, \\
\lambda_1(1, \eta_1) &= \lambda_2(1, \eta_2) = \tfrac{1}{2}(1 - \rho_L), \\
\lambda_1(0, \eta_2) &= \lambda_2(0, \eta_1) = \tfrac{1}{2}\rho_H \text{ and} \\
\lambda_1(1, \eta_2) &= \lambda_2(1, \eta_1) = \tfrac{1}{2}(1 - \rho_H).
\end{aligned}$$

As it is widely known in this class of models, under the presence of Arrow securities the implied consumption plans depend on the aggregate resources of the entire economy. It

[8] In a standard New-Monetarist setting, the welfare measure results from aggregating buyers and sellers expected utilities. Therefore, the extents of labor effort exerted by both agents cancel out and (by construction) the disutility of labor has no effect on welfare.

is easy to show that, in a parallel way to Gollier (1996), this *mutuality principle* relies on the existence of complete financial markets for aggregate risk and its ability to hedge diversifiable (liquidity) risks. To reflect this, I construct an equilibrium under the additional assumption of symmetry (i.e. all ex-ante decisions are the same). Consequently, all the equilibrium variables remain the same across ex-ante types and aggregate states of nature. It is straightforward to obtain the equilibrium values corresponding to

$$\begin{aligned}
\bar{r} &\equiv \bar{r}(\eta_1) = \bar{r}(\eta_2), \\
\bar{m}^{night} &\equiv \bar{m}_1^{night} = \bar{m}_2^{night}, \\
\bar{b}^{night} &\equiv \bar{b}_1^{night} = \bar{b}_2^{night}, \\
\bar{m}^{day} &\equiv \bar{m}_1^{day}(\eta_1) = \bar{m}_1^{day}(\eta_2) = \bar{m}_2^{day}(\eta_1) = \bar{m}_2^{day}(\eta_2) \text{ and} \\
\bar{a}^{day} &\equiv \bar{a}_1^{day}(\eta_1) = \bar{a}_1^{day}(\eta_2) = \bar{a}_2^{day}(\eta_1) = \bar{a}_2^{day}(\eta_2),
\end{aligned}$$

by solving the following equations (see appendix C):

$$\begin{aligned}
1 &= \frac{\beta}{\mu} u'(\bar{m}^{day}), && (5.1) \\
1 &= \beta \bar{r} u'(\bar{a}^{day}), && (5.2) \\
\frac{1}{\mu} \bar{m}^{night} &= \rho \bar{m}^{day}, && (5.3) \\
\bar{r} \bar{b}^{night} &= (1-\rho) \bar{a}^{day} \text{ and} && (5.4) \\
\frac{\bar{m}^{night}}{\bar{m}^{night} + \bar{b}^{night}} &= \delta && (5.5)
\end{aligned}$$

where $\rho \equiv \frac{1}{2}\rho_L + \frac{1}{2}\rho_H$ denotes the "average" proportion of depositors in anonymous (i.e. cash-only) transactions. In this equilibrium, the linear technology implies that $\bar{w} = 1$, the real demand for fiat currency remains constant and therefore the price dynamics is determined by monetary policy $\bar{\phi}'/\bar{\phi} = \bar{\phi}/\bar{\phi}_{-} = 1/\mu$. Also, the trading of Arrow securities implies that the consumption under anonymous and monitored transactions are each perfectly smoothed across aggregate states and then $\bar{q}(\eta_2)/\bar{q}(\eta_1) = 1$ which in turn implies $\bar{p}\bar{r} = 1/\mu$. Other properties of this equilibrium can be obtained from 5.1-5.5. First, it becomes evident that the introduction of 5.5 along with the currency-to-liability ratio δ is necessary to close the model by pinning down the real return $\bar{r}$. On the other hand, from 5.1 and 5.3 the existence of both $\bar{m}^{day}$ and $\bar{m}^{night}$ is always guaranteed. Also, notice that although the individual demand for cash $\bar{m}^{day}$ positively depends on μ, the relationship between the aggregate demand for cash $\bar{m}^{night}$ and μ depends on the coefficient of Relative Risk Aversion (RRA) associated to the utility function u. A similar situation applies to 5.2 and 5.4 when deriving the equilibrium relationship between $\bar{b}^{night}$ and $\bar{r}$. These results, which are obtained by direct calculation, are summarized as follows.

Proposition 1. *Suppose that for the economy described above there exists a stationary competitive equilibrium with complete financial markets that exhibits symmetry and satisfies the mutuality principle. For such equilibrium, let μ and $\bar{m}^{night}$ denote the money growth rate and* *the equilibrium real cash holdings, respectively. Also, let $\bar{r}$ and $\bar{b}^{night}$ denote the equilibrium real gross return and bond holdings, respectively. Then,*

$$\frac{\partial \bar{m}^{night}}{\partial \mu} \lesseqqgtr 0 \textit{ if and only if } RRA\left(\frac{\bar{m}^{night}}{\mu\rho}\right) \equiv -\frac{u''\left(\frac{\bar{m}^{night}}{\mu\rho}\right)}{u'\left(\frac{\bar{m}^{night}}{\mu\rho}\right)}\frac{\bar{m}^{night}}{\mu\rho} \lesseqqgtr 1 \textit{ and}$$

$$\frac{\partial \bar{b}^{night}}{\partial \bar{r}} \gtreqqless 0 \textit{ if and only if } RRA\left(\frac{\bar{r}\bar{b}^{night}}{1-\rho}\right) \equiv -\frac{u''\left(\frac{\bar{r}\bar{b}^{night}}{1-\rho}\right)}{u'\left(\frac{\bar{r}\bar{b}^{night}}{1-\rho}\right)}\frac{\bar{r}\bar{b}^{night}}{1-\rho} \lesseqqgtr 1.$$

The main result of this section (i.e. the mutuality principle) constitutes a benchmark that will be compared with the results arising from a counter-factual, almost identical, economy that exhibits a marginal change in the asset market structure.

6 A case of incomplete financial markets

The previous characterization constitutes a benchmark to be compared against counter-factual economies possessing a different financial-markets structure. In this section I describe an alternative economy that exhibits almost the same setup as in section 5 but now there is a lack of complete financial markets (see Figure 4). Specifically,

$$\text{number of state-contingent assets} = 0 < 2 = \text{number of aggregate states of nature.}$$

Hereafter, some prior notation is further simplified for a sake of clarity. First, by construction, remember that if $\eta' = \eta_1$ then banks servicing ex-ante type $i = 1$ depositors face a fraction ρ_L of these being in cash-only transactions and banks servicing ex-ante type $i = 2$ depositors face a fraction ρ_H of these being in cash-only transactions. Conversely, if $\eta' = \eta_2$ then banks servicing ex-ante type $i = 1$ depositors face a fraction ρ_H of these being in cash-only transactions and banks servicing ex-ante type $i = 2$ depositors face a fraction ρ_L of these being in cash-only transactions. This description implies that, regardless of the specific aggregate state $\eta' \in H$, there will always be one bank experiencing a low need for cash ρ_L and one bank experiencing a high need for cash ρ_H. It will be shown that based on the probability structure already adopted and, without loss of generality (since spot prices and real returns are state-independent within the new equilibrium concept), ex-post individual consumption levels on either ρ_L or ρ_H.

6.1 Asset markets

I mainly adapt the description in section 4 for the case in which banks only accumulate cash and/or bonds during the night and re-balance their portfolios during the day at a given non-contingent relative price of bonds. There exist a spot market for cash and a spot market for bonds during the day. Since fiat currency is the numeraire, its real price will be denoted by $p_{\text{cash}} = 1$ and the price of the government bond $p_{\text{bond}} > 0$ is the number of units of the consumption good during the day needed to purchase one unit of the bond. Let $(p_{\text{cash}}, p_{\text{bond}}) = (1, p)$ denote the vector of asset prices during the day. Only banks have access to this spot (inter-bank) market and the government bonds cannot be liquidated during the day.

6.2 Banks' behavior in equilibrium

Profit-maximizing banks offer a deposit contract each. Once again, perfect competition and free entry imply that each bank's deposit contract maximizes the expected utility of its corresponding depositor and earns no profits. Given τ, $(w, \phi'/\phi)$ and (p, r'), the deposit contract offered to an ex-ante type i individual (depositor) solves[9]

$$\begin{aligned}\max_{(L_i, m_i^{night}, b_i^{night}, m_i^{day}(\rho_L), a_i^{day}(\rho_L), m_i^{day}(\rho_H), a_i^{day}(\rho_H)) \in \mathbb{R}_+^7} &\Big\{-L_i \\ &+\beta\tfrac{1}{2}\left[\rho_L u\left(m_i^{day}(\rho_L)\right) + (1-\rho_L)u\left(a_i^{day}(\rho_L)\right)\right] \\ &+\beta\tfrac{1}{2}\left[\rho_H u\left(m_i^{day}(\rho_H)\right) + (1-\rho_H)u\left(a_i^{day}(\rho_H)\right)\right]\Big\} \quad (6.1)\end{aligned}$$

subject to

$$\begin{aligned} m_i^{night} + b_i^{night} &\leq wL_i - \tau, & (6.2)\\ a_i^{day}(\rho_L) &\geq m_i^{day}(\rho_L), & (6.3)\\ a_i^{day}(\rho_H) &\geq m_i^{day}(\rho_H) & (6.4)\end{aligned}$$

and

$$\begin{aligned} \rho_L m_i^{day}(\rho_L) + (1-\rho_L)pa_i^{day}(\rho_L) &\leq \tfrac{\phi'}{\phi}m_i^{night} + pr'b_i^{night} \text{ and} & (6.5)\\ \rho_H m_i^{day}(\rho_H) + (1-\rho_H)pa_i^{day}(\rho_H) &\leq \tfrac{\phi'}{\phi}m_i^{night} + pr'b_i^{night}. & (6.6)\end{aligned}$$

Restriction (6.2) states that the real value of assets cannot exceed the net labor income from individuals. Also, the truth-telling incentive constraints (6.3) and (6.4) are analogous versions of (4.9) and (4.10) for the case in which the gross real return on the government bond is state-independent. Finally, from the point of view of a bank, financial markets are incomplete since there are two budget constraints, (6.5) and (6.6), instead of a single budget constraint that reflects the bank's ability to transfer consumption across aggregate states. These constraints can be summarized by

$$\begin{bmatrix} -wL_i + \tau \\ \rho_L m_i^{day}(\rho_L) + (1-\rho_L)\, pa_i^{day}(\rho_L) \\ \rho_H m_i^{day}(\rho_H) + (1-\rho_H)\, pa_i^{day}(\rho_H) \end{bmatrix} \leq \mathbf{W} \times \begin{bmatrix} m_i^{night} \\ b_i^{night} \end{bmatrix}$$

[9]For a derivation of the depositor's ex-ante expected utility, see appendix D.

where

$$\mathbf{W} = \begin{bmatrix} - \underset{(1\times 2)}{\mathbf{q}} \\ \hdashline \underset{(2\times 2)}{\mathbf{A}} \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ \hdashline \phi'/\phi & pr' \\ \phi'/\phi & pr' \end{bmatrix}$$

In the above representation, the first (second) column of $\mathbf{W}$ reflects the fact that banks are able to get fiat currency (government bonds) in order to transfer goods across periods. However, during the day the real value of currency (bonds) remains at ϕ'/ϕ (pr'), regardless of the aggregate state of nature. Formally, financial markets are incomplete as $\det(\mathbf{A}) = 0$.

6.3 Public sector: monetary and (passive) fiscal policy

Regarding the behavior of the consolidated public sector, most of the structure in section 4 remains unaltered. The key difference with respect to such case relies on the fact that both the real tax per individual and the gross nominal return of government bonds, denoted by τ and g respectively, are state-independent. The consolidated government budget constraint is now given by

$$\phi\,(M+B) + \tau = \phi(M_- + gB_-), \text{ for } t = 1, 2, \ldots \tag{6.7}$$

along with the assumption that private agents are endowed with no outstanding liabilities at the beginning of the first period:

$$\phi_0\,(M_0 + B_0) + \tau_0 = 0. \tag{6.8}$$

Again, I assume that the monetary authority commits to a policy such that the total stock of nominal currency M grows at a constant rate $\mu > 0$,

$$M' = \mu M, \tag{6.9}$$

and the ratio of currency to total nominal government debt remains fixed at $\delta \in (0, 1)$,

$$\frac{M}{M+B} = \delta. \tag{6.10}$$

The previous assumptions require the path of lump-sum taxes to change passively in order to support monetary policy. Therefore, a similar derivation to those in appendix A shows that the expressions (6.7)-(6.10) imply that the lump-sum taxes are determined according to

$$\tau = -\frac{\phi M}{\delta}\left(1 - \frac{1}{\mu}\right) + \frac{\phi}{\phi_-}\phi_- M_- \left(\frac{1}{\delta} - 1\right)\left[\frac{\phi_-}{\phi} r - 1\right], \; t = 1, 2, \ldots, \text{ and} \tag{6.11}$$

$$\tau_0 = -\frac{\phi_0 M_0}{\delta}. \tag{6.12}$$

6.4 Labor and asset market-clearing conditions

As described in subsection 4.6, given the real wage w firms choose the product-labor combination (Y, N) to maximize profits 4.18 subject to the technological constraint 4.19 and this leads to their labor demand on the right-hand side of 6.13. I also assume that during the day the banks, in order to re-balance their portfolios, meet up in an interbank (Walrasian) market where the relative price of government bonds p is determined and no individual agent has access to this market. Equations (6.14)-(6.17) summarize the market-clearing conditions for both assets during the night and the next day. For a sake of exposition, let each left-hand (right-hand) side term denote the supply (demand) in the corresponding market:

$$\begin{aligned}
\tfrac{1}{2}L_1 + \tfrac{1}{2}L_2 &= N, && (6.13)\\
\phi M &= \tfrac{1}{2}m_1^{night} + \tfrac{1}{2}m_2^{night}, && (6.14)\\
\phi B &= \tfrac{1}{2}b_1^{night} + \tfrac{1}{2}b_2^{night}, && (6.15)\\
\tfrac{\phi'}{\phi}[\tfrac{1}{2}m_1^{night} + \tfrac{1}{2}m_2^{night}] &= \tfrac{1}{2}\rho_L m^{day}(\rho_L) + \tfrac{1}{2}\rho_H m^{day}(\rho_H) \text{ and} && (6.16)\\
r'[\tfrac{1}{2}b_1^{night} + \tfrac{1}{2}b_2^{night}] &= \tfrac{1}{2}\rho_L a^{day}(\rho_L) + \tfrac{1}{2}\rho_H a^{day}(\rho_H). && (6.17)
\end{aligned}$$

Equation 6.14 states that during the day the real supply of fiat currency must equal the demand for real money balances across banks servicing ex-ante depositors. A similar description applies to equation 6.15 where the real supply of bonds must equal the demand for these assets by banks. The real demand for assets during the night determines its real supply during the day, when banks trade with each other in the interbank market, which is reflected in 6.16 for the real demand for fiat currency. Finally, equation 6.17 applies the same logic to the real demand for interest-bearing assets during the day. The assumed aggregate uncertainty structure simplifies the calculation of the right-hand side terms in 6.16 and 6.17 for a sake of tractability (see appendix E for details).

7 Competitive equilibrium with incomplete markets

7.1 Definition

Equipped with this specific structure for incomplete financial markets, in this section I define its corresponding competitive equilibrium. For this purpose, I use the hat notation $\hat{x}$ to represent the equilibrium value of x.

Definition 2. *Given the initial condition (ϕ_0, M_0) and a monetary policy (μ, δ), a stationary* ***competitive equilibrium with incomplete financial markets*** *consists of a price of bonds $\hat{p}$, price dynamics $\hat{\phi}'/\hat{\phi}$, a real wage $\hat{w}$, a gross real return $\hat{r}$, a feasible allocation $(\hat{Y}, \hat{N})$, contingent deposit contracts $\{(\hat{L}_i, \hat{m}_i^{night}, \hat{b}_i^{night}, \hat{m}_i^{day}(\rho_L), \hat{a}_i^{day}(\rho_L), \hat{m}_i^{day}(\rho_H), \hat{a}_i^{day}(\rho_H))\}_{i=1,2}$, a tax $\hat{\tau}$ for periods $t = 1, 2, \ldots$ and an initial tax $\hat{\tau}_0$ such that:*

1. *The gross real return is bounded:* $\hat{\phi}'/\hat{\phi} \leq \hat{r} \leq 1/\beta$.

2. *Given the equilibrium wage* $\hat{w}$*, the feasible allocation* $(\hat{Y}, \hat{N})$ *solves the profit maximization 4.18 subject to the technological constraint 4.19.*

3. *For* $i = 1, 2$*, given the values* $\hat{\tau}$*,* $\hat{w}$*,* $\hat{\phi}'/\phi$*,* $\hat{p}$ *and* $\hat{r}$*, the contingent deposit contract offered to the ex-ante type* i *individuals* $(\hat{L}_i, \hat{m}_i^{night}, \hat{b}_i^{night}, \hat{m}_i^{day}(\rho_L), \hat{a}_i^{day}(\rho_L), \hat{m}_i^{day}(\rho_H), \hat{a}_i^{day}(\rho_H))$ *solves the problem of maximizing their expected utility in 6.1 subject to 6.2-6.6.*

4. *Monetary policy rules 6.9 and 6.10 hold.*

5. *The lump sum tax* $\hat{\tau}$ *satisfies 6.11 for* $r = \hat{r}$ *and* $\hat{\tau}_0 = -\phi_0 M_0/\delta$ *satisfies 6.12.*

6. *Labor and asset markets clear: 6.13-6.17 hold.*

In a similar fashion to definition 1, in a competitive equilibrium with incomplete financial markets: 1) the nominal interest rate is bounded, 2) firms maximize profits, 3) banks take prices and returns as given and provide a deposit contract that maximizes depositors' utility, 4) monetary policy rules hold, 5) the government's budget constraints hold and 6) labor market clears, the government supply of fiat currency and bonds meets the banks' ex-ante demand for these assets during the night and also the banks' holdings of fiat currency and government bonds meet their ex-post demand for these assets during the day.

Some comments are in order. Specifically, as it also happens in section 5, the specific probability structure assumed satisfies the consistency condition 3.2 and, in some sense, allows me to resemble the probability structure in Champ, Smith, and Williamson (1996) for a random variable with a finite set of possible values. However, it is important to clarify the underlying structure by Allen and Gale (2004) that is employed here. First, I assumed that the marginal distribution of the *proportion of individuals in anonymous transactions* is the same across *ex-ante types*. This assumption in turn implies that during the day any individual can calculate her expected utility based on the marginal distribution of the shock her *ex-ante type* will experience, as it can be seen in 6.1. Second, I assume that the cross-sectional distribution of shocks is the same for every aggregate state or, equivalently, that for every possible proportion of individuals in anonymous transactions there is a constant number of ex-ante types (irrespective of who they are) drawing such fraction. This assumption implies that during the day there is no variation in both the real demand for fiat currency and government bonds since each demand displays an expected-value representation, as it is reflected in 6.16 and 6.17 and this property is critical in order to preserve the stationarity of the relevant concept of equilibrium.

7.2 Characterization and some properties

The characterization to the above equilibrium under symmetry is shown in Appendix F. Condition F.1 has an analogous interpretation to (B.7). If $\hat{p}\hat{r} > 1/\mu$ then the return on bonds is greater than the return on cash between the night and the next day and no bank will hold the latter. Conversely, if $\hat{p}\hat{r} < 1/\mu$ then the return on cash is greater than the return on bonds between night and day and no bank will hold bonds. Therefore, equilibrium requires that $\hat{p}\hat{r} = 1/\mu$ during the night. Conditions F.2 and F.3 are the Euler equation for holdings of real currency and government bonds, respectively. Condition F.4 is the pricing equation for bonds in the inter-bank market. Conditions F.5, F.6 and F.7 denote the binding budget constraints. The expressions F.8 and F.9 denote the two monetary policy rules and the expressions F.10 and F.11 reflect the passive fiscal policy. Finally, equations F.12, F.13, F.14, F.15 and F.16 denote the market-clearing conditions for the labor, cash and bond markets during the night and cash and bond markets during the day, respectively.

Once again, in this equilibrium the linear technology implies that the real wage equals $\hat{w} = 1$ and the price dynamics is in turn determined by monetary policy $\hat{\phi}\prime/\hat{\phi} = \hat{\phi}/\hat{\phi}_{-} = 1/\mu$. This latter result, along with F.1, implies that the relative price of bonds satisfies $\hat{p} = 1/(\mu\hat{r}) \leq 1$ since the nominal interest is non-negative. This situation leads to two possible scenarios.

Figure 6 depicts some equilibrium properties for the case in which $\hat{p} < 1$. In this case, it is worth to mention that the value for each bank's ex-ante portfolio equals $\hat{m}^{night}/(\mu\delta)$. The horizontal axis represents the ex-post real amount of cash during the day whereas the vertical axis represents the ex-post real amount of claims on bank accounts during the day. On the other hand, all points contained in the 45-degree line represent those cash-claims combinations for which the incentive constraint $a^{day} \geq m^{day}$ binds. Given that the instantaneous utility function u is strictly increasing, the condition F.7 implies that any equilibrium allocation must lie within the shaded area (on the left of the 45-degree line where $a^{day} > m^{day}$) and that the slope corresponding to the pricing equation $\hat{p} = u'(a^{day})/u'(m^{day})$ is greater than unity under both states of nature. Since $\rho_L < \rho_H$ the slope of the budget constraint for the bank facing ρ_L (red line) is lower in absolute value than those of the budget constraint for the bank facing ρ_H (blue line) and therefore $\hat{m}^{day}(\rho_H) > \hat{m}^{day}(\rho_L)$. That is, banks with higher needs for cash effectively obtain more cash in the inter-bank market. On the other hand, a similar description applies to Figure 7 which depicts some equilibrium properties for the case in which $\hat{p} = 1$. In this case, the curve $\hat{p} = u'(a^{day})/u'(m^{day})$ overlaps with the 45-degree line and therefore $\hat{m}^{day}(\rho_L) = \hat{m}^{day}(\rho_H) = \hat{a}^{day}(\rho_L) = \hat{a}^{day}(\rho_H)$.

Based on the previous description, it is easy to notice that the ability to support perfect-risk sharing ultimately depends on whether government bonds are as liquid as fiat currency or, equivalently, $\hat{p}$ (which determines the nominal interest rate $\mu\hat{r}$) equals 1. Since the relative price of bonds is in turn an endogenous variable, this raises the question of what is happening in the inter-bank market. The answer to this question ultimately depends on the values of the "average" need for cash $\rho \equiv \frac{1}{2}\rho_L + \frac{1}{2}\rho_H$ and the currency-to-liability ratio δ. Specifically, it easy to show that if $\delta = \rho$ then both allocations coincide (see Appendix G).

The above analysis showed the conditions that characterize an equilibrium with complete financial markets. Also, under the lack of complete financial markets economic agents accept a deposit contract by banks in order to (imperfectly) diversify their liquidity risk. In this sense, it was shown that the existence of incomplete financial markets represents a distortion. However, as it will be shown, the existence of this market failure also provides a justification for imposing a welfare-increasing liquidity requirement on banks.

8 Regulated equilibrium with incomplete markets

8.1 Definition

The previous characterization for the case of incomplete financial markets allows me to conclude that the welfare arising from such equilibrium is, in general, lower than those corresponding to the case of complete markets. This leaves open the question on whether there is an intervention that leads to a welfare-improving allocation. For the case of an endowment economy, Allen and Gale (2004) study the effect of regulation of banks by taking as given the ex-ante portfolio choice by banks and analyze the equilibrium determination of 1) the ex-post component of the deposit contract and 2) the spot price of the long term asset (which in my framework is given by the government bond). I follow the aforementioned approach for the production economy and define a regulated equilibrium in an analogous way. For this purpose, I use the tilde notation $\widetilde{x}$ to represent the equilibrium value of x.

Definition 3. *Given (ϕ_0, M_0), μ, $1/\mu \leq \widetilde{r} \leq 1/\beta$ and $\{(\widetilde{m}_i^{night}, \widetilde{b}_i^{night})\}_{i=1,2}$ a* ***regulated equilibrium with incomplete financial markets*** *consists of a real wage $\widetilde{w}$, price dynamics $\widetilde{\phi}'/\widetilde{\phi}$, relative price of bonds $\widetilde{p}$, allocations $(\widetilde{Y}, \widetilde{N})$, extents of labor effort $\{\widetilde{L}_i\}_{i=1,2}$, ex-post components corresponding to deposit contracts $\left\{\left(\widetilde{m}_i^{day}(\rho_L), \widetilde{a}_i^{day}(\rho_H), \widetilde{m}_i^{day}(\rho_L), \widetilde{a}_i^{day}(\rho_H)\right)\right\}_{i=1,2}$, a tax $\widetilde{\tau}$ for periods $t = 1, 2, \ldots$ and an initial tax $\widetilde{\tau}_0$ such that:*

1. *Given the equilibrium wage $\widetilde{w}$, the feasible allocation $(\widetilde{Y}, \widetilde{N})$ solves the profit maximization problem 4.18 subject to the technological constraint 4.19.*

2. *For $i = 1, 2$, given $\widetilde{\tau}$, $\widetilde{w}$, $\widetilde{\phi}'/\widetilde{\phi}$, $\widetilde{p}$, $\widetilde{r}$ and the bank's ex-ante component $(\widetilde{L}_i, \widetilde{m}_i^{night}, \widetilde{b}_i^{night})$ satisfying 6.2, the ex-post component $\left(\widetilde{m}_i^{day}(\rho_L), \widetilde{a}_i^{day}(\rho_H), \widetilde{m}_i^{day}(\rho_L), \widetilde{a}_i^{day}(\rho_H)\right)$ solves the problem of maximizing the expected utility in 6.1 subject to 6.3-6.6.*

3. *The monetary policy rule 6.9 holds.*

4. *The lump sum tax $\widetilde{\tau}$ satisfies 6.11 for $r = \widetilde{r}$ and $\widetilde{\tau}_0 = -\phi_0 M_0/\widetilde{\delta}$ satisfies 6.12, where $\widetilde{\delta} \equiv \widetilde{m}^{night}/(\widetilde{m}^{night} + \widetilde{b}^{night})$, $\widetilde{m}^{night} \equiv \frac{1}{2}\widetilde{m}_1^{night} + \frac{1}{2}\widetilde{m}_2^{night}$ and $\widetilde{b}^{night} \equiv \frac{1}{2}\widetilde{b}_1^{night} + \frac{1}{2}\widetilde{b}_2^{night}$.*

5. *Labor and asset markets clear: 6.13-6.17 hold.*

Recall that both equilibria in sections 5 and 7 imply that $pr = 1/\mu$. Otherwise, if pr is lower (greater) than $1/\mu$, then fiat currency dominates (is dominated by) government bonds and therefore no bank is willing to accumulate government bonds (fiat currency) at all. Although a regulated equilibrium does not require $pr = 1/\mu$ to hold, the incentives for the banks just described still prevail. If a regulator requires banks to hold a minimum amount of cash (bonds) then each bank will desire to hold that minimum amount as well. In such a case, the ex-ante portfolio solves the maximization problem 6.1 subject to the additional constraints $m_i^{night} \geq \underline{m}$ and $b_i^{night} \leq \overline{b}$ ($b_i^{night} \geq \underline{b}$ and $m_i^{night} \leq \overline{m}$) for appropriately chosen values of $\underline{m}$ and $\overline{b}$ ($\underline{b}$ and $\overline{m}$). Therefore, in this regulated equilibrium banks choose the deposit contract that maximizes the expected utility of its depositors but also subject to a implementation constraint that requires them to hold a minimum amount of cash if $\widetilde{pr} > 1/\mu$ (bonds if $\widetilde{pr} < 1/\mu$). After setting $\widetilde{r} = \hat{r}$[10], the main purpose of these requirements is to influence the resulting equilibrium price $\widetilde{p}$ in the inter-bank market. For this purpose, I examine whether it is possible to further increase depositor's welfare (indirect utility) by imposing binding bounds on banks' holdings of fiat currency and bonds. It is easy to show that, in a competitive equilibrium with incomplete markets, the depositor's expected utility in 6.1 can be expressed as

$$\begin{aligned}\hat{W} \equiv -\frac{1}{\mu}(\hat{m}^{night} + \hat{b}^{night}) - \frac{1}{\mu}\hat{b}^{night}(\mu\hat{r} - 1) \\ +\beta\frac{1}{2}\left[\rho_L u\left(\hat{m}^{day}(\rho_L)\right) + (1-\rho_L)u\left(\hat{a}^{day}(\rho_L)\right)\right] \\ +\beta\frac{1}{2}\left[\rho_H u\left(\hat{m}^{day}(\rho_H)\right) + (1-\rho_H)u\left(\hat{a}^{day}(\rho_H)\right)\right] \quad (8.1)\end{aligned}$$

where first two terms are related to the fact that the fiscal policy is passive. Since $\hat{w} = 1$, all the labor income is spent on asset accumulation and taxes. The first term includes the effect of accumulating a gross amount of assets denoted by $\hat{m}^{night} + \hat{b}^{night}$ whereas the second term reflect the effect of the portfolio composition ($\hat{b}^{night}$) on the labor disutility. Finally, the remaining terms correspond to the discounted expected utility during the next day.

Now, assume that the economy's original prices and quantities correspond to an competitive equilibrium with incomplete markets. Specifically, the gross real return equals $\hat{r}$ and the aggregate portfolio (in real terms) is given by the cash-bond combination $(\hat{m}^{night}, \hat{b}^{night})$. I construct a regulated equilibrium in which the new aggregate portfolio $(\widetilde{m}^{night}, \widetilde{b}^{night})$ satisfies $\widetilde{m}^{night} + \widetilde{b}^{night} = \hat{m}^{night} + \hat{b}^{night}$ or, equivalently, the real value of the new portfolio remains constant and therefore any cash requirement only has a composition effect. Also, assume that the new currency-to-liability ratio $\widetilde{\delta} \equiv \widetilde{m}^{night}/(\widetilde{m}^{night} + \widetilde{b}^{night})$ is lower than ρ. Since the denominator of $\widetilde{\delta}$ remains constant, an increase in the real amount of cash held by banks $\widetilde{m}^{night}$ implies that bonds are relatively scarcer in the inter-bank market which in

[10]Since the condition $pr = 1/\mu$ no longer holds, the regulated equilibrium is undetermined as there exists one equilibrium for each given real return in the interval $[1/\mu, 1/\beta]$. I specifically choose $\widetilde{r} = \hat{r}$ to analyze the case in which the liquidity requirements are combined with a real return fixed at its original level (that is, without any portfolio regulation). Notice that the latter condition in turn implies that the nominal interest rate is maintained at a certain level by the government, which is a practice commonly adopted by Central Banks.

turn leads to a higher price p. Since now $\hat{p} < \widetilde{p} < 1$, government bonds are more valuable once the portfolio regulation is imposed. In the previous utility decomposition, the relative price of bonds p plays a critical role. Also, by construction, the first term remains constant whereas the second includes a composition effect for a given real gross return. Finally, the main effect of a higher value of p arises from an improved risk diversification across states of nature and it is easy to show that $\partial \hat{W}/\partial p > 0$ (see Appendix H).

9 A numerical example

I compare the numerical results for the three equilibrium definitions. For this, I rely on the class of utility functions with constant relative risk aversion (CRRA) coefficient: $u(x) = Ax^{1-\sigma}/(1-\sigma)$ for $x \geq 0$, $A > 0$ and $0 < \sigma < 1$. The main appeal of this specification relies on the possibility to obtain a closed-form solution for each equilibrium (see Appendix I). For the equilibrium with complete markets to exist, $\delta \leq \rho$ must be imposed as the nominal interest rate must be non-negative. Such condition implies that $\bar{p} \leq 1$ or, equivalently, in the inter-bank market the government bonds are as valued as currency at most (when $\bar{p} = 1$ bonds are as liquid as cash).

Table 2 reports the results of the numerical exercise for the chosen parametrization where the leftmost column contains the results for complete markets, the central column reports the results with incomplete markets and the rightmost column so does for the regulated equilibrium. The discount factor is set at $\beta = 0.8$ to provide a wide range for the real return with complete markets as $\bar{r} \leq 1/\beta = 1.25$. For the utility function, the scale parameter A is set at 1 and the coefficient of relative risk aversion is set at 0.25. To emphasize the role of banks in the liquidity transformation of bonds, I set $\rho_L = 0.1$ and $\rho_H = 0.2$ and these values represent an economy in which, on average, 15 percent of bank depositors need only cash for transactions which in turn reflects that banks provide endogenous liquidity to a high percentage of depositors. The policy parameter $\mu = 1.025$ reflects a money growth rate of 2.5 percent per period. Finally, $\delta = 0.08$ reflects that, although 8 percent of total government liabilities are composed by cash, the relative amount of currency provided cannot satisfy the proportion $\rho = 0.15$ required by banks. This only feature makes bonds strictly less valued than cash ($\bar{p} < 1$). It is also worth to mention that, although increasing δ would make resource allocations to coincide (see Appendix G), a currency-to-liability ratio $\delta < \rho$ aims to show the welfare gains from regulating the banks' portfolio.

As expected, the equilibrium with complete markets exhibits perfect risk-sharing (i.e. prices and allocations are constant across states of nature): the relative price of Arrow securities $q(\eta_2)/q(\eta_1)$ equals 1 and, in the inter-bank market, the relative price of bonds (in units of the consumption good during the day) is state-independent and equals 0.7898. This in turn means that bonds' liquidity is transformed by banks but they are not as liquid as currency. The real return on assets is state-independent as well, $\bar{r} = 1.2352$, and implies a positive nominal interest rate. The real amount of (perfectly divisible) cash $\bar{m}^{night} = 0.0571$

and bonds $\bar{b}^{night} = 0.6561$ held by banks, along with the lump-sum taxes $\bar{\tau} = 0.1529$, determine the extent of effort $\bar{L} = 0.8661$ by individuals in the labor market. These quantities, along with the implied welfare measure of $\bar{W} = 0.0848$, constitute a benchmark to be compared with the results under alternative financial market structures. Finally, it is worth to emphasize that currency and bonds co-exist in positive quantities as an additional unit of bonds (in real terms) provides $\overline{pr} = 0.9756$ units of consumption (during the next day) which equals the $1/\mu = 0.9756$ units provided after accumulating another unit of fiat currency.

The central column displays the results for an economy under the same parametrization but in which the only marginal change consists on the lack of markets for Arrow securities. Since $\delta = 0.08 < 0.15 = \rho$, the proportion of currency provided by the government is lower than the proportion of agents who need cash for transactions and then $\hat{p} = 0.7870 < 1$. Banks provide $\hat{a}^{day}(\rho_L) = 0.9409 > 0.3609 = \hat{m}^{day}(\rho_L)$ and $\hat{a}^{day}(\rho_H) = 0.9947 > 0.3816 = \hat{m}^{day}(\rho_H)$ in order to induce self-selection among their depositors. In this regard, perfect risk-sharing is not attainable and banks are endogenously induced to accumulate higher amounts of assets $\hat{m}^{night} = 0.0576$ and $\hat{b}^{night} = 0.6625$ than those under complete markets. This precautionary motive, along with the lump-sum taxes $\hat{\tau} = 0.1574$, determine a higher extent of effort $\hat{L} = 0.8775$ by individuals. It can then be concluded that the lack of contingent markets affects depositors' welfare through two channels: 1) it prevents perfect risk-sharing and 2) forces workers to exert additional (precautionary) effort. The welfare measure $\hat{W} = 0.0827$ represents 97.48 percent of the welfare measure with complete markets.

The last column displays the results for a regulated equilibrium based on the one under incomplete markets. I support a portfolio $(\widetilde{m}^{night}, \widetilde{b}^{night})$ such that the total amount of assets remains constant or, equivalently, $\widetilde{m}^{night} + \widetilde{b}^{night} = \hat{m}^{night} + \hat{b}^{night} = 0.7201$ which in turn implies that requirements are specified only in terms of the portfolio composition. In the new portfolio, I impose a lower bound on the cash holdings of 1 percent higher than the unrestricted amount of currency or $\widetilde{m}^{night} \geq \underline{m} = (1+1)\hat{m}^{night} = 0.0582$. Given this, for the total portfolio to remain constant I impose an upper bound on the real bond holdings such that $\widetilde{b}^{night} \leq \bar{b} = (1-0.09)\hat{b}^{night} = 0.6619$. Notice also that all the parameters remain unaltered excepting for the currency-to-liability ratio which is now higher and equals $\widetilde{m}^{night}/(\widetilde{m}^{night} + \widetilde{b}^{night}) = 0.0808$. This is not a trivial requirement since the government must support its portfolio requirement by providing banks with the resources to do so. As it can be anticipated, this implicit "increase in δ" aims to make bonds scarcer then more valued (as they become more liquid) in the inter-bank market. Finally, the real return is set at $\widetilde{r} = \hat{r} = 1.2397$ as in the unrestricted equilibrium with incomplete markets.

Under the portfolio regulation, the relative price of bonds increases to $\widetilde{p} = 0.78914$ and therefore the real value of an additional unit of bonds during the day $\widetilde{pr} = 0.9783$ exceeds those for an additional unit of currency $1/\mu = 0.9756$. This fact makes both bounds on banks' portfolio to bind. The real value of each bank's portfolio equals $(1/\mu)\widetilde{m}^{night} + \widetilde{pr}\widetilde{b}^{night} = 0.7043$ which exceeds the corresponding value without regulation $(1/\mu)\hat{m}night + prbnight = 0.7025$. Additionally, the higher (relative) price of bonds induces a substitution effect as the cash-to-claim ratio during the day increases from 0.3836 to 0.3877. To summarize, there are positive income and substitution effects on the demand for cash during the day. Finally, given the passive fiscal policy, the extent of labor effort decreases from 0.8775 to 0.8773. By construction, even under contingent deposit contracts, a liquidity requirement is not welfare-improving because it decreases the likelihood of bank runs but because it increases the liquidity of government bonds.

10 Conclusions

In this paper, I developed a simple model of endogenous money in which banks transform the liquidity of government bonds and provide claims on their own accounts as a medium of exchange (in addition to fiat currency). For a given economic setup under complete financial markets for aggregate liquidity risk, the equilibrium exhibits perfect risk-sharing for any combination of the policy parameters. On the other hand, when these markets are incomplete, perfect risk-sharing is (un)attainable as the relative amount of fiat currency provided by the public sector is lower than (equal to) those needed by the private sector. Individuals cannot fully smooth consumption across states of nature and therefore exert higher labor effort to accumulate a higher amount of assets. These two channels reduce welfare. In general, the government can make bonds as valued as currency and support perfect risk-sharing by increasing its currency-to-liability ratio and therefore making the nominal interest to equal zero. By construction, under incomplete markets a minimum requirement in terms of banks' holding of fiat currency has the same welfare-increasing effect of a marginal increase of the currency-to-liability ratio on the relative price of bonds which become more liquid. Therefore, bank runs are not a necessary condition for justifying liquidity regulation since the incompleteness of financial markets represent the distortion.

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Appendix A Taxes under passive fiscal policy

Equation (4.12) implies that, under passive fiscal policy, taxes are determined according to

$$
\begin{aligned}
\tau_t(\eta_t) &= -\phi_t \left(M_t + B_t\right) + \phi_t \left[M_{t-1} + g_t(\eta_t) B_{t-1}\right] \\
&= -\phi_t \frac{M_t + B_t}{M_t} M_t + \phi_t \left[M_{t-1} + g_t(\eta_t) B_{t-1}\right] \\
&= -\phi_t \frac{1}{\delta} M_t + \phi_t \left[M_{t-1} + g_t(\eta_t) B_{t-1}\right] \\
&= -\frac{\phi_t M_t}{\delta} \left(1 - \frac{1}{\mu} + \frac{1}{\mu}\right) + \phi_t \left[M_{t-1} + g_t(\eta_t) B_{t-1}\right] \\
&= -\frac{\phi_t M_t}{\delta} \left(1 - \frac{1}{\mu}\right) - \frac{\phi_t M_t}{\delta} \frac{1}{\mu} + \phi_t \left[M_{t-1} + g_t(\eta_t) B_{t-1}\right] \\
&= -\frac{\phi_t M_t}{\delta} \left(1 - \frac{1}{\mu}\right) - \phi_t \frac{\mu M_{t-1}}{\delta \mu} + \phi_t \left[\frac{\delta}{\delta} M_{t-1} + g_t(\eta_t) \frac{1-\delta}{\delta} M_{t-1}\right] \\
&= -\frac{\phi_t M_t}{\delta} \left(1 - \frac{1}{\mu}\right) - \frac{\phi_t M_{t-1}}{\delta} + \frac{\phi_t M_{t-1}}{\delta} \left[\delta + (1-\delta)\, g_t(\eta_t)\right] \\
&= -\frac{\phi_t M_t}{\delta} \left(1 - \frac{1}{\mu}\right) + \frac{\phi_t M_{t-1}}{\delta} \left[-1 + \delta + (1-\delta)\, g_t(\eta_t)\right] \\
&= -\frac{\phi_t M_t}{\delta} \left(1 - \frac{1}{\mu}\right) + \frac{\phi_t M_{t-1}}{\delta} (1-\delta) \left[g_t(\eta_t) - 1\right] \\
&= -\frac{\phi_t M_t}{\delta} \left(1 - \frac{1}{\mu}\right) + \frac{\phi_t}{\phi_{t-1}} \phi_{t-1} M_{t-1} \left(\frac{1}{\delta} - 1\right) \left[\frac{\phi_{t-1}}{\phi_t} r_t(\eta_t) - 1\right].
\end{aligned}
$$

Appendix B Competitive equilibrium characterization with complete markets

The equations that characterize the equilibrium are given by

$$\bar{m}_1^{night} + \bar{b}_1^{night} = \bar{w}\bar{L}_1 - \bar{\tau}(\eta_1), \tag{B.1}$$

$$\bar{m}_1^{night} + \bar{b}_1^{night} = \bar{w}\bar{L}_1 - \bar{\tau}(\eta_2), \tag{B.2}$$

$$\bar{m}_2^{night} + \bar{b}_2^{night} = \bar{w}\bar{L}_2 - \bar{\tau}(\eta_1), \tag{B.3}$$

$$\bar{m}_2^{night} + \bar{b}_2^{night} = \bar{w}\bar{L}_2 - \bar{\tau}(\eta_2), \tag{B.4}$$

$$\begin{aligned}
\bar{q}(\eta_1) & \left[\lambda_1(0,\eta_1)\bar{m}_1^{day}(\eta_1) + \lambda_1(1,\eta_1)\bar{p}(\eta_1)\bar{a}_1^{day}(\eta_1)\right] \\
& + \bar{q}(\eta_2)\left[\lambda_1(0,\eta_2)\bar{m}_1^{day}(\eta_2) + \lambda_1(1,\eta_2)\bar{p}(\eta_2)\bar{a}_1^{day}(\eta_2)\right] \\
& = \bar{q}(\eta_1)\left[\tfrac{1}{2}\tfrac{\phi'}{\phi}\bar{m}_1^{night} + \tfrac{1}{2}\bar{p}(\eta_1)\bar{r}(\eta_1)\bar{b}_1^{night}\right] + \bar{q}(\eta_2)\left[\tfrac{1}{2}\tfrac{\phi'}{\phi}\bar{m}_1^{night} + \tfrac{1}{2}\bar{p}(\eta_2)\bar{r}(\eta_2)\bar{b}_1^{night}\right],
\end{aligned} \tag{B.5}$$

$$\begin{aligned}
\bar{q}(\eta_1) & \left[\lambda_2(0,\eta_1)\bar{m}_2^{day}(\eta_1) + \lambda_2(1,\eta_1)\bar{p}(\eta_1)\bar{a}_2^{day}(\eta_1)\right] \\
& + \bar{q}(\eta_2)\left[\lambda_2(0,\eta_2)\bar{m}_2^{day}(\eta_2) + \lambda_2(1,\eta_2)\bar{p}(\eta_2)\bar{a}_2^{day}(\eta_2)\right] \\
& = \bar{q}(\eta_1)\left[\tfrac{1}{2}\tfrac{\phi'}{\phi}\bar{m}_2^{night} + \tfrac{1}{2}\bar{p}(\eta_1)\bar{r}(\eta_1)\bar{b}_2^{night}\right] + \bar{q}(\eta_2)\left[\tfrac{1}{2}\tfrac{\phi'}{\phi}\bar{m}_2^{night} + \tfrac{1}{2}\bar{p}(\eta_2)\bar{r}(\eta_2)\bar{b}_2^{night}\right],
\end{aligned} \tag{B.6}$$

$$
\begin{aligned}
\frac{\phi'}{\phi} &= \frac{\bar{q}(\eta_1)\bar{p}(\eta_1)\bar{r}(\eta_1) + \bar{q}(\eta_2)\bar{p}(\eta_2)\bar{r}(\eta_2)}{\bar{q}(\eta_1) + \bar{q}(\eta_2)}, && \text{(B.7)}\\
1 &= \tfrac{1}{2}\beta\tfrac{\phi'}{\phi}u'(\bar{m}_1^{day}(\eta_1)) + \tfrac{1}{2}\beta\tfrac{\phi'}{\phi}u'(\bar{m}_1^{day}(\eta_2)), && \text{(B.8)}\\
1 &= \tfrac{1}{2}\beta\tfrac{\phi'}{\phi}u'(\bar{m}_2^{day}(\eta_1)) + \tfrac{1}{2}\beta\tfrac{\phi'}{\phi}u'(\bar{m}_2^{day}(\eta_2)), && \text{(B.9)}\\
1 &= \tfrac{1}{2}\beta\bar{r}(\eta_1)u'(\bar{a}_1^{day}(\eta_1)) + \tfrac{1}{2}\beta\bar{r}(\eta_2)u'(\bar{a}_1^{day}(\eta_2)), && \text{(B.10)}\\
1 &= \tfrac{1}{2}\beta\bar{r}(\eta_1)u'(\bar{a}_2^{day}(\eta_1)) + \tfrac{1}{2}\beta\bar{r}(\eta_2)u'(\bar{a}_2^{day}(\eta_2)), && \text{(B.11)}\\
\bar{p}(\eta_1) &= \frac{u'(\bar{a}_1^{day}(\eta_1))}{u'(\bar{m}_1^{day}(\eta_1))} = \frac{u'(\bar{a}_2^{day}(\eta_1))}{u'(\bar{m}_2^{day}(\eta_1))}, && \text{(B.12)}\\
\bar{p}(\eta_2) &= \frac{u'(\bar{a}_1^{day}(\eta_2))}{u'(\bar{m}_1^{day}(\eta_2))} = \frac{u'(\bar{a}_2^{day}(\eta_2))}{u'(\bar{m}_2^{day}(\eta_2))}, && \text{(B.13)}\\
\frac{\bar{q}(\eta_2)}{\bar{q}(\eta_1)} &= \frac{u'(\bar{m}_1^{day}(\eta_2))}{u'(\bar{m}_1^{day}(\eta_1))} = \frac{u'(\bar{m}_2^{day}(\eta_2))}{u'(\bar{m}_2^{day}(\eta_1))}, && \text{(B.14)}\\
M' &= \mu M, && \text{(B.15)}\\
M &= \delta(M+B), && \text{(B.16)}\\
\bar{\tau}(\eta_1) &= -\phi M/\delta\,(1-1/\mu) + \phi M_-\,(1/\delta - 1)\,[\bar{r}(\eta_1)\phi_-/\phi - 1], && \text{(B.17)}\\
\bar{\tau}(\eta_2) &= -\phi M/\delta\,(1-1/\mu) + \phi M_-\,(1/\delta - 1)\,[\bar{r}(\eta_2)\phi_-/\phi - 1], && \text{(B.18)}\\
\bar{\tau}_0 &= -\phi_0 M_0/\delta, && \text{(B.19)}\\
\tfrac{1}{2}\bar{L}_1 + \tfrac{1}{2}\bar{L}_2 &= \bar{N}, && \text{(B.20)}\\
\phi M &= \tfrac{1}{2}\bar{m}_1^{night} + \tfrac{1}{2}\bar{m}_2^{night} \equiv \bar{m}^{night}, && \text{(B.21)}\\
\phi B &= \tfrac{1}{2}\bar{b}_1^{night} + \tfrac{1}{2}\bar{b}_2^{night} \equiv \bar{b}^{night}, && \text{(B.22)}\\
\tfrac{\phi'}{\phi}\bar{m}^{night} &= \lambda_1(0,\eta_1)\bar{m}_1^{day}(\eta_1) + \lambda_2(0,\eta_1)\bar{m}_2^{day}(\eta_1), && \text{(B.23)}\\
\tfrac{\phi'}{\phi}\bar{m}^{night} &= \lambda_1(0,\eta_2)\bar{m}_1^{day}(\eta_2) + \lambda_2(0,\eta_2)\bar{m}_2^{day}(\eta_2), && \text{(B.24)}\\
\bar{r}(\eta_1)\bar{b}^{night} &= \lambda_1(1,\eta_1)\bar{a}_1^{day}(\eta_1) + \lambda_2(1,\eta_1)\bar{a}_2^{day}(\eta_1), && \text{(B.25)}\\
\bar{r}(\eta_2)\bar{b}^{night} &= \lambda_1(1,\eta_2)\bar{a}_1^{day}(\eta_2) + \lambda_2(1,\eta_2)\bar{a}_2^{day}(\eta_2), && \text{(B.26)}
\end{aligned}
$$

$$
\begin{aligned}
&\left[\lambda_1(0,\eta_1)\bar{m}_1^{day}(\eta_1) + \lambda_2(0,\eta_1)\bar{m}_2^{day}(\eta_1)\right] + \bar{p}(\eta_1)\left\{\lambda_1(1,\eta_1)\bar{a}_1^{day}(\eta_1) + \lambda_2(1,\eta_1)\bar{a}_2^{day}(\eta_1)\right\}\\
&\qquad = \tfrac{\phi'}{\phi}\left[\tfrac{1}{2}\bar{m}_1^{night} + \tfrac{1}{2}\bar{m}_2^{night}\right] + \bar{p}(\eta_1)\bar{r}(\eta_1)\left\{\tfrac{1}{2}\bar{b}_1^{night} + \tfrac{1}{2}\bar{b}_2^{night}\right\} \text{ and}
\end{aligned}
\tag{B.27}
$$

$$
\begin{aligned}
&\left[\lambda_1(0,\eta_2)\bar{m}_1^{day}(\eta_2) + \lambda_2(0,\eta_2)\bar{m}_2^{day}(\eta_2)\right] + \bar{p}(\eta_2)\left\{\lambda_1(1,\eta_2)\bar{a}_1^{day}(\eta_2) + \lambda_2(1,\eta_2)\bar{a}_2^{day}(\eta_2)\right\}\\
&\qquad = \tfrac{\phi'}{\phi}\left[\tfrac{1}{2}\bar{m}_1^{night} + \tfrac{1}{2}\bar{m}_2^{night}\right] + \bar{p}(\eta_2)\bar{r}(\eta_2)\left\{\tfrac{1}{2}\bar{b}_1^{night} + \tfrac{1}{2}\bar{b}_2^{night}\right\}.
\end{aligned}
\tag{B.28}
$$

Appendix C Mutuality principle under complete markets

In a stationary *competitive equilibrium with complete financial markets*, the aggregate demand for fiat currency (in real terms) is constant. Therefore, (B.21) and (B.15) imply $\bar{\phi}'/\bar{\phi} = 1/\mu$. Also, given the definitions of $\bar{m}^{day}$, $\bar{a}^{day}$ and $\bar{r}$, equations (B.8)-(B.9) and (B.10)-(B.11) collapse to (5.1) and (5.2), respectively. Additionally, it is easy to show that the market-clearing conditions (B.23)-(B.24) and (B.25)-(B.26) collapse to (5.3) and (5.4), respectively. These market-clearing conditions, along with the monetary policy rule (B.16) and (B.22) lead to (5.5) in main text. Therefore (5.1)-(5.5) can be solved for $(\bar{m}^{night}, \bar{b}^{night}, \bar{m}^{day}, \bar{a}^{day}, \bar{r})$. Given the previous results, (B.14) implies $\bar{q}(\eta_2)/\bar{q}(\eta_1) = 1$ and $\bar{p}$ is obtained from either (B.12) or (B.13). On the other hand, $\bar{\tau}$ is obtained from either (B.17) or (B.18). This in turn allows to calculate $\bar{L}$ and $\bar{N}$ by using the ex-ante budget constraints and the labor market-clearing condition, respectively. Finally, the resulting prices and allocations obtained satisfy the two budget constraints (B.5)-(B.6) and the market-clearing conditions (B.27)-(B.28).

Appendix D Expected utility under incomplete markets

The way in which the assumed probabilistic structure leads to the specific form of the ex-ante expected utility in (6.1) is illustrated by means of a simple example where, without loss of generality, $w = 1$ and $\tau = 0$. Assume a unit mass of individuals grouped in two *ex-ante types* ($n = 2$): type 1 ($i \in [0, \mu_1]$ with mass μ_1 satisfying $0 < \mu_1 < 1$) and type 2 ($i \in (\mu_2, 1]$ with mass μ_2 satisfying $\mu_2 = 1 - \mu_1$).

Consider the case in which, for each *ex-ante type*, the ex-post *proportion of depositors in anonymous transactions* can only take one of two values ($K = 2$): ρ_1 (low) and ρ_2 (high) with $\rho_1 < \rho_2$. Each realization of the aggregate shock is denoted by a pair $\boldsymbol{\eta} = (\eta_1, \eta_2)$ where η_1 (η_2) denotes the proportion of type 1 (type 2) ex-ante individuals in anonymous transactions and this proportion belongs to $\{\rho_1, \rho_2\}$. This implies that the set of four ($n \times K = 4$) possible aggregate shocks is $H = \{\boldsymbol{\eta}^{\mathbf{1}}, \boldsymbol{\eta}^{\mathbf{2}}, \boldsymbol{\eta}^{\mathbf{3}}, \boldsymbol{\eta}^{\mathbf{4}}\}$ where $\boldsymbol{\eta}^{\mathbf{1}} = (\rho_1, \rho_1)$, $\boldsymbol{\eta}^{\mathbf{2}} = (\rho_1, \rho_2)$, $\boldsymbol{\eta}^{\mathbf{3}} = (\rho_2, \rho_1)$ and $\boldsymbol{\eta}^{\mathbf{4}} = (\rho_2, \rho_2)$.

For a type i ex-ante individual, let $\lambda_i(0, \boldsymbol{\eta})$ ($\lambda_i(1, \boldsymbol{\eta})$) denote the probability that such individual is engaged in an anonymous (monitored) transaction, conditional on the aggregate state $\boldsymbol{\eta}$. The probability of being a type i ex-ante individual is μ_i. Consistency requires that

$$\begin{aligned}
\text{for } \boldsymbol{\eta} &= \boldsymbol{\eta}^{\mathbf{1}}\colon \begin{cases} \mu_1 = \lambda_1(0, \boldsymbol{\eta}^{\mathbf{1}}) + \lambda_1(1, \boldsymbol{\eta}^{\mathbf{1}}) \\ \mu_2 = \lambda_2(0, \boldsymbol{\eta}^{\mathbf{1}}) + \lambda_2(1, \boldsymbol{\eta}^{\mathbf{1}}) \end{cases}, \\
\text{for } \boldsymbol{\eta} &= \boldsymbol{\eta}^{\mathbf{2}}\colon \begin{cases} \mu_1 = \lambda_1(0, \boldsymbol{\eta}^{\mathbf{2}}) + \lambda_1(1, \boldsymbol{\eta}^{\mathbf{2}}) \\ \mu_2 = \lambda_2(0, \boldsymbol{\eta}^{\mathbf{2}}) + \lambda_2(1, \boldsymbol{\eta}^{\mathbf{2}}) \end{cases}, \\
\text{for } \boldsymbol{\eta} &= \boldsymbol{\eta}^{\mathbf{3}}\colon \begin{cases} \mu_1 = \lambda_1(0, \boldsymbol{\eta}^{\mathbf{3}}) + \lambda_1(1, \boldsymbol{\eta}^{\mathbf{3}}) \\ \mu_2 = \lambda_2(0, \boldsymbol{\eta}^{\mathbf{3}}) + \lambda_2(1, \boldsymbol{\eta}^{\mathbf{3}}) \end{cases} \text{ and} \\
\text{for } \boldsymbol{\eta} &= \boldsymbol{\eta}^{\mathbf{4}}\colon \begin{cases} \mu_1 = \lambda_1(0, \boldsymbol{\eta}^{\mathbf{4}}) + \lambda_1(1, \boldsymbol{\eta}^{\mathbf{4}}) \\ \mu_2 = \lambda_2(0, \boldsymbol{\eta}^{\mathbf{4}}) + \lambda_2(1, \boldsymbol{\eta}^{\mathbf{4}}) \end{cases}.
\end{aligned}$$

Let the set $H_{i,k} \equiv \{\boldsymbol{\eta} \in H | \eta_i = \rho_k\}$ contain all those states of nature in which a proportion ρ_k of type i ex-ante depositors engages in anonymous meetings. Therefore, $H_{1,1} = \{\boldsymbol{\eta}^{\mathbf{1}}, \boldsymbol{\eta}^{\mathbf{2}}\}$, $H_{1,2} = \{\boldsymbol{\eta}^{\mathbf{3}}, \boldsymbol{\eta}^{\mathbf{4}}\}$, $H_{2,1} = \{\boldsymbol{\eta}^{\mathbf{1}}, \boldsymbol{\eta}^{\mathbf{3}}\}$ and $H_{2,2} = \{\boldsymbol{\eta}^{\mathbf{2}}, \boldsymbol{\eta}^{\mathbf{4}}\}$. Furthermore, define the marginal probabilities

$$\begin{aligned}
\text{Prob}(\eta_1 = \rho_1) &\equiv \textstyle\sum_{\eta \in H_{1,1}} \lambda_1(0, \boldsymbol{\eta}) = \lambda_1(0, \boldsymbol{\eta}^{\mathbf{1}}) + \lambda_1(0, \boldsymbol{\eta}^{\mathbf{2}}), \\
\text{Prob}(\eta_1 = \rho_2) &\equiv \textstyle\sum_{\eta \in H_{1,2}} \lambda_1(0, \boldsymbol{\eta}) = \lambda_1(0, \boldsymbol{\eta}^{\mathbf{3}}) + \lambda_1(0, \boldsymbol{\eta}^{\mathbf{4}}), \\
\text{Prob}(\eta_2 = \rho_1) &\equiv \textstyle\sum_{\eta \in H_{2,1}} \lambda_2(0, \boldsymbol{\eta}) = \lambda_2(0, \boldsymbol{\eta}^{\mathbf{1}}) + \lambda_2(0, \boldsymbol{\eta}^{\mathbf{3}}) \text{ and} \\
\text{Prob}(\eta_2 = \rho_2) &\equiv \textstyle\sum_{\eta \in H_{2,2}} \lambda_2(0, \boldsymbol{\eta}) = \lambda_2(0, \boldsymbol{\eta}^{\mathbf{2}}) + \lambda_2(0, \boldsymbol{\eta}^{\mathbf{4}}),
\end{aligned}$$

which imply that the expected utility for an ex-ante type 1 depositor can be written as

$$
\begin{aligned}
& E\left[\text{utility}|\text{ex-ante type 1}\right] \\
& = -m_1^{night} - b_1^{night} \\
& \quad + \beta\lambda_1\left(0,\boldsymbol{\eta}^{\mathbf{1}}\right)\left\{\rho_1 u\left(m_1^{day}\left(\rho_1\right)\right) + \left(1-\rho_1\right) u\left(a_1^{day}\left(\rho_1\right)\right)\right\} \\
& \quad + \beta\lambda_1\left(0,\boldsymbol{\eta}^{\mathbf{2}}\right)\left\{\rho_1 u\left(m_1^{day}\left(\rho_1\right)\right) + \left(1-\rho_1\right) u\left(a_1^{day}\left(\rho_1\right)\right)\right\} \\
& \quad + \beta\lambda_1\left(0,\boldsymbol{\eta}^{\mathbf{3}}\right)\left\{\rho_2 u\left(m_1^{day}\left(\rho_2\right)\right) + \left(1-\rho_2\right) u\left(a_1^{day}\left(\rho_2\right)\right)\right\} \\
& \quad + \beta\lambda_1\left(0,\boldsymbol{\eta}^{\mathbf{4}}\right)\left\{\rho_2 u\left(m_1^{day}\left(\rho_2\right)\right) + \left(1-\rho_2\right) u\left(a_1^{day}\left(\rho_2\right)\right)\right\} \\
& = -m_1^{night} - b_1^{night} \\
& \quad + \beta\underbrace{\left[\lambda_1\left(0,\boldsymbol{\eta}^{\mathbf{1}}\right) + \lambda_1\left(0,\boldsymbol{\eta}^{\mathbf{2}}\right)\right]}_{\text{Prob}(\eta_1=\rho_1)}\left\{\rho_1 u\left(m_1^{day}\left(\rho_1\right)\right) + \left(1-\rho_1\right) u\left(a_1^{day}\left(\rho_1\right)\right)\right\} \\
& \quad + \beta\underbrace{\left[\lambda_1\left(0,\boldsymbol{\eta}^{\mathbf{3}}\right) + \lambda_1\left(0,\boldsymbol{\eta}^{\mathbf{4}}\right)\right]}_{\text{Prob}(\eta_1=\rho_2)}\left\{\rho_2 u\left(m_1^{day}\left(\rho_2\right)\right) + \left(1-\rho_2\right) u\left(a_1^{day}\left(\rho_2\right)\right)\right\}
\end{aligned}
$$

and that the expected utility for an ex-ante type 2 depositor can be written as

$$
\begin{aligned}
& E\left[\text{utility}|\text{ex-ante type 2}\right] \\
& = -m_2^{night} - b_2^{night} \\
& \quad + \beta\lambda_2\left(0,\boldsymbol{\eta}^{\mathbf{1}}\right)\left\{\rho_1 u\left(m_2^{day}\left(\rho_1\right)\right) + \left(1-\rho_1\right) u\left(a_2^{day}\left(\rho_1\right)\right)\right\} \\
& \quad + \beta\lambda_2\left(0,\boldsymbol{\eta}^{\mathbf{2}}\right)\left\{\rho_2 u\left(m_2^{day}\left(\rho_2\right)\right) + \left(1-\rho_2\right) u\left(a_2^{day}\left(\rho_2\right)\right)\right\} \\
& \quad + \beta\lambda_2\left(0,\boldsymbol{\eta}^{\mathbf{3}}\right)\left\{\rho_1 u\left(m_2^{day}\left(\rho_1\right)\right) + \left(1-\rho_1\right) u\left(a_2^{day}\left(\rho_1\right)\right)\right\} \\
& \quad + \beta\lambda_2\left(0,\boldsymbol{\eta}^{\mathbf{4}}\right)\left\{\rho_2 u\left(m_2^{day}\left(\rho_2\right)\right) + \left(1-\rho_2\right) u\left(a_2^{day}\left(\rho_2\right)\right)\right\} \\
& = -m_2^{night} - b_2^{night} \\
& \quad + \beta\underbrace{\left[\lambda_2\left(0,\boldsymbol{\eta}^{\mathbf{1}}\right) + \lambda_2\left(0,\boldsymbol{\eta}^{\mathbf{3}}\right)\right]}_{\text{Prob}(\eta_2=\rho_1)}\left\{\rho_1 u\left(m_2^{day}\left(\rho_1\right)\right) + \left(1-\rho_1\right) u\left(a_2^{day}\left(\rho_1\right)\right)\right\} \\
& \quad + \beta\underbrace{\left[\lambda_2\left(0,\boldsymbol{\eta}^{\mathbf{2}}\right) + \lambda_2\left(0,\boldsymbol{\eta}^{\mathbf{4}}\right)\right]}_{\text{Prob}(\eta_2=\rho_2)}\left\{\rho_2 u\left(m_2^{day}\left(\rho_2\right)\right) + \left(1-\rho_2\right) u\left(a_2^{day}\left(\rho_2\right)\right)\right\}.
\end{aligned}
$$

It is assumed that the marginal distribution of shocks is the same across ex-ante types. Namely,

$$
\begin{aligned}
\text{Prob}(\eta_1 = \rho_1) &= \text{Prob}(\eta_2 = \rho_1) \equiv \lambda_1 \text{ and} \\
\text{Prob}(\eta_1 = \rho_2) &= \text{Prob}(\eta_2 = \rho_2) \equiv \lambda_2.
\end{aligned}
$$

Provided that $\lambda_1 + \lambda_2 = 1$, the former expression is equivalent to state the following: each ex-ante depositor knows that, with probability λ_1, the probability of an anonymous transaction is ρ_1 and, with probability λ_2, the probability of an anonymous transaction equals ρ_2. Therefore, the ex-ante expected utility of a representative depositor can be written in compact form as follows

$$
\begin{aligned}
&E\left[\text{utility}|\text{ex-ante type } i\right] \\
&= -m_i^{night} - b_i^{night} + \beta \sum_{k=1}^{K} \lambda_k \left\{\rho_k u\left(m_i^{day}\left(\rho_k\right)\right) + \left(1-\rho_k\right) u\left(a_i^{day}\left(\rho_k\right)\right)\right\},\ i = 1, \ldots, n
\end{aligned}
$$

for $n = 2$ and $K = 2$. It is straightforward to show that this case can be generalized for arbitrary values of n and K.

Appendix E Ex-post aggregate demand under incomplete markets

In the definition of a competitive equilibrium with incomplete financial markets, the two market-clearing conditions during the day involve an expected-value representation of the aggregate demand for fiat currency and government bonds (both in real terms). To clarify this result, let m_j^{day} denote the real demand for fiat currency corresponding to the individual $j \in [0,1]$. Once the aggregate uncertainty has been resolved, the banks' total demand for fiat currency can be rewritten as

$$\begin{aligned}
\int_{\{j\in[0,1]\}} m_j^{day} d\mathcal{L} &= \sum_{k=L,H} \int_{\{j\in[0,1]:j \text{ belongs to the ex-ante type } i \text{ such that } \eta_i=\rho_k\}} m_j^{day} d\mathcal{L} && \text{(E.1)} \\
&= \sum_{k=L,H} \int_{\{j\in[0,1]:j \text{ belongs to the ex-ante type } i \text{ such that } \eta_i=\rho_k\}} m^{day}(\rho_k) d\mathcal{L} && \text{(E.2)} \\
&= \sum_{k=L,H} m^{day}(\rho_k) \int_{\{j\in[0,1]:j \text{ belongs to the ex-ante type } i \text{ such that } \eta_i=\rho_k\}} d\mathcal{L} && \text{(E.3)} \\
&= \sum_{k=L,H} m^{day}(\rho_k) \sum_{\{i:\eta_i=\rho_k\}} \left(\frac{1}{2}\right) \rho_k && \text{(E.4)} \\
&= \sum_{k=L,H} \frac{\#\{i \in \{1,2\} : \eta_i = \rho_k\}}{2} \rho_k m^{day}(\rho_k) && \text{(E.5)} \\
&= \sum_{k=L,H} \frac{1}{2} \rho_k m^{day}(\rho_k) && \text{(E.6)} \\
&= \frac{1}{2}\rho_L m^{day}(\rho_L) + \frac{1}{2}\rho_H m^{day}(\rho_H). && \text{(E.7)}
\end{aligned}$$

In (E.1) the individuals are grouped according to the shock they experience whereas (E.2) specifies the individual demand in each case. In (E.3) terms are rearranged and in (E.4) the total mass of ex-ante individuals facing ρ_k is quantified. An equivalent representation is used in (E.5) where it can be seen the role of the assumption regarding the cross-sectional distribution of shocks in (E.6). Finally, (E.7) displays the expression in main text which expresses $\int_{\{j\in[0,1]\}} m_j^{day} d\mathcal{L}$ as the expected value of the fiat-currency demand per bank or, equivalently, $E[\rho m^{day}(\rho)]$. A similar procedure can be applied to obtain the banks' total demand for interest-bearing assets.

Appendix F Competitive equilibrium characterization with incomplete markets

The equations that characterize the equilibrium are given by

$$\frac{1}{\mu} = \hat{p}\hat{r}, \tag{F.1}$$

$$1 = \frac{1}{2}\frac{\beta}{\mu}u'\left(\hat{m}^{day}(\rho_L)\right) + \frac{1}{2}\frac{\beta}{\mu}u'\left(\hat{m}^{day}(\rho_H)\right), \tag{F.2}$$

$$1 = \frac{1}{2}\beta\hat{r}u'\left(\hat{a}^{day}(\rho_L)\right) + \frac{1}{2}\beta\hat{r}u'\left(\hat{a}^{day}(\rho_H)\right), \tag{F.3}$$

$$\hat{p} = \frac{u'(\hat{a}^{day}(\rho_L))}{u'(\hat{m}^{day}(\rho_L))} = \frac{u'(\hat{a}^{day}(\rho_H))}{u'(\hat{m}^{day}(\rho_H))}, \tag{F.4}$$

$$\hat{m}^{night} + \hat{b}^{night} = \hat{w}\hat{L} - \tau, \tag{F.5}$$

$$\frac{1}{\mu}\hat{m}^{night} + \hat{p}\hat{r}\hat{b}^{night} = \rho_L\hat{m}^{day}(\rho_L) + (1-\rho_L)\hat{p}\hat{a}^{day}(\rho_L), \tag{F.6}$$

$$\frac{1}{\mu}\hat{m}^{night} + \hat{p}\hat{r}\hat{b}^{night} = \rho_H\hat{m}^{day}(\rho_H) + (1-\rho_H)\hat{p}\hat{a}^{day}(\rho_H), \tag{F.7}$$

$$\frac{M'}{M} = \mu, \tag{F.8}$$

$$\frac{M+B}{M} = \delta, \tag{F.9}$$

$$\hat{\tau} = -\frac{\phi M}{\delta}\left(1-\frac{1}{\mu}\right) + \frac{1}{\mu}\phi_{-}M_{-}\left(\frac{1}{\delta}-1\right)[\mu\hat{r}-1], \tag{F.10}$$

$$\text{and } \bar{\tau}_0 = -\frac{\phi_0 M_0}{\delta}, \tag{F.11}$$

$$\hat{L} = \hat{N}, \tag{F.12}$$

$$\phi M = \hat{m}^{night}, \tag{F.13}$$

$$\phi B = \hat{b}^{night}, \tag{F.14}$$

$$\frac{\hat{\phi}'}{\hat{\phi}}\hat{m}^{night} = \tfrac{1}{2}\rho_L\hat{m}^{day}(\rho_L) + \tfrac{1}{2}\rho_H\hat{m}^{day}(\rho_H) \text{ and} \tag{F.15}$$

$$\hat{r}\hat{b}^{night} = \tfrac{1}{2}(1-\rho_L)\hat{a}^{day}(\rho_L) + \tfrac{1}{2}(1-\rho_H)\hat{a}^{day}(\rho_H). \tag{F.16}$$

Appendix G Risk-sharing under incomplete markets

I show that $\delta = \rho$ supports a competitive equilibrium with incomplete financial markets exhibiting perfect risk-sharing and $\hat{p} = 1$. Specifically, after assuming $\hat{m}^{day}(\rho_L) = \hat{m}^{day}(\rho_H) = \hat{m}^{day}$ and $\hat{a}^{day}(\rho_L) = \hat{a}^{day}(\rho_H) = \hat{a}^{day}$, the equation (F.2) reduces to $1 = \frac{\beta}{\mu} u'(\hat{m}^{day})$ whereas the equation (F.3) reduces to $1 = \beta \hat{r} u'(\hat{a}^{day})$. On the other hand, $\hat{p} = 1$ implies the *Friedman rule* (i.e. $\mu \hat{r} = 1$) and $\hat{m}^{day} = \hat{a}^{day}$ and therefore both budget constraints (F.6) and (F.7) collapse to $\hat{m}^{day} = \frac{1}{\mu\delta} \hat{m}^{night}$. For this degenerated budget constraint to be consistent with the market-clearing condition $\rho \hat{m}^{day} = \frac{1}{\mu} \hat{m}^{night}$, it has to be the case that $\delta = \rho$.

Appendix H Welfare effects of regulation

For the welfare decomposition in main text, the real amount of assets $m^{night} + b^{night}$ remains constant and, given that the real return r is fixed, a lower amount of government bonds reduces the disutility of labor effort. Therefore, the response of welfare to a change in p is given by

$$\begin{aligned}
\frac{\partial \hat{W}}{\partial p} &= \beta \sum_{k=L,H} \frac{1}{2}\left[\rho_k u'\left(\hat{m}^{day}(\rho_k)\right)\frac{\partial \hat{m}^{day}(\rho_k)}{\partial p} + (1-\rho_k)u'\left(\hat{a}^{day}(\rho_k)\right)\frac{\partial \hat{a}^{day}(\rho_k)}{\partial p}\right] \\
&= \beta \sum_{k=L,H} \frac{1}{2}\left[\rho_k \frac{1}{\hat{p}} u'\left(\hat{a}^{day}(\rho_k)\right)\frac{\partial \hat{m}^{day}(\rho_k)}{\partial p} + (1-\rho_k)u'\left(\hat{a}^{day}(\rho_k)\right)\frac{\partial \hat{a}^{day}(\rho_k)}{\partial p}\right] \\
&= \beta \sum_{k=L,H} \frac{1}{2}\frac{1}{\hat{p}} u'\left(\hat{a}^{day}(\rho_k)\right)\left[\rho_k \frac{\partial \hat{m}^{day}(\rho_k)}{\partial p} + (1-\rho_k)\hat{p}\frac{\partial \hat{a}^{day}(\rho_k)}{\partial p}\right] \\
&= \beta \sum_{k=L,H} \frac{1}{2}\frac{1}{\hat{p}} u'\left(\hat{a}^{day}(\rho_k)\right)\left[\hat{r}\hat{b}^{night} - (1-\rho_k)\hat{a}^{day}(\rho_k)\right] \\
&= \frac{\beta}{\hat{p}} \sum_{k=L,H} \tfrac{1}{2} u'\left(\hat{a}^{day}(\rho_k)\right)\left[\hat{r}\hat{b}^{night} - (1-\rho_k)\hat{a}^{day}(\rho_k)\right].
\end{aligned}$$

Now, given that the degree of relative risk aversion is less then one, (F.4) implies that

$$\hat{m}^{day}(\rho_k) < \hat{p}\hat{a}^{day}(\rho_k), \text{ for } k = L, H.$$

This in turn implies that $\hat{m}^{day}(\rho_k)$ and $(1-\rho_k)\hat{a}^{day}(\rho_k)$ move in opposite directions. To see this, note that, within a fixed equilibrium, an increase in $(1-\rho_k)$ must lead to a decrease in consumption at both transaction types (this follows directly from the fact that $\hat{m}^{day}(\rho_k) < \hat{p}\hat{a}^{day}(\rho_k)$). Then $\rho_k \hat{m}^{day}(\rho_k)$ falls as $(1-\rho_k)$ increases and so in order to satisfy each budget constraint it must be the case that $(1-\rho_k)\hat{p}\hat{a}^{day}(\rho_k)$ increases. Assuming the distributions are not degenerate, it must be the case that

$$\begin{aligned}
&\frac{\beta}{\hat{p}} \sum_{k=L,H} \tfrac{1}{2} u'\left(\hat{a}^{day}(\rho_k)\right)\left[\hat{r}\hat{b}^{night} - (1-\rho_k)\hat{a}^{day}(\rho_k)\right] \\
&\qquad > \frac{\beta}{\hat{p}} \sum_{k=L,H} \tfrac{1}{2} u'\left(\hat{a}^{day}(\rho_k)\right) \sum_{k=L,H} \tfrac{1}{2}\left[\hat{r}\hat{b}^{night} - (1-\rho_k)\hat{a}^{day}(\rho_k)\right] = 0.
\end{aligned}$$

Appendix I Closed-form solutions

Under instantaneous utility functions of the form $u(x) = Ax^{1-\sigma}/(1-\sigma)$ for $x \geq 0$, where $A > 0$ and $0 < \sigma < 1$, it is easy to show by direct calculation that

1. the prices and private allocations corresponding to a symmetric **competitive equilibrium with complete financial markets** are given by

$$
\begin{aligned}
\bar{w} &= 1, \\
\frac{\bar{\phi}'}{\bar{\phi}} &= \frac{1}{\mu}, \\
\bar{p} &= \left(\frac{1/\rho - 1}{1/\delta - 1}\right)^{\sigma/(1-\sigma)}, \\
\bar{r} &= \frac{1}{\mu}\left(\frac{1/\delta - 1}{1/\rho - 1}\right)^{\sigma/(1-\sigma)}, \\
\bar{m}^{night} &= \mu\rho\left(\frac{\beta}{\mu}A\right)^{1/\sigma}, \\
\bar{b}^{night} &= \left(\frac{1}{\delta} - 1\right)\mu\rho\left(\frac{\beta}{\mu}A\right)^{1/\sigma}, \\
\bar{m}^{day} &= \bar{m}^{day}(\rho_L) = \bar{m}^{day}(\rho_H) = \left(\frac{\beta}{\mu}A\right)^{1/\sigma} \quad \text{and} \\
\bar{a}^{day} &= \bar{a}^{day}(\rho_L) = \bar{a}^{day}(\rho_H) = \left(\frac{1/\delta - 1}{1/\rho - 1}\right)^{1/(1-\sigma)}\left(\frac{\beta}{\mu}A\right)^{1/\sigma},
\end{aligned}
$$

2. the prices and private allocations corresponding to a symmetric **competitive equilibrium with incomplete financial markets**, after defining

$$
D \equiv \frac{-(2\delta - 1)\left(\frac{\rho_H}{1-\rho_H} - \frac{\rho_L}{1-\rho_L}\right) + \sqrt{\left[(2\delta - 1)\left(\frac{\rho_H}{1-\rho_H} - \frac{\rho_L}{1-\rho_L}\right)\right]^2 + 4\frac{\rho_H}{1-\rho_H}\frac{\rho_L}{1-\rho_L}}}{2\frac{\rho_H}{1-\rho_L}},
$$

are given by

$$
\begin{aligned}
\hat{w} &= 1, \\
\frac{\hat{\phi}'}{\hat{\phi}} &= \frac{1}{\mu}, \\
\hat{p} &= \left[\frac{(\rho_L + \rho_H D)/(2\delta) - \rho_L}{1 - \rho_L}\right]^{\sigma/(1-\sigma)}, \\
\hat{r} &= \frac{1}{\mu}\left[\frac{1 - \rho_L}{(\rho_L + \rho_H D)/(2\delta) - \rho_L}\right]^{\sigma/(1-\sigma)}, \\
\hat{m}^{night} &= \frac{\mu}{2}(\rho_L + \rho_H D)\left[\frac{1}{2}\frac{\beta}{\mu}A(1 + D^{-\sigma})\right]^{1/\sigma}, \\
\hat{b}^{night} &= \left(\frac{1}{\delta} - 1\right)\frac{\mu}{2}(\rho_L + \rho_H D)\left[\frac{1}{2}\frac{\beta}{\mu}A(1 + D^{-\sigma})\right]^{1/\sigma}, \\
\hat{m}^{day}(\rho_L) &= \left[\frac{1}{2}\frac{\beta}{\mu}A(1 + D^{-\sigma})\right]^{1/\sigma}, \\
\hat{a}^{day}(\rho_L) &= \left[\frac{(\rho_L + \rho_H D)/(2\delta) - \rho_L}{1 - \rho_L}\right]^{1/(1-\sigma)}\left[\frac{1}{2}\frac{\beta}{\mu}A(1 + D^{-\sigma})\right]^{1/\sigma}, \\
\hat{m}^{day}(\rho_H) &= \left[\frac{1}{2}\frac{\beta}{\mu}A(1 + D^{\sigma})\right]^{1/\sigma} \quad \text{and} \\
\hat{a}^{day}(\rho_H) &= \left[\frac{(\rho_L/D + \rho_H)/(2\delta) - \rho_H}{1 - \rho_H}\right]^{1/(1-\sigma)}\left[\frac{1}{2}\frac{\beta}{\mu}A(1 + D^{\sigma})\right]^{1/\sigma}, \text{ and}
\end{aligned}
$$

3. given $\tilde{r}$, $\tilde{m}^{night}$ and $\tilde{b}^{night}$, the prices and remaining private allocations corresponding to a symmetric **regulated equilibrium with incomplete financial markets** are

given by

$$
\begin{aligned}
\tilde{w} &= 1, \\
\frac{\tilde{\phi}'}{\tilde{\phi}} &= \frac{1}{\mu}, \\
\tilde{m}^{day}(\rho_L) &= \frac{\frac{1}{\mu}\tilde{m}^{night} + \tilde{p}\tilde{r}\tilde{b}^{night}}{\rho_L + (1-\rho_L)\tilde{p}^{1-1/\sigma}}, \\
\tilde{a}^{day}(\rho_L) &= p^{-\frac{1}{\sigma}} \left[\frac{\frac{1}{\mu}\tilde{m}^{night} + \tilde{p}\tilde{r}\tilde{b}^{night}}{\rho_L + (1-\rho_L)\tilde{p}^{1-1/\sigma}} \right], \\
\tilde{m}^{day}(\rho_H) &= \frac{\frac{1}{\mu}\tilde{m}^{night} + \tilde{p}\tilde{r}\tilde{b}^{night}}{\rho_H + (1-\rho_H)\tilde{p}^{1-1/\sigma}} \\
\tilde{a}^{day}(\rho_H) &= p^{-\frac{1}{\sigma}} \left[\frac{\frac{1}{\mu}\tilde{m}^{night} + \tilde{p}\tilde{r}\tilde{b}^{night}}{\rho_H + (1-\rho_H)\tilde{p}^{1-1/\sigma}} \right]
\end{aligned}
$$

and $\tilde{p}$ satisfies

$$
\frac{1}{2}\rho_L \left[\frac{\frac{1}{\mu}\tilde{m}^{night} + \tilde{p}\tilde{r}\tilde{b}^{night}}{\rho_L + (1-\rho_L)\tilde{p}^{1-1/\sigma}} \right] + \frac{1}{2}\rho_H \left[\frac{\frac{1}{\mu}\tilde{m}^{night} + \tilde{p}\tilde{r}\tilde{b}^{night}}{\rho_H + (1-\rho_H)\tilde{p}^{1-1/\sigma}} \right] = \tilde{m}^{night}.
$$

Based on the previous closed-form solutions the taxes τ, labor effort L and welfare measures W can be directly computed for each equilibrium concept.

Figure 1: United States: M1 components (%): 1959-2015

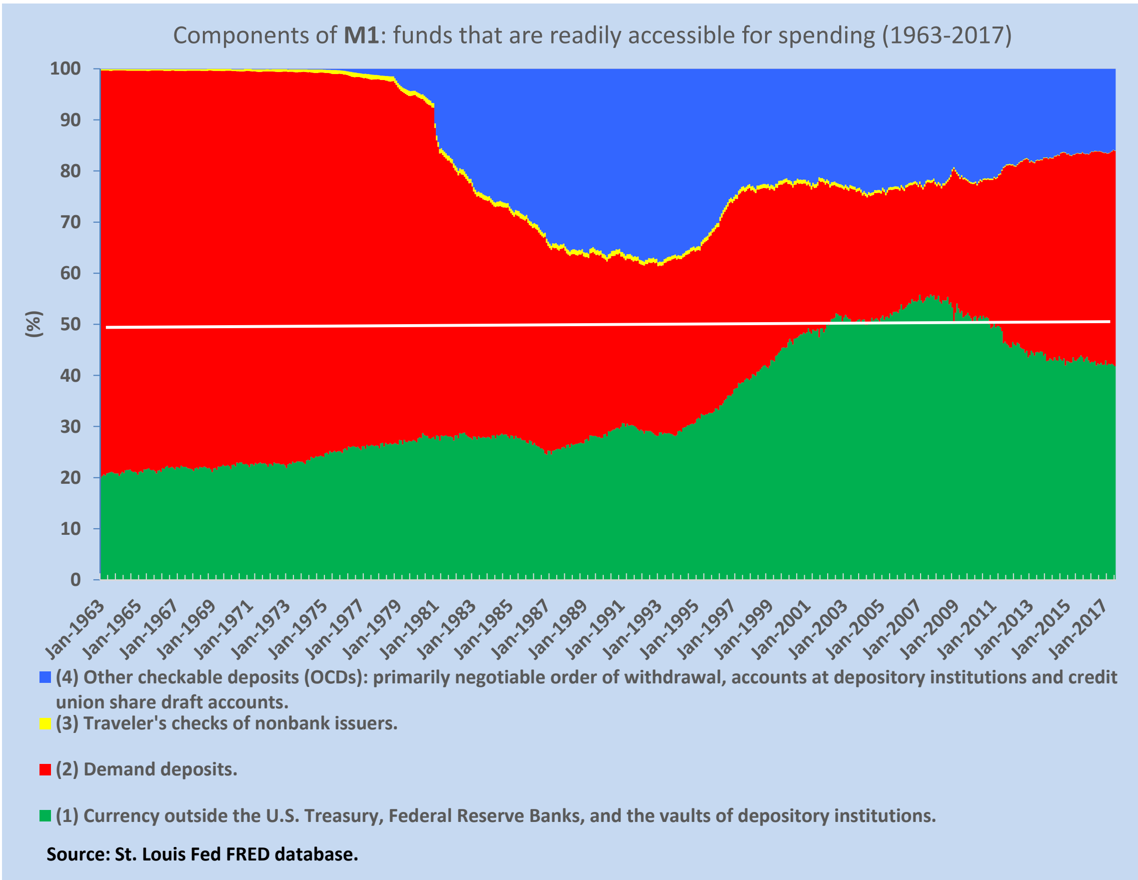


Figure 2: Instantaneous utility function u

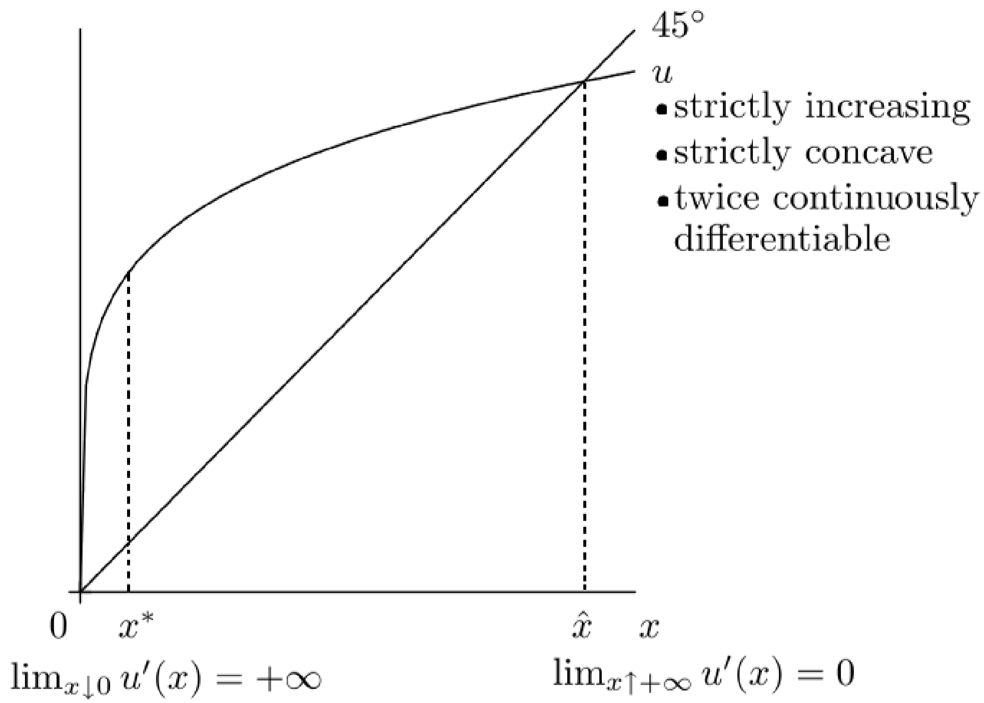
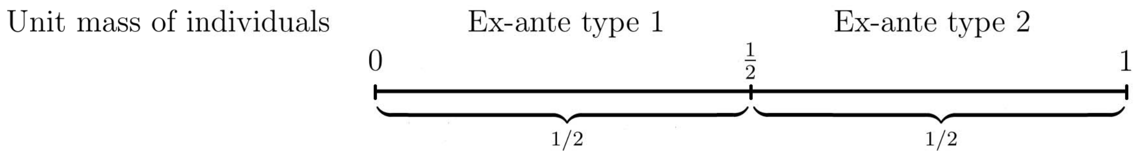
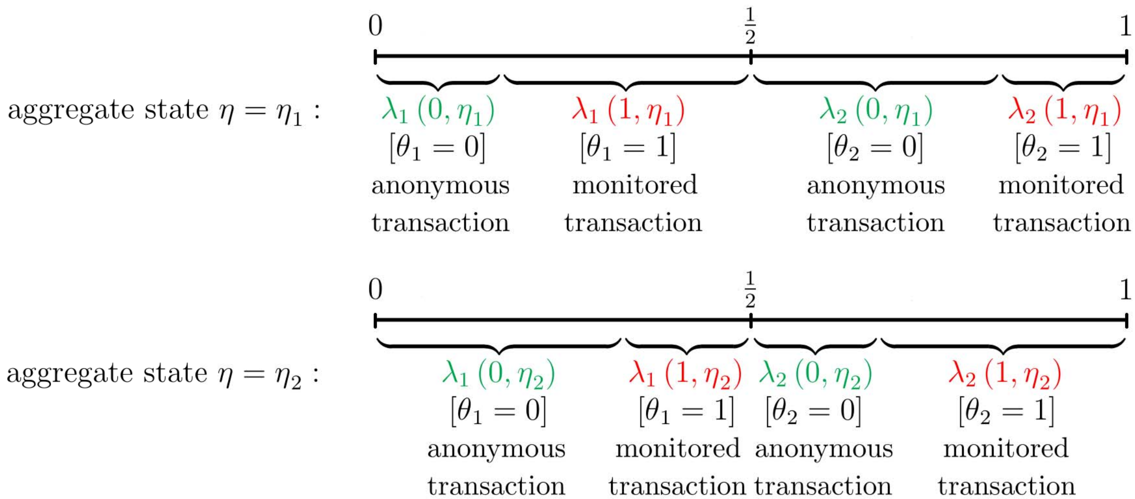


Figure 3: Risk about the aggregate distribution of transaction types

a) *Ex-ante* aggregate distribution



b) *Ex-post* aggregate distributions



c) *Ex-post* aggregate distribution under $0 < \rho_L < \rho_H < 1$

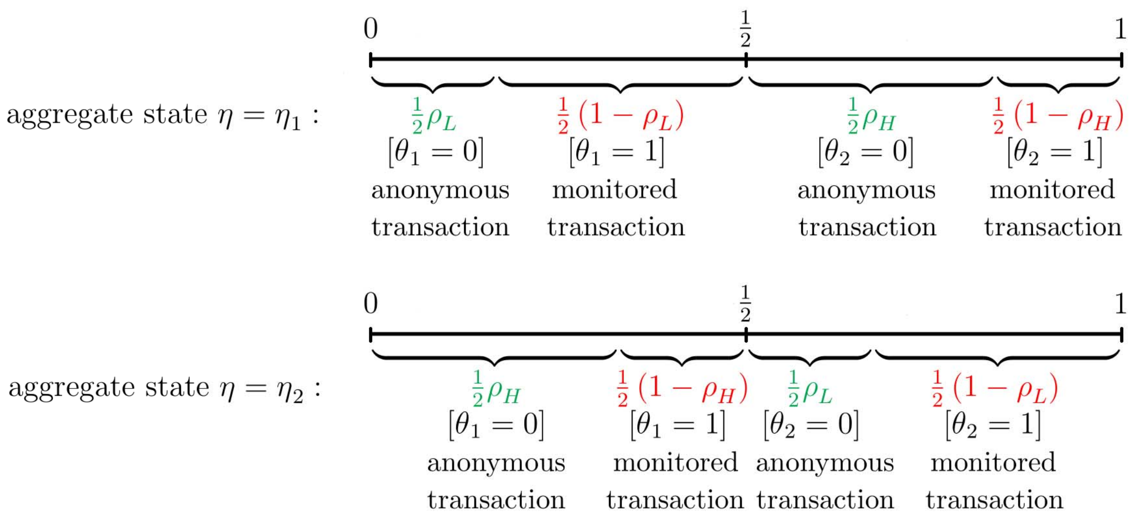


Figure 4: Environment with complete markets

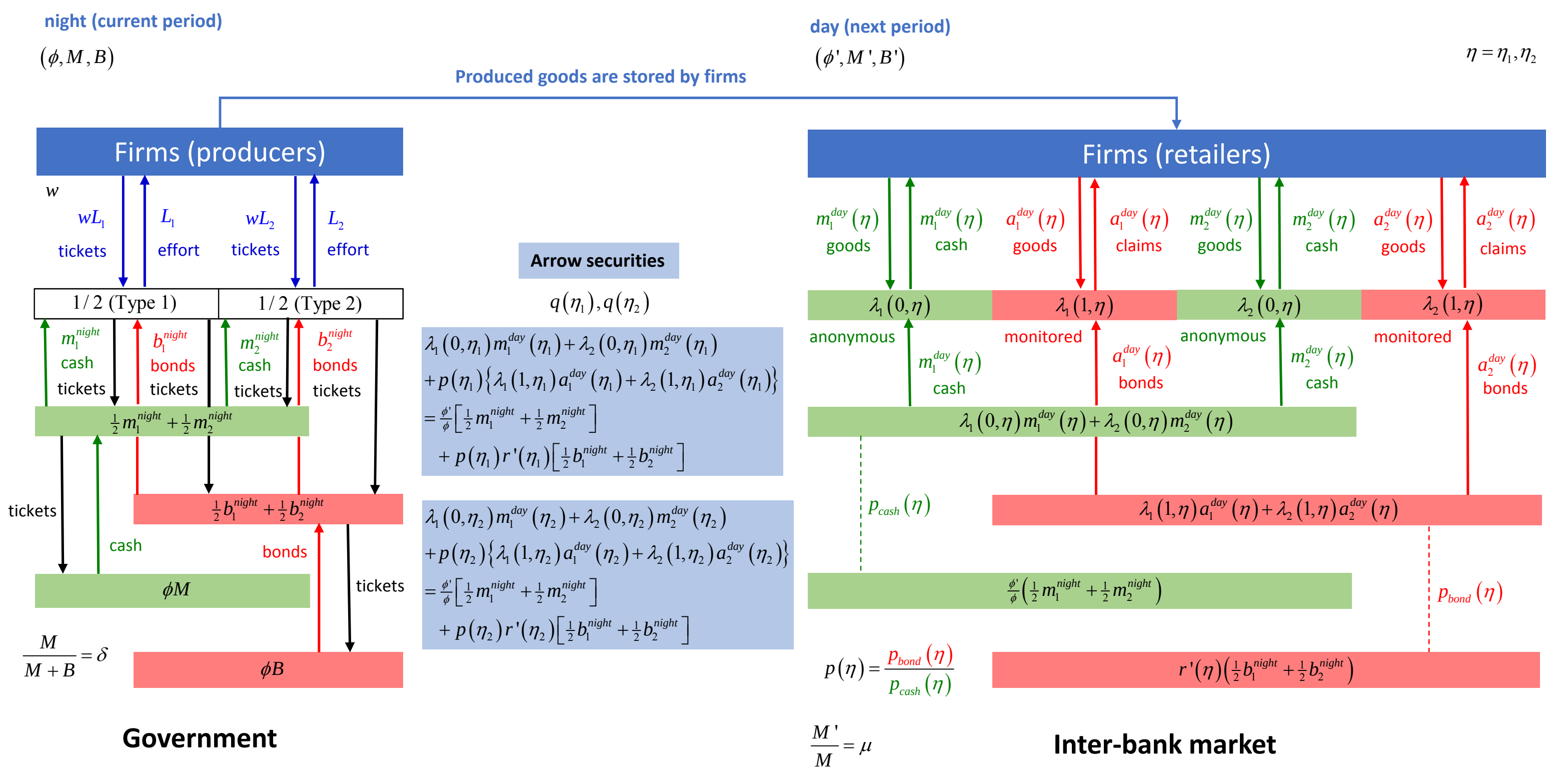


Figure 5: Environment with incomplete markets

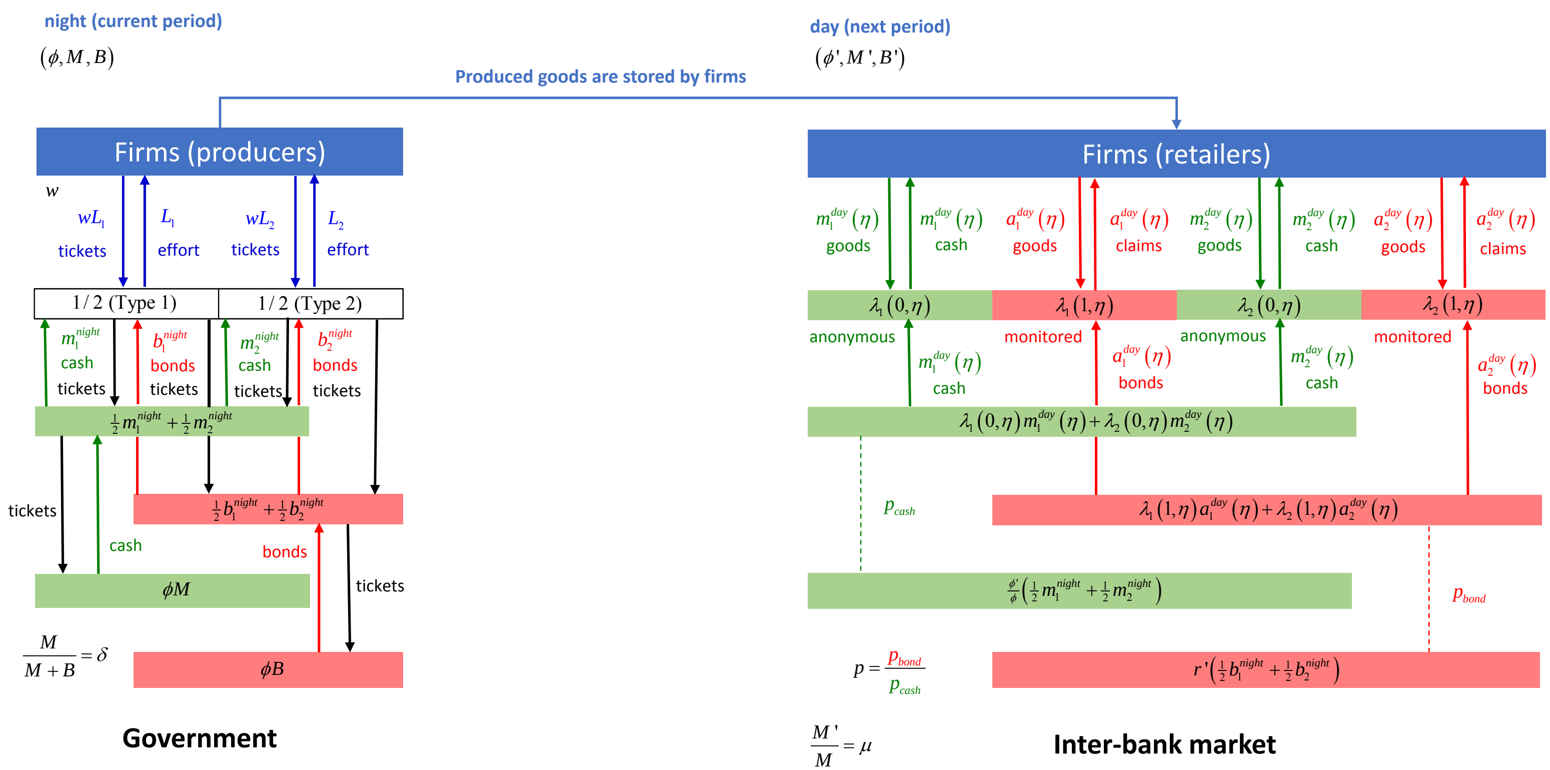


Figure 6: Budget constraints under incomplete markets I ($\delta < \rho$)

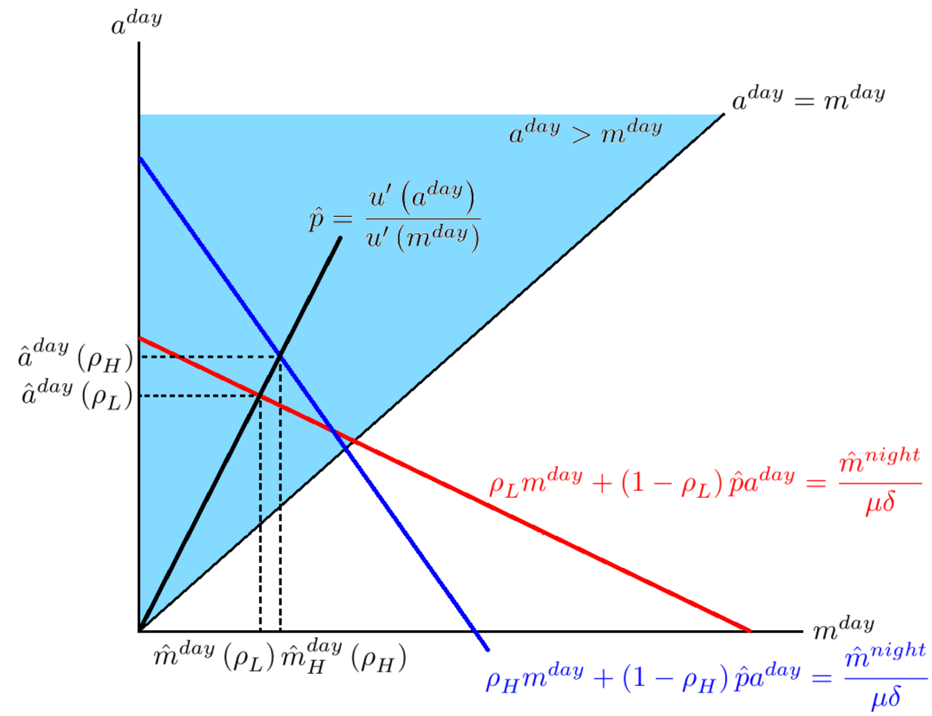


Figure 7: Budget constraints under incomplete markets II ($\delta = \rho$)

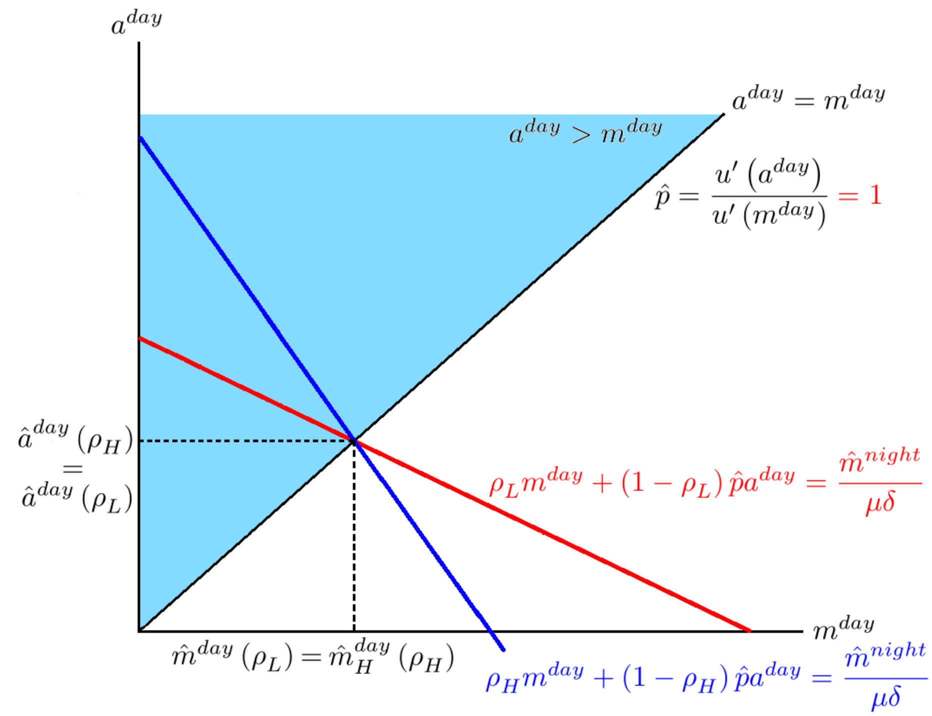


Table 1: Regulatory frameworks

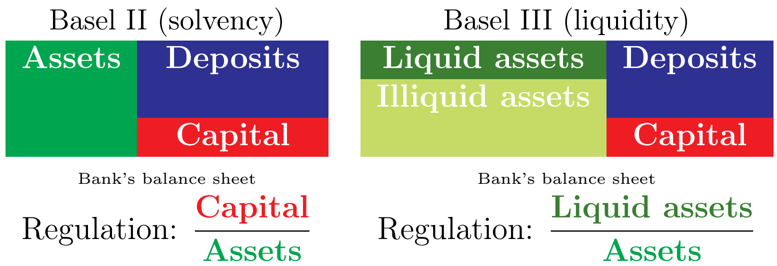


Table 2: Numerical example ($RRA = \sigma < 1$)

Variable	Equilibrium with complete markets	Equilibrium with incomplete markets	Regulated equilibrium
Discount factor: β	0.8000	0.8000	0.8000
Utility function: A	1.0000	1.0000	1.0000
Utility function: σ	0.2500	0.2500	0.2500
Low proportion: ρ_L	0.1000	0.1000	0.1000
High proportion: ρ_H	0.2000	0.2000	0.2000
Money growth rate: μ	1.0250	1.0250	1.0250
Currency-to-bond ratio: δ	0.0800	0.0800	0.0808
Price of Arrow security			
$q(\eta_2)/q(\eta_1)$	1.0000	-.-	-.-
Spot price (interbank market)			
$p(\eta_1)$	0.7898	-.-	-.-
$p(\eta_2)$	0.7898	-.-	-.-
p	-.-	0.7870	0.7891
$1/\mu$	0.9756	0.9756	0.9756
Real return (govt. bonds)			
$r(\eta_1)$	1.2352	-.-	-.-
$r(\eta_2)$	1.2352	-.-	-.-
r	-.-	1.2397	1.2397
$1/\beta$	1.2500	1.2500	1.2500
Fiat currency (night)			
$\underline{\boldsymbol{m}}^{night}$	-.-	-.-	0.0582
m^{night}	0.0571	0.0576	0.0582
$\overline{\boldsymbol{m}}^{night}$	-.-	-.-	-.-
pr	0.9756	0.9756	0.9783
Govt. bonds (night)			
$\underline{\boldsymbol{b}}^{night}$	-.-	-.-	-.-
b^{night}	0.6561	0.6625	0.6619
$\overline{\boldsymbol{b}}^{night}$	-.-	-.-	0.6619
Cash (day)			
$m^{day}(\eta_1)$	0.3711	0.3609	0.3647
$m^{day}(\eta_2)$	0.3711	0.3816	0.3853
Claims (day)			
$a^{day}(\eta_1)$	0.9534	0.9409	0.9403
$a^{day}(\eta_2)$	0.9534	0.9947	0.9936
Tax (night)			
τ	0.1529	0.1574	0.1572
Welfare measure: W	0.0848	0.0827	0.0828
(as a % of $W^{\text{complete markets}}$)	100.00	97.48	97.66