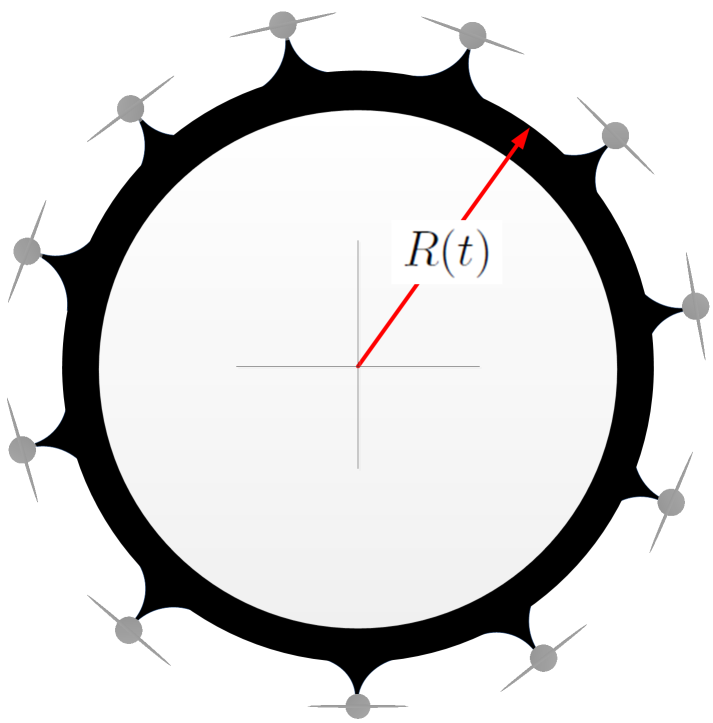
# *Vacuum Energy and the Quantized Electron*

*March 23, 2017*



**Abstract**

The success of vacuum gauge electrodynamics hinges on the ability to demonstrate that the flow of the vacuum field away from the source electron bears a direct equivalence to Dirac electron theory. This can be shown by evaluating the total stress tensor at the electron radius, or by finding the zero of the vacuum Lagrangian. In addition to this, the quantized Dirac field Hamiltonian and other operators can be derived from the classical electron total energy tensor.

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# 1 Theory of a Classical Electron

A successful theory of a stable classical electron follows from an application of the causality principle to its electromagnetic field which leads directly to the vacuum gauge condition

$$|\eta| = \sqrt{\mathrm{E}^2 - \mathrm{B}^2} \tag{1.1}$$

Aside from the de-coupling of velocity and acceleration fields into independent theories, the velocity fields also become momentum flux fields related to the traditional velocity electric and magnetic fields thru the simple relations

$$\boldsymbol{\pi}_{\mathrm{E}} = \sigma_e \mathbf{E} \tag{1.2a}$$

$$\boldsymbol{\pi}_{\mathrm{B}} = \sigma_e \mathbf{B} \tag{1.2b}$$

and satisfying the set of Maxwell-Lorentz equations given by

$$\boldsymbol{\nabla} \cdot \boldsymbol{\pi}_{\mathrm{E}} = 4\pi\rho_\pi \qquad \boldsymbol{\nabla} \times \boldsymbol{\pi}_{\mathrm{B}} = \frac{4\pi}{c}\mathbf{J}_\pi + \frac{1}{c}\frac{\partial \boldsymbol{\pi}_{\mathrm{E}}}{\partial t} \tag{1.3a}$$

$$\boldsymbol{\nabla} \times \boldsymbol{\pi}_{\mathrm{E}} = -\frac{1}{c}\frac{\partial \boldsymbol{\pi}_{\mathrm{B}}}{\partial t} \qquad \boldsymbol{\nabla} \cdot \boldsymbol{\pi}_{\mathrm{B}} = 0 \tag{1.3b}$$

No such momentum flux is associated with the acceleration fields which keep their identity but are now linked to a divergence free acceleration current density $J_a^\nu$ proportional to the four-acceleration of the particle.

Velocity potentials determined by the vacuum gauge condition are null potentials

$$A^\nu = \frac{eR^\nu}{\rho^2} \tag{1.4}$$

In the neighborhood of the charged particle they appear as displacements of the vacuum producing stresses and strains on the surrounding medium. Specifically, the quantity $\eta^{\mu\nu} = \partial^\mu A^\nu$ defines a ***vacuum strain tensor*** whose scalar contraction is exactly the vacuum gauge condition while the quantity $\Delta^{\mu\nu}$ defines the associated ***vacuum tensor***:

$$\Delta^{\mu\nu} = \eta^{\mu\nu} - \mathrm{g}^{\mu\nu}\eta \tag{1.5}$$

Together, vacuum strain and stress tensors determine the velocity Lagrangian

$$\mathcal{L}_{vac} = -\frac{1}{8\pi}\Delta^{\mu\nu}\eta_{\mu\nu} + \frac{1}{2}\sigma_e\,\eta - \frac{1}{c}j_e^{*\mu}A_\mu \tag{1.6}$$

specifically designed to propagate the vacuum while having equations of motion which are precisely the Maxwell-Lorentz equations of the classical electron. This Lagrangian admits a total stress tensor which can be shown to be

$$\mathcal{T}^{\mu\nu} = \tfrac{1}{4}\mathrm{g}^{\mu\nu}\mathcal{R} - \mathcal{R}^{\mu\nu} + \Lambda^{\mu\nu} \tag{1.7}$$

The quadratic portion has an exact equivalence to the electromagnetic theory and is gauge invariant

$$\Theta^{\mu\nu} = \tfrac{1}{4}\mathrm{g}^{\mu\nu}\mathcal{R} - \mathcal{R}^{\mu\nu} \tag{1.8}$$

In contrast, the propagation term $\Lambda^{\mu\nu}$ must be constructed from vacuum gauge potentials only.

**Stability and Radiated Stress:** For motion relative to an arbitrary inertial frame covariant representations of the symmetric stress tensor and the vacuum tensor can be written

$$\Theta^{\mu\nu} = \frac{1}{4\pi}\eta^2\left[\beta^\mu\beta^\nu - \mathcal{U}^\mu\mathcal{U}^\nu - \tfrac{1}{2}\mathrm{g}^{\mu\nu}\right] \tag{1.9a}$$

$$\Lambda^{\mu\nu} = \tfrac{1}{2}\sigma_e\eta\left[2\mathcal{U}^\mu\mathcal{U}^\nu - \beta^\mu\beta^\nu + \beta^\mu\mathcal{U}^\nu\right] \tag{1.9b}$$

A covariant theory of stability can be established by defining the unitless quantities

$$\mathcal{G}_1^{\mu\nu} \equiv 2\beta^\mu\beta^\nu - 2\mathcal{U}^\mu\mathcal{U}^\nu - \mathrm{g}^{\mu\nu} \tag{1.10a}$$

$$\mathcal{G}_2^{\mu\nu} \equiv \mathcal{U}^\nu\beta^\mu + \beta^\mu\beta^\nu - \mathrm{g}^{\mu\nu} = -\partial^\nu R^\mu \tag{1.10b}$$

This allows the symmetric stress tensor and the vacuum tensor to be written

$$\Theta_1^{\mu\nu} = \frac{1}{8\pi}\eta^2\mathcal{G}_1^{\mu\nu} \qquad\qquad \Lambda^{\mu\nu} = -\frac{1}{2}\sigma_e\eta\left[\mathcal{G}_1^{\mu\nu} - \mathcal{G}_2^{\mu\nu}\right] \tag{1.11}$$

and the total stress is

$$\mathcal{T}^{\mu\nu} = -(\mathcal{L}_o + \mathcal{L}_\Lambda)\mathcal{G}_1^{\mu\nu} + \mathcal{L}_\Lambda\mathcal{G}_2^{\mu\nu} \tag{1.12}$$

Stability is determined by the condition $\rho = r_e$ for which the first term on the right vanishes. What remains is radiated vacuum stress having the value

$$\mathcal{E}_{rad}^{\mu\nu} \equiv \mathcal{L}_\Lambda\mathcal{G}_2^{\mu\nu}\bigg|_{\rho=r_e} = \frac{e^2}{8\pi r_e^4}\left[\mathcal{U}^\nu\beta^\mu + \beta^\mu\beta^\nu - \mathrm{g}^{\mu\nu}\right] \tag{1.13}$$

This tensor has no energy component in the rest frame.

**Properties of $\mathcal{E}_{rad}^{\mu\nu}$:** The divergence of $\mathcal{T}^{\mu\nu}$ applied to the first index over all space is

$$\partial_\mu\mathcal{T}^{\mu\nu} = -\frac{1}{c}j_e^{*\lambda}\eta^\nu{}_\lambda \tag{1.14}$$

But if we decide—by dilating the vacuum—that quadratic stresses cannot exist inside the sphere of radius $\rho = r_e$, then this divergence will be zero. This means that the integral over the four-volume of the hyper-cylinder in figure 1 is also zero and Gauss' law can then be written

$$\int \partial_\mu\mathcal{T}^{\mu\nu}R^2 d\rho d\Omega\, cd\tau = \oint_{cyl}\mathcal{T}^{\mu\nu}d\mathcal{S}_\mu = 0 \tag{1.15}$$

In the spherically-based coordinate system the three appropriate surface integrals are

$$\int_{\mathcal{S}_1}\Lambda^{\mu\nu}d\sigma_\mu^s + \int_{\mathcal{S}_2}\Lambda^{\mu\nu}d\sigma_\mu^s + \int_{\mathcal{S}_3}\mathcal{E}_{rad}^{\mu\nu}d\sigma_\mu^\tau = 0 \tag{1.16}$$

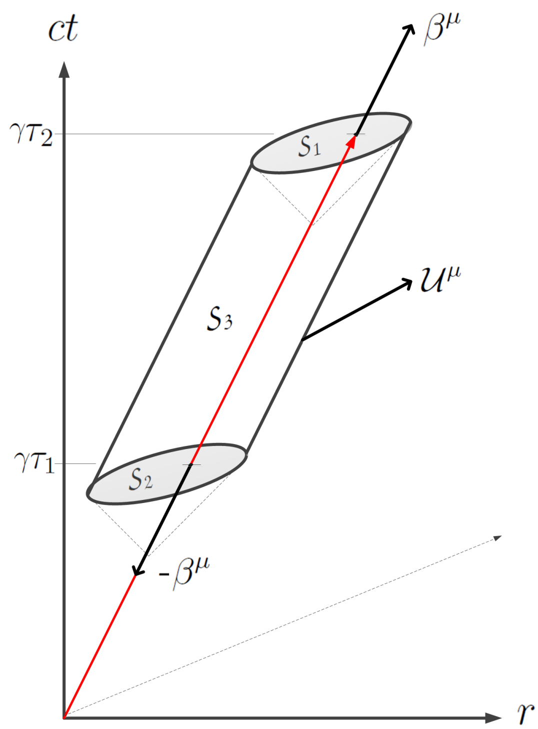


Figure 1: *Divergence theorem applied to the total stress tensor. Red line is the world line of the particle.*

The first two integrals give the energy-momentum four-vector of the particle with different signs so that the sum over the hyper-surfaces $\mathcal{S}_1$ and $\mathcal{S}_2$ is zero. This requires the integral of $\mathcal{E}^{\mu\nu}_{rad}$ to be zero which can easily be verified by integrating over solid angle.

Gauss' law applied to the second index is more interesting. The four-volume integral is not zero anymore since the divergence of the vacuum tensor includes a $\delta$-function:

$$\partial_\nu \Lambda^{\mu\nu} = 2\pi\sigma_e j_e^{*\nu}/c \tag{1.17}$$

In this case the divergence theorem is

$$\int \partial_\nu \Lambda^{\mu\nu} d^4 v = \oint_{cyl} \mathcal{T}^{\mu\nu} d\mathcal{S}_\nu = mc^2 \beta^\mu \Delta\tau/\tau_e \tag{1.18}$$

Here again the integrals over the spacelike planes cancel but this time the flux integral over $\mathcal{S}_3$ is

$$-\int_{\mathcal{S}_3} \mathcal{E}^{\mu\nu}_{rad} d\sigma^\tau_\nu = mc^2 \beta^\mu \Delta\tau/\tau_e \tag{1.19}$$

This is the total flux of vacuum power radiated from the particle radius over a proper time $\Delta\tau$.

# 2 Quantization of the Vacuum Gauge Electron

To establish the link between the vacuum gauge electron and the Dirac theory it is useful to determine how the vacuum tensor responds to the $\gamma$-matrices. Applying $\gamma^\mu\gamma^\nu$ to the vacuum strain produces

$$\gamma^\mu\gamma^\nu\partial_\mu A_\nu = I \cdot \mathrm{g}^{\mu\nu}\partial_\mu A_\nu - i\sigma^{\mu\nu}\partial_\mu A_\nu \equiv \eta_{\scriptscriptstyle D} \tag{2.1}$$

This equation also determines an operator version of the vacuum tensor

$$\mathit{\Delta}_{\scriptscriptstyle D} \equiv -i\sigma^{\mu\nu}\partial_\mu A_\nu \tag{2.2}$$

which is solely a function of the classical velocity fields of the particle:

$$\mathit{\Delta}_{\scriptscriptstyle D} = \begin{bmatrix} i\boldsymbol{\sigma}\cdot\mathbf{B} & \boldsymbol{\sigma}\cdot\mathbf{E} \\ \boldsymbol{\sigma}\cdot\mathbf{E} & i\boldsymbol{\sigma}\cdot\mathbf{B} \end{bmatrix} \tag{2.3}$$

Applying Dirac spinor fields to $\mathit{\Delta}_{\scriptscriptstyle D}$ then shows that

$$\bar{\psi}\mathit{\Delta}_{\scriptscriptstyle D}\psi = 0 \tag{2.4}$$

and this result can be extended to the case where the left and right spinor fields are different. One might also re-formulate the vacuum Lagrangian as a Dirac operator:

$$\mathcal{L}_{\scriptscriptstyle D} = -\frac{1}{8\pi}\mathit{\Delta}_{\scriptscriptstyle D}\cdot\eta_{\scriptscriptstyle D} = -\frac{1}{8\pi}\left[\eta\mathit{\Delta}_{\scriptscriptstyle D} - \mathit{\Delta}_{\scriptscriptstyle D}^2\right] \tag{2.5}$$

When applying Dirac spinors to both sides of $\mathcal{L}_{\scriptscriptstyle D}$ off-diagonal elements are eliminated since the classical velocity fields of the electron satisfy $\mathbf{E}\cdot\mathbf{B} = 0$. The end result is a re-production of four copies of the original vacuum Lagrangian

$$\bar{\psi}\mathcal{L}_{\scriptscriptstyle D}\psi = I\cdot\mathcal{L}_{vac}\bar{\psi}\psi \tag{2.6}$$

The propagation term may also be included in $\mathcal{L}_{vac}$ and the previous equation is still valid. It seems that any attempt to include fundamental quantities of vacuum gauge electrodynamics into Dirac electron theory is bound for failure. This may be a welcome result however since any new predictions resulting from the vacuum gauge run the risk of destroying its credibility.

## 2.1 QED Lagrangian

A simple (but somewhat superficial) algorithm for deriving the QED Lagrangian is to combine the Dirac gamma matrices with the total stress tensor and integrate over the timelike surface element in (1.16) producing the operator integral

$$\hat{\mathcal{O}} = -\frac{1}{c}\int_{\rho=r_e}\gamma^\mu\,\mathcal{T}_{\mu\nu}\,d\sigma_\tau^\nu \tag{2.7}$$

Comparison with equation (1.12) shows that the only contribution to this integral comes from $\mathcal{E}^{\mu\nu}_{rad}$. The previous integral can therefore be reduced to

$$\hat{\mathcal{O}} = -\frac{1}{c}\int_{\rho=r_e} \gamma^\mu \, \mathcal{E}^{rad}_{\mu\nu}\, \mathcal{U}^\nu R^2 d\Omega \, cd\tau \tag{2.8}$$

Carrying out integrals over solid angle leaves only contributions in the direction of the four-velocity. Integrating the proper time to the electron radius $[0, \tau_e]$ and including an integration constant with an appropriate sign then allows the operator to be written

$$\hat{\mathcal{O}} = \gamma^\mu p_\mu \pm mc \tag{2.9}$$

The Dirac Lagrangian follows from the usual substitution

$$p_\mu \longrightarrow i\hbar\partial_\mu \tag{2.10}$$

along with the introduction of a pair of conjugate spinor fields. An interaction term can be generated by replacing the four-gradient with the covariant derivative

$$\partial_\mu \longrightarrow \partial_\mu - i\frac{e}{\hbar c}A'_\mu \tag{2.11}$$

where $A'_\mu$ is an external potential. If this potential is associated with its own external free field then the complete QED Lagrangian (for the particle only) follows as

$$\mathcal{L}_{QED} = i\hbar c\,\bar{\psi}\,\partial_\nu\gamma^\nu\psi - mc^2\bar{\psi}\psi - e\bar{\psi}\gamma^\nu\psi A'_\nu - F'^{\mu\nu}F'_{\mu\nu}/16\pi \tag{2.12}$$

This calculation exemplifies the notion that the physics of the vacuum gauge electron is of no consequence to the theory of quantum electrodynamics.

## 2.2 Dirac Electron from Radiated Stress

A more sophisticated approach linking $\mathcal{E}^{\mu\nu}_{rad}$ to Dirac electron theory begins with the four-volume integral

$$\mathcal{S}^{\mu\nu} = \int \mathcal{T}^{\mu\nu}\rho^2 d\rho d\Omega' cd\tau \tag{2.13}$$

Witholding integration over $\rho$ and integrating proper time to the electron radius leads to

$$\left.\frac{d\mathcal{S}^{\mu\nu}}{d\rho}\right|_{\rho=r_e} = \int \mathcal{E}^{\mu\nu}_{rad}\, r_e^2 d\Omega' \, cd\tau = mc^2[\beta^\mu\beta^\nu - \mathrm{g}^{\mu\nu}] \tag{2.14}$$

This determines the particles' classical energy-momentum tensor $\mathcal{E}^{\mu\nu}_{part}$ with the inclusion of the integration constant.

Unfortunately, the functional form of $\mathcal{E}^{\mu\nu}_{rad}$ in equation (1.13)—although technically correct—is inadequate as a tensor representing radiated stress. To understand why, it is important to realize that it must be constrained by the causality sphere. The implication is that radiated stress must be determined from binary combinations of timelike and spacelike Fourier modes of potentials of the form

$$A^{\nu}_{e:\omega}(\rho) = \sqrt{\frac{2}{\pi}} \cdot e\tilde{\phi}_{\omega} \frac{e^{-i\omega\rho/c}}{\omega\rho} \beta^{\nu} \tag{2.15a}$$

$$A^{\nu}_{\ell:\omega}(\rho) = \sqrt{\frac{2}{\pi}} \cdot e\tilde{\phi}_{\omega} \frac{e^{-i\omega\rho/c}}{\omega\rho} \mathcal{U}^{\nu} \tag{2.15b}$$

A Fourier mode $[\mathcal{E}^{\mu\nu}_{rad}]_{\omega}$ follows from a general equation

$$[\mathcal{E}^{\mu\nu}_{rad}]_{\omega} = \frac{1}{a_e} \left[A^{\nu}_{\ell} A^{\mu}_{e} + A^{\mu}_{e} A^{\nu}_{e} - (A^{\lambda}_{e} A^{e}_{\lambda} + A^{\lambda}_{\ell} A^{e}_{\lambda}) \mathrm{g}^{\mu\nu}\right]_{\omega} \qquad \rho = r_e \tag{2.16}$$

where terms multiplying the metric are inserted to enforce

$$[\mathcal{E}^{\mu\nu}_{rad}]_{\omega} \longrightarrow [Z^{\mu\nu} - Z\mathrm{g}^{\mu\nu}]_{\omega} \qquad \rho = r_e \tag{2.17}$$

This gives the appropriate form of $\mathcal{E}^{\mu\nu}_{rad}$ in equation (2.14) which can be expanded as

$$\left.\frac{d\mathcal{S}^{\mu\nu}}{d\rho}\right|_{\rho=r_e} = \frac{mc^2}{4\pi r_e^3} \int [\mathcal{U}^{\nu}\beta^{\mu} + \beta^{\mu}\beta^{\nu} - \beta^{\lambda}(\beta_{\lambda} + \mathcal{U}_{\lambda})\mathrm{g}^{\mu\nu}] r_e^2 d\Omega' c d\tau \tag{2.18}$$

To obtain Dirac electron theory it will be necessary to require that stress radiated by the classical particle cannot be known to the quantum mechanical theory. This must be true for each Fourier mode of the classical field so that the integrand above must be zero. One practical possibility for a vanishing integrand is to make the quantum mechanical substitution

$$\boxed{\mathcal{U}^{\nu} \longrightarrow \pm\gamma^{\nu}} \tag{2.19}$$

while replacing the four-velocity using $p^{\nu} = mc\beta^{\nu}$. The integral now reads

$$\left.\frac{d\mathcal{S}^{\mu\nu}}{d\rho}\right|_{\rho=r_e} = \frac{c}{4\pi} \int [\pm\gamma^{\nu}p^{\mu} + \frac{1}{mc}p^{\mu}p^{\nu} - mc\,\mathrm{g}^{\mu\nu} \mp \gamma^{\lambda}p_{\lambda}\mathrm{g}^{\mu\nu}] d\Omega' \tag{2.20}$$

Taking the trace of the integrand introduces factors of 3 indicative of radiated vacuum energy in each of three spatial directions. The resulting scalar equaton acting on $\psi$ is either the particle or anti-particle Dirac equation

$$(\gamma^{\nu}p_{\nu} \pm mc)\psi = 0 \tag{2.21}$$

and becomes the Dirac Lagrangian when operated on the left by the conjugate wave function. More precisely, when (2.20) is operated on the left and right by Dirac spinors the integrand can be written in terms of the Dirac energy-momentum tensor

$$\frac{1}{mc}p^{\mu}p^{\nu}\bar{\psi}\psi \pm \mathsf{T}^{\dagger\mu\nu}_{Dirac} = 0 \tag{2.22}$$

where

$$\mathsf{T}^{\mu\nu}_{Dirac} = \bar{\psi}[\gamma^{\mu}p^{\nu} - \mathrm{g}^{\mu\nu}(\gamma^{\lambda}p_{\lambda} \pm mc)]\psi \tag{2.23}$$

## 2.3 Dirac Electron from the Vacuum Lagrangian

Another approach toward a derivation of the Dirac Lagrangian is to begin with the free term vacuum Lagrangian in equation (1.6) which vanishes at $\rho = r_e$—equivalent to the condition

$$\eta - 4\pi\sigma_e = 0 \tag{2.24}$$

This is just the vacuum gauge condition evaluated at the classical radius but it already looks like the Dirac Lagrangian. Suppose we write

$$\mathcal{L}_{[r_e]} = \partial_\nu A^\nu_\ell - 4\pi\sigma_e \tag{2.25}$$

which generates the stress tensor

$$T^{\mu\nu}_{[r_e]} = \partial^\nu A^\mu_\ell - \mathrm{g}^{\mu\nu}(\partial_\nu A^\nu_\ell - 4\pi\sigma_e) \tag{2.26}$$

Quantization follows by associating $A^\nu_\ell$ with unobservable rotations of the particles' spin magnetic moment. This is possible by trading out the gauge field in either of equations (2.25) or (2.26) for Dirac matrices via

$$\boxed{e\tau_e\, A^\nu_\ell \longrightarrow \pm 2i\hbar\,\gamma^\nu} \tag{2.27}$$

and then operating on the left and right with Dirac spinors. Roughly speaking, equation (2.27) says that the action of the gauge field for the classical time $\tau_e$ has a quantum mechanical counterpart associated with the production of two fundamental units of angular momentum. Instead of inserting it into (2.25) one can also apply $\partial_\nu$ directly to both sides of (2.27) and evaluate at $\rho = r_e$. This will become the $e^+\, e^-$ Dirac equation when operated on by $\psi$.

One more possibility is to begin by integrating equation (2.24) over solid angle

$$\int_\Omega [\eta - 4\pi\sigma_e]\, d\Omega = 0 \tag{2.28}$$

This approach requires the velocity potentials to be written in terms of dilatation functions

$$A^\nu = 4\pi\sigma_e(\theta,\phi)\, u^\nu(R) \tag{2.29}$$

and will lead to $\partial_\nu u^\nu - 1 = 0$. The solution here is the condition $R = r_e$ but quantization follows by making the correspondence

$$\boxed{mc\, u^\nu \longrightarrow \pm i\hbar\gamma^\nu} \tag{2.30}$$

Applying Dirac spinor fields once again yields

$$\mathcal{L}_{Dirac} = i\hbar c\,\bar\psi\,\partial_\lambda\gamma^\lambda\psi \pm mc^2\bar\psi\psi = 0 \tag{2.31}$$

Since the value of this Lagrangian is zero, the energy-momentum tensor in (2.23) simplifies to

$$\mathsf{T}^{\mu\nu}_{_{Dirac}} = i\hbar c\,\bar{\psi}\,\partial^{\mu}\gamma^{\nu}\psi \tag{2.32}$$

Now suppose a normalized wave function is chosen to be

$$\psi = \frac{1}{\sqrt{2mc^2}} u(E,\boldsymbol{p})e^{-i(Et-\boldsymbol{p}\cdot\boldsymbol{r})/\hbar} \tag{2.33}$$

Inserting this into (2.32) shows that the energy-momentum tensor is

$$\mathsf{T}^{\mu\nu}_{_{Dirac}} = mc^2\beta^{\mu}\beta^{\nu} \tag{2.34}$$

To conclude this section it is important to discuss details of both quantization formulas (2.27) and (2.30). Both sides of these equations have units of angular momentum, but it is easy to show that the left side of the equation—evaluated at the electron radius—is smaller by the factor $\frac{1}{2}\alpha_f$ where $\alpha_f$ is the fine structure constant. The physical interpretation of this ratio in the context of a radiating field is not well understood.

## 2.4 Second Quantized Dirac Field

Having derived the stress tensor for Dirac particles and anti-particles, it becomes essential to use vacuum gauge electrodynamics to derive the quantized Dirac field Hamiltonian. To accomplish this it is first necessary to write vacuum gauge potentials for the $e^+\,e^-$ pair shown in figure 2. For this system, the position vector of each particle is

$$\boldsymbol{w}_+ = \boldsymbol{w}_+(\tau) \tag{2.35a}$$

$$\boldsymbol{w}_- = \boldsymbol{w}_-(\tau) \tag{2.35b}$$

Since the pair created and annihilated itself, their positions must be identical at time $\tau = 0$ and $\tau = \tau_o$:

$$\boldsymbol{w}_+(0) = \boldsymbol{w}_-(0) \tag{2.36a}$$

$$\boldsymbol{w}_+(\tau_o) = \boldsymbol{w}_-(\tau_o) \tag{2.36b}$$

Add to these constraints the retardation condition, which is generally different for each particle

$$R^{\nu}_{+} = x^{\nu} - w^{\nu}_{+}(t_r) \tag{2.37a}$$

$$R^{\nu}_{-} = x^{\nu} - w^{\nu}_{-}(t_r) \tag{2.37b}$$

Relative to the moving frame $S$, the vacuum gauge potentials $A^{\nu}_{_{total}}(ct,\boldsymbol{r})$ are the sum of vacuum gauge potentials associated with each particle. In addition, since

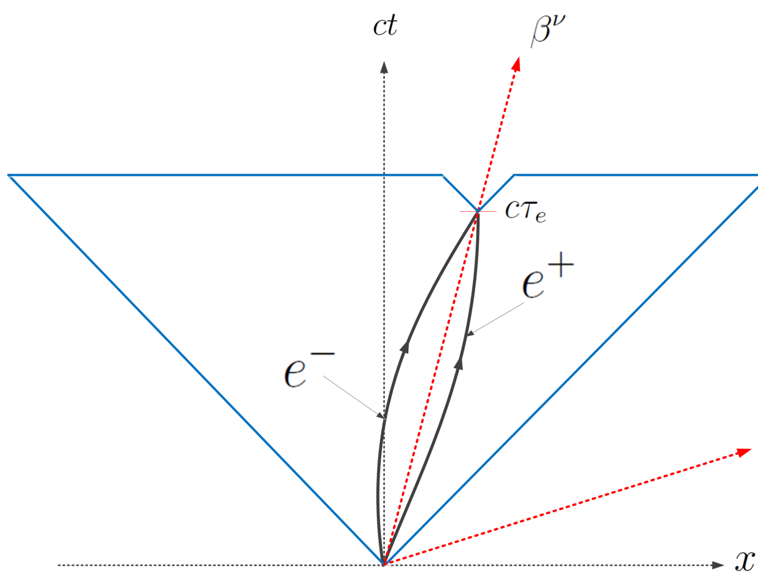


Figure 2: *Classical system of an $e^+ e^-$ pair which annihilates after time $\tau_o$. The blue line marks the limits of the causal fields as viewed in the frame S.*

both particles share the same causality requirements at both ends of their life span, the form of the potentials is

$$A^\nu_{total}(ct, \boldsymbol{r}) = \left[A^\nu_{v+} + A^\nu_{a+} + A^\nu_{v-} + A^\nu_{a+}\right] \cdot \vartheta(z) \cdot \vartheta(\tau_o - z) \tag{2.38}$$

where $z$ contains the particle radius and is defined by

$$z \equiv \tau - \rho/c + \tau_e \tag{2.39}$$

Derivatives then determine the appropriate form of the causal field stength tensor

$$F^{\mu\nu}_{total}(ct, \boldsymbol{r}) = [F^{\mu\nu}_+ + F^{\mu\nu}_-] \cdot \vartheta(z) \cdot \vartheta(\tau_o - z) \tag{2.40}$$

A causal stress tensor for the pair is

$$\mathcal{T}^{\mu\nu}_{total} = [\Theta^{\mu\nu}_+ + \Theta^{\mu\nu}_- + \Lambda^{\mu\nu}_+ + \Lambda^{\mu\nu}_-] \cdot \vartheta(z) \cdot \vartheta(\tau_o - z) \tag{2.41}$$

Dropping the $\pm$ subscripts for now and concentrating on a single particle, its total energy four-vector and radiated field follows from the flux integral

$$\begin{aligned} \mathcal{E}^\mu_{total} &= \int \mathcal{T}^{\mu\nu} d\sigma^s_\nu \\ &= -\beta^\mu \int \mathcal{L}_{vac} \cdot \vartheta(z) \cdot \vartheta(\tau_o - z) \rho^2 d\rho d\Omega' = \mathcal{E}^\mu_{part} + \mathcal{E}^\mu_{vac} \end{aligned} \tag{2.42}$$

After simple integrals over solid angle, the particle term is

$$\mathcal{E}^{\mu}_{part} = \frac{e^2}{2}\beta^{\mu}\int_{r_e}^{r_e+c\tau}\frac{1}{\rho^2}\cdot\vartheta(z)\cdot\vartheta(\tau_o - z)d\rho \tag{2.43}$$

Now let $u$ be the combination of causality functions and integrate by parts. The particle term is now

$$\mathcal{E}^{\mu}_{part} = mc^2\beta^{\mu}\cdot\vartheta(\tau)\cdot\vartheta(\tau_o-\tau) + \frac{e^2}{2}\beta^{\mu}\cdot\chi(\tau) \tag{2.44}$$

where $\chi(\tau)$ is

$$\chi(\tau) = \int_{\tau}^{0}\frac{\delta(z)\cdot\vartheta(\tau_o-z) - \vartheta(z)\cdot\delta(\tau_o-z)}{\tau+\tau_e-z}dz \tag{2.45}$$

This integral marks the causal appearance and disappearance of the dilatated vacuum in the near neighborhood of the particle during creation and annihilation. As the integral shows however both terms will dissipate like $1/\tau$ so that $\chi(\tau)\to 0$ for large enough times.

The vacuum term extends limits of $\rho$ to the origin and reads

$$\mathcal{E}^{\mu}_{vac} = -\frac{1}{2}\sigma_e e\beta^{\mu}\int_0^{c\tau+r_e}\vartheta(z)\cdot\vartheta(\tau_o-z)d\rho d\Omega' \tag{2.46}$$

One possibility for a rigorous calculation is to write $\vartheta(z)$ in terms of Fourier modes and then integrate by parts. On the other hand, an easy way to perform the integral is to note that the limits of integration cover the region where the integrand is one. The result is simply

$$\mathcal{E}^{\mu}_{vac} = -[\tfrac{1}{2}\gamma\dot{\rho}c\,\tau + mc^2]\beta^{\mu} \tag{2.47}$$

Now include the necessary integration constant $mc^2\beta^{\mu}$ and combine equations (2.47) and (2.43) to determine the four-energy of the particle at time $\tau$:

$$\mathcal{E}^{\mu}_{total}(\tau) = [mc^2\cdot\vartheta^{\dagger}\cdot\vartheta - \tfrac{1}{2}\gamma\dot{\rho}c\,\tau]\beta^{\mu} \tag{2.48}$$

In this equation the time dependence of the causality functions has been suppressed and they have been re-labeled in a way to characterize their true function as creation and annihilation operators. The invariant Hamiltonian follows from a contraction with $\beta^{\mu}$ but of interest here will be the time component of $\mathcal{E}^{\mu}_{total} = (\mathcal{E}, c\boldsymbol{\mathcal{P}})$ given by

$$\boxed{\mathcal{E}(\tau) = \gamma mc^2\cdot\vartheta^{\dagger}\cdot\vartheta - \tfrac{1}{2}\gamma\dot{\rho}c\,\tau} \tag{2.49}$$

and it is worth mentioning that the value of $\tau$ can be any value in the range $[0,\tau_o]$.

To derive the quantized Dirac field Hamiltonian it will be necessary to first write energy terms for both the electron and the positron. It will also be necessary evaluate each energy term for the minimal classical time $\Delta\tau = \tau_e$. In this case both energy terms have a functional value of zero

$$e^- : \quad \mathcal{E}^- = \gamma mc^2 \cdot \vartheta^\dagger \cdot \vartheta - \gamma mc^2 \tag{2.50a}$$

$$e^+ : \quad \mathcal{E}^+ = \gamma mc^2 \cdot \vartheta^\dagger \cdot \vartheta - \gamma mc^2 \tag{2.50b}$$

One can argue here that the factors of $\gamma$ for each particle in the two-particle system should be different but this is not an issue since the quantized field operator sums over all possible values of momentum. Now replace the last term in each equation with

$$\gamma mc^2 \longrightarrow \langle 0|E_{\boldsymbol{k}}|0\rangle \tag{2.51}$$

where the quantum mechanical energy is given by $E_{\boldsymbol{k}} = \hbar\omega_{\boldsymbol{k}}$. Also replace the classical creation and annihilation operators with the well known quantum mechanical counterparts

$$e^- : \quad \sqrt{\gamma mc^2} \cdot \vartheta^\dagger \longrightarrow \langle \boldsymbol{k}, s| \equiv \langle 0| \sqrt{2E_{\boldsymbol{k}}} \cdot b_{\boldsymbol{k}}^{s\dagger} \tag{2.52a}$$

$$e^- : \quad \sqrt{\gamma mc^2} \cdot \vartheta \longrightarrow |\boldsymbol{k}, s\rangle \equiv \sqrt{2E_{\boldsymbol{k}}} \cdot b_{\boldsymbol{k}}^{s}|0\rangle \tag{2.52b}$$

$$e^+ : \quad \sqrt{\gamma mc^2} \cdot \vartheta^\dagger \longrightarrow \langle \boldsymbol{k}, s| \equiv \langle 0| \sqrt{2E_{\boldsymbol{k}}} \cdot d_{\boldsymbol{k}}^{s\dagger} \tag{2.52c}$$

$$e^+ : \quad \sqrt{\gamma mc^2} \cdot \vartheta \longrightarrow |\boldsymbol{k}, s\rangle \equiv \sqrt{2E_{\boldsymbol{k}}} \cdot d_{\boldsymbol{k}}^{s}|0\rangle \tag{2.52d}$$

Adding the two quantized Hamiltonians together, dividing by a factor of 2, and summing over all possible momenta and spins then derives $\langle 0|H_{QFT}|0\rangle$ where the quantized Dirac Field Hamiltonian operator is given by

$$H_{QFT} = \sum_k \left[ \sum_{s=1,2} E_{\boldsymbol{k}} (b_{\boldsymbol{k}}^{s\dagger}\, b_{\boldsymbol{k}}^{s} + d_{\boldsymbol{k}}^{s\dagger}\, d_{\boldsymbol{k}}^{s} - 1) \right] \tag{2.53}$$

A similar calculation using the space components of equation (2.49) will derive the momentum operator

$$\boldsymbol{P}_{QFT} = \sum_k \left[ \sum_{s=1,2} \boldsymbol{P} (b_{\boldsymbol{k}}^{s\dagger}\, b_{\boldsymbol{k}}^{s} + d_{\boldsymbol{k}}^{s\dagger}\, d_{\boldsymbol{k}}^{s}) \right] \tag{2.54}$$

where $\boldsymbol{P} = \hbar\boldsymbol{k}$.

The factor of two which connects the classical and quantum theories can be explained by the schematic in figure 3. In the classical picture the emission of vacuum energy is described by the gauge field $A_\ell^\nu$ —a radial function which does not change the classical path of the particle. The classical limit is something like simultaneous back-to-back emissions of particles. This is not required by the quantum theory where the directional change of the spin vector emits a single quantum of energy.

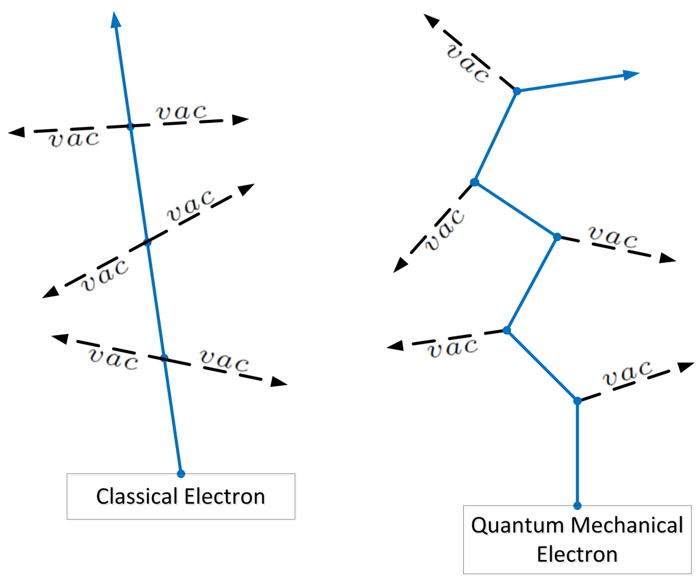


Figure 3: *Comparison of classical and quantum theories of the electron.*

## 2.5 Quantum Theory of the Vacuum Field

The classical theory only knows how to radiate classical waves over all frequencies as dictated by its causality function. Moreover, the energy radiated between frequencies $\omega$ and $\omega + \delta\omega$ must be vanishingly small. It may be possible to re-interpret Fourier modes of the step as probabilities for the emission of quanta, but we do not see how this can be done. Another possibility is to limit the values of the emitted frequencies to a discrete set. Unfortunately, this destroys the ability to construct the causality step which relies on a continuous spectrum. On the other hand, quantizing the radiation field requires the observer to view the field as streams of particles which is fundamentally different than the propagation of longitudinal waves. If this is the case it might be expected that the classical causality step might lose its meaning. However causality can still be enforced by simply imposing the requirement that all radiated quanta must exist within the particles finite light cone.

In a theory of a discrete set of emissions, first suppose that the number of vacuum quanta radiated per unit time is twice the Dirac frequency, or $2\nu_D$. If we require this theory to re-produce the inertial power formula of the classical theory then the average energy of the emitted quanta must be given by

$$\langle \mathsf{E}_{vac} \rangle = \frac{P_{in}}{2\nu_D} = \frac{1}{2}\frac{hc}{r_e} \tag{2.55}$$

In other words, longitudinal oscillations of the particle radius can be associated with the emission of a vacuum particle having a wavelength $\lambda \sim r_e$. If indeed, all particles have this same energy then the ratio of the mass-energy of the particle to the energy of the emitted quanta is given by[1]

$$\frac{mc^2}{\langle \mathsf{E}_{vac} \rangle} = \frac{\alpha_f}{2\pi} \tag{2.57}$$

where $\alpha_f$ is the fine structure constant. In this scenario, the energy of emitted particles is therefore several orders of magnitude larger than the mass-energy of the particle itself.

Now impose the requirement that the electron may only radiate at a discrete set of frequencies given by $\omega = n\omega_o$ where $n$ is a quantum number and $\omega_o$ is a fundamental frequency which has yet to be determined. The value of $\Delta\omega$ is

$$\Delta\omega = n\omega_o - (n-1)\omega_o = \omega_o \tag{2.58}$$

Fourier modes of the vacuum gauge potentials still have the same overall form as the continuum theory, but it will be useful to introduce an overall unitless constant $\zeta$ so that potential fields at index $n$ can be written

$$A^{\nu}_{e:n} = e\sqrt{\frac{2}{\zeta}} \cdot \frac{\sin\omega(\tau - \rho/c + \tau_e)}{\omega\rho} \cdot \beta^{\nu}\Delta\omega \tag{2.59a}$$

$$A^{\nu}_{\ell:n} = e\sqrt{\frac{2}{\zeta}} \cdot \frac{\sin\omega(\tau - \rho/c + \tau_e)}{\omega\rho} \cdot \mathcal{U}^{\nu}\Delta\omega \tag{2.59b}$$

The associated energy flux tensor is

$$\mathrm{S}^{\mu\nu}_n = \frac{1}{2}\pi^{\mu\nu}_n c \tag{2.60}$$

with the momentum flux given by

$$\pi^{\mu\nu}_n = \frac{1}{a_e}[A^{\mu}_{\ell:n}, A^{\nu}_{e:n}] \tag{2.61}$$

Electric and magnetic components of $\mathrm{S}^{\mu\nu}_n$ may be written

$$(\mathbf{S_E})_n = \frac{e^2c}{a_e\zeta}\left[\frac{\hat{\boldsymbol{n}} - \boldsymbol{\beta}}{1 - \boldsymbol{\beta}\cdot\hat{\boldsymbol{n}}}\right] \cdot \frac{\sin^2 n\omega_o(\tau - \rho/c + \tau_e)}{n^2\rho^2} \tag{2.62a}$$

$$(\mathbf{S_B})_n = \frac{e^2c}{a_e\zeta}\left[\frac{\boldsymbol{\beta} \times \hat{\boldsymbol{n}}}{1 - \boldsymbol{\beta}\cdot\hat{\boldsymbol{n}}}\right] \cdot \frac{\sin^2 n\omega_o(\tau - \rho/c + \tau_e)}{n^2\rho^2} \tag{2.62b}$$

[1] The right of side of this equation is etched in Schwingers' tombstone and allows the electron g-factor and its first order anomaly to be written in terms of the flow of the vacuum field

$$g = 2\left[1 + \frac{mc^2}{\langle \mathsf{E}_{vac} \rangle}\right] \tag{2.56}$$

It is conceivable that $\langle \mathsf{E}_{vac} \rangle$ might be adjusted to cover all orders of the anomalous magnetic moment.

The total power radiated comes only from the electric component and may be determined by first averaging over a single oscillation of the $n^{th}$ mode

$$\langle \mathbf{S_E} \rangle_n = \frac{e^2 c}{2a_e n^2 \rho^2 \zeta} \left[ \frac{\hat{\boldsymbol{n}} - \boldsymbol{\beta}}{1 - \boldsymbol{\beta} \cdot \hat{\boldsymbol{n}}} \right] \tag{2.63}$$

The total power radiated is then

$$P = \sum_{n=1}^{\infty} \oint (\mathbf{S_E})_n \cdot \hat{\boldsymbol{n}} \rho^2 d\Omega' = \frac{mc^2}{\tau_e \zeta} \sum_{n=1}^{\infty} \frac{1}{n^2} \tag{2.64}$$

and if this power is required to be the inertial power $P_{in}$ of the continuum theory, this will conveniently identify $\zeta$ as the Riemann zeta-function $\zeta[2] = \pi^2/6$.

A schematic showing the general $1/\rho^2$ dependence of the energy flux in (2.62) is available in figure 4 and is determined by performing the sum

$$\chi = \sum_{n=1}^{\infty} \frac{\sin^2 n\omega_o z}{n^2} \tag{2.65}$$

over any interval between two successive zeros of the function. Choosing and arbitrary interval, the sum is

$$\chi = \frac{\pi^2}{2} \cdot \frac{\omega_o z}{\pi} \left[ 1 - \frac{\omega_o z}{\pi} \right] \tag{2.66}$$

For a given $\tau$ this is an inverted parabola of which several are shown in the figure. The width of each bump is the same but cannot be determined yet since $\omega_o$ is still unknown.

To this point nothing has been quantized—the continuum classical theory has only been replaced by a countably infinite set of modes. Suppose however that energy at each wave number is composed of vacuum qaunta with energy given by

$$E_n = \frac{1}{2} n \hbar \omega_o \tag{2.67}$$

The total number of quanta scattered per unit time at each index $n$ is then

$$N_n = \frac{P_n}{E_n} = \frac{2P_{in}}{\zeta[2] \hbar \omega_o n^3} \tag{2.68}$$

The only other requirement will be to equate the total quanta radiated per second to twice the Dirac frequency.

$$2\nu_D = \sum_{n=1}^{\infty} N_n = \frac{\zeta[3]}{\zeta[2]} \cdot \frac{2P_{in}}{\hbar \omega_o} \tag{2.69}$$

The ratio of zeta-functions is a constant $\alpha_o \sim 0.73$ and equation (2.69) solves for the fundamental frequency

$$\omega_o = \frac{2\pi \alpha_o}{\tau_e} \tag{2.70}$$

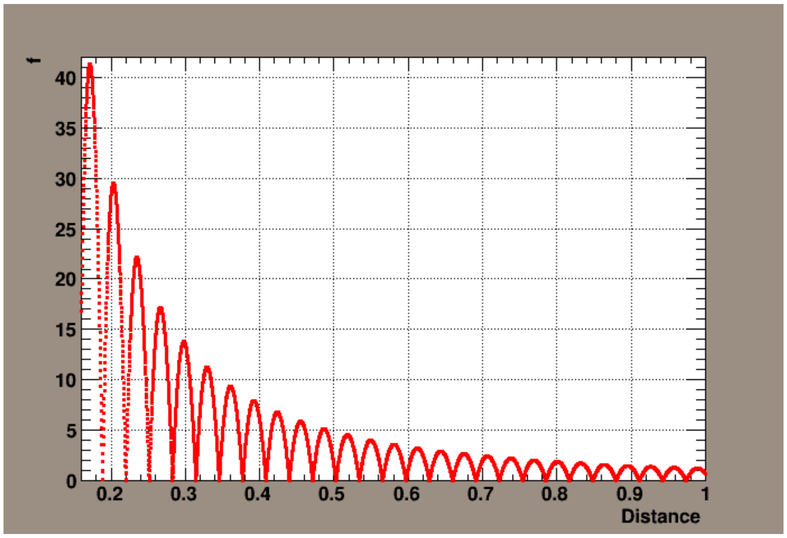


Figure 4: *Plot showing the granularity of the energy flux in the discrete wavelength theory. The distance between adjacent zeros is on the order of the electron radius $r_e$.*

The quantum mechanical energy formula can then be re-written

$$E_n = \frac{n\alpha_o}{2}\frac{hc}{r_e} \tag{2.71}$$

where the quantized wavelengths are given by

$$\lambda_n = \frac{r_e}{\alpha_o n} \tag{2.72}$$

This formula allows for the interpretation of the emission of vacuum particles in terms of harmonic oscillations of the particle radius. The classical radius is simply a superposition of longitudinal pulses summed over all wave vectors. Approximately 83 percent of the emissions are at the fundamental frequency having an energy of $E \sim 321$ MeV. For the 500th harmonic (overtone?) the the rate of emission is reduced by a factor of $10^8$ but there are still $10^{12}$ particles emitted each second. The particle radius associated with each wave vector is proportional to the wavelength and given by

$$r_n = \frac{r_e}{2\pi\alpha_o n} \tag{2.73}$$

If we let $r_n/c \rightarrow \Delta\tau$ and $E_n \rightarrow \Delta E$ then

$$\Delta E \Delta\tau = \frac{\hbar}{2} \tag{2.74}$$

which saturates the lower bound of the uncertainty principle.

A final calculation is the average energy of vacuum radiation determined from

$$\langle \mathsf{E}_{vac} \rangle = \frac{\sum_{n=1}^{\infty} N_n E_n}{\sum_{n=1}^{\infty} N_n} = \frac{1}{2}\frac{hc}{r_e} \tag{2.75}$$

which agrees with (2.55).

# 3 Model for a De Sitter Universe

Suppose an electrically neutral universe explodes at time $c\tau = 0$ producing a collection of $N/2$ electrons and $N/2$ positrons with individual charges $q_i = \pm e$. It may also be assumed that the number density of particles created in any given direction is approximately constant. Each particle can be described by a position vector

$$\boldsymbol{w}_i(c\tau) \quad \text{where} \quad \boldsymbol{w}_i(0) = 0 \qquad i = 1, ..., N \tag{3.1}$$

and a retarded position coordinate

$$\boldsymbol{R}_i = \boldsymbol{r} - \boldsymbol{w}_i(ct_r) \qquad i = 1, ..., N \tag{3.2}$$

## 3.1 Propagation of Vacuum Gauge Potentials

According to vacuum gauge theory each particle will be described by a scalar field

$$\varphi_i = -q_i \ln \rho_i \cdot \vartheta(\Lambda) \tag{3.3}$$

Where $\Lambda = \Lambda(\boldsymbol{R}, c\tau)$ is a currently unknown spherically symmetric causality function which can allow the initial universe to expand faster than light. Summing over all electrons and positrons in the universe and observing that they are all constrained by the same causality sphere produces the resultant field

$$\varphi_{unv}(\boldsymbol{r}, t) = \sum_{i=1}^{N} \varphi_i = \ln \prod_{i=1}^{N} \rho_i^{-q_i} \cdot \vartheta(\Lambda) \tag{3.4}$$

A first derivative for general accelerated motions picks up delta functions for each particle

$$\partial^\nu \varphi_{unv} = \sum_{i=1}^{N} \{(A_v^\nu)_i + (A_a^\nu)_i - (A_e^\nu)_i\} \cdot \vartheta - \sum_{i=1}^{N} q_i \ln \rho \cdot \partial^\nu \vartheta_i \tag{3.5}$$

but the second derivative can be written

$$\partial^\mu \partial^\nu \varphi_{unv} = \sum_{i=1}^{N} \{(\partial^\mu A_v^\nu)_i + (\partial^\mu A_a^\nu)_i - (\partial^\mu A_e^\nu)_i\} \cdot \vartheta_i + S^{\mu\nu} \tag{3.6}$$

where $S^{\mu\nu}$ is the collection of remaining terms symmetric in the two indicies. Now form the derivative

$$\partial_\mu \left[\partial^\mu \partial^\nu \varphi_{unv} - \partial^\nu \partial^\mu \varphi_{unv}\right] = 0 \tag{3.7}$$

This set of operations can be written in terms of the potentials as

$$\sum_{i=1}^{N} \{\Box^2 A_v^\nu + \Box^2 A_a^\nu - \Box^2 A_e^\nu - \partial^\nu \partial_\mu A_v^\mu - \partial^\nu \partial_\mu A_a^\mu\}_i \cdot \vartheta_i = 0 \tag{3.8}$$

The point charge four current $(J_e^{*\nu})_i$ replaces the wave operator acting the Liénard-Wiechert potentials and if the total vacuum gauge potential is defined by

$$A_{unv}^\mu = \sum_{i=1}^{N} \{(A_v^\nu)_i + (A_a^\nu)_i\} \cdot \vartheta_i \tag{3.9}$$

then a simple set of equations describing initial universe can be written

$$\Box^2 A_{unv}^\nu - \partial^\nu \partial_\mu A_{unv}^\mu = \frac{4\pi}{c} J_{e\ unv}^{*\nu} \tag{3.10}$$

where $J_e^{*\nu}$ is a sum of point charge current densities at various locations within the causal spacetime. Both fields $\varphi_{unv}(\boldsymbol{r}, t)$ and $A_{unv}^\nu(\boldsymbol{r}, t)$ also permeate all points of the causal spacetime and diverge at the source infinity located at the position of each particle. The velocity and acceleration terms can be also be separated with the introduction of the acceleration current. Each potential field is still neatly nested within the causality sphere and independent equations of motion can be written

$$\Box^2 A_{v\ unv}^\nu - \partial^\nu \partial_\mu A_{v\ unv}^\mu - \frac{4\pi}{c} J_{e\ unv}^{*\nu} = 0 \tag{3.11a}$$

$$\Box^2 A_{a\ unv}^\nu - \partial^\nu \partial_\mu A_{a\ unv}^\mu - \frac{4\pi}{c} J_{a\ unv}^{\nu} = 0 \tag{3.11b}$$

Completely independent of the calculation in equation (3.10), now require the N particle universe to be associated with a large input vacuum power $P_{in}$. Since radiated vacuum energy from each particle is an invariant, the total energy initially available at time $\tau$ is simply

$$\mathcal{E} = -\tfrac{1}{2} N \dot{\rho} c \tau \tag{3.12}$$

Unfortunately, the initial vacuum power will not be able to sustain itself since individual $e^+ e^-$ pairs will quickly begin to annihilate. If complete annihilation occurs at some later time $\tau_o$ this will mark the end of a phase transition producing an amount of vacuum energy

$$\mathcal{E}_o = \int_0^{\tau_o} P_{in}(\tau) d\tau \tag{3.13}$$

Both velocity and acceleration current densities in equations (3.11) will vanish at $\tau_o$ along with the total vacuum gauge potentials $A_{unv}^\nu$ leaving a universe composed of a swarm of vacuons, photons, and neutrinos.

## 3.2 Dark Energy and Dark Matter

With the exception of light mass neutrinos, the presence of vacuouns and photons constitutes a universe everywhere moving at the speed of light. A de Sitter Vacuum arises by recalling that whatever amount of vacuum has been initially produced, it will be characterized by one positive and one negative vacuum constant making it a porous medium with the ability to briefly tear and then repair itself. A particle physicist might refer to this as the creation and annihilation of $e^+ e^-$ virtual pairs. From the point of view of the classical theory, a spatial volume (cube of side $l$) anywhere in the universe will still be randomly radiating a small amount of additional vacuum energy in all directions. This means the volume will be associated with a small amount of

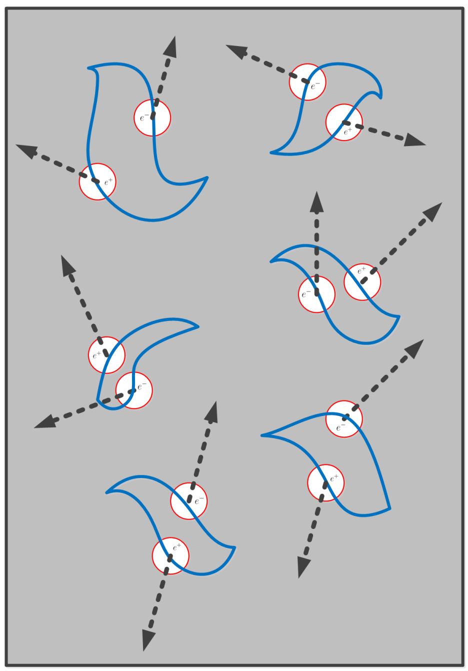

Figure 5: *Graphic illustrating paths (in blue) of short-lived pairs of electrons and positrons. In accordance with the classical theory short lived particle/anti-particle pairs must be associated with the emission of a small amount of vacuum energy. Not shown is ambient sea of zero inertia vacuum energy which innundates the entire volume.*

observable inertia and dark energy content $\mathsf{E}$. A graphic showing general features of this volume is illustrated in 5.

An equation describing the dark energy available in the universe at time $\tau$ can be established under the assumption that the dark energy density is proportional to the vacuum energy density so that for some constant $\alpha$

$$\frac{d\mathsf{E}}{d\tau} = \alpha\mathsf{E} \qquad \mathsf{E}(\tau_o) = \mathsf{E}_o \tag{3.14}$$

with solution

$$\mathsf{E}(\tau) = \mathsf{E}_o e^{\alpha\tau} \tag{3.15}$$

In other words, the vacuum continues to self-generate even in the absence of vacuum power.

The field equations for a universe dominated by a cosmological constant are

$$\mathsf{R}^{\mu\nu} - \tfrac{1}{2}\mathrm{g}^{\mu\nu}\mathsf{R} = \mathrm{g}^{\mu\nu}\Lambda \tag{3.16}$$

Including an initial radius $a_o$ determined by the initial vacuum input, solutions to

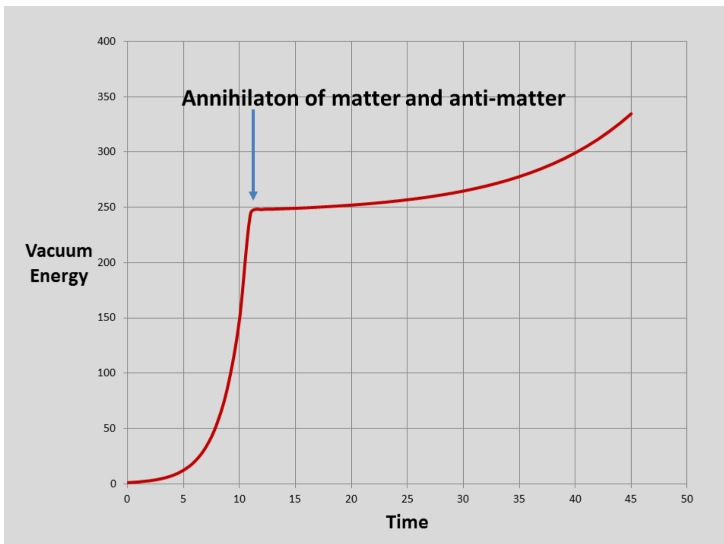


Figure 6: *Proliferation of vacuum energy in the N-particle universe.*

the field equations require a scale parameter $a(\tau)$ to evolve according to

$$a(\tau) = a_o e^{\sqrt{\Lambda/3}\,c\tau} \tag{3.17}$$

A formula for the cosmological constant at large times follows by making the connection $\alpha^2 = 3\Lambda c^2$ and writing the constant observable dark energy density as

$$\varrho_{DE} = \frac{3\mathsf{E}(\tau)}{4\pi a(\tau)^3} = \frac{\Lambda c^4}{8\pi G} \tag{3.18}$$

Equations for both $\Lambda$ and the Hubble parameter are then

$$\Lambda = \frac{6\mathsf{E}_o G}{a_o^3 c^4} \qquad\qquad \mathrm{H} = \sqrt{\frac{2\mathsf{E}_o G}{a_o^3 c^3}} \tag{3.19}$$

In this calculation there is no need to assume the proliferation of the spacetime for no apparent reason at all. Empty space does not expand without the proliferation of vacuum energy. Also, the cosmological theory of the vacuum has been deduced completely independent of the field equations. This parallels the vacuum gauge theory of the electron where the propagation term is simply added to the theory without any measurable consequence.

A qualitative plot of the vacuum energy present in the N-particle universe is illustrated in figure 6 where the production of vacuum energy goes thru a phase transition at the point of particle-anti-particle annihilation. This might occur when the initial expanding universe cools to a critical temperature associated with the de-coupling of photon radiation

**Dark Energy and Dark Matter:** It may be possible to interpret dark matter in the neighborhood of a galaxy of stars as an effect of radiating vacuum energy. Clearly the density of radiated energy will fall off roughly like $r^{-2}$, and continue far beyond the visible outer reaches of the galaxy.

As before, suppose that the ability of the vacuum to create virtual particle-antiparticle pairs is proportional to the vacuum energy density. This implies observable vacuum inertia, equivalent to dark energy also falling off like $r^{-2}$. In this picture, dark matter and dark energy are therefore a result of the same unobservable process of the spontaneous and unobservable emission of vacuum energy. An illustration on the title page has been included showing the dark energy density in black reaching local maxmima in each of several galactic neighborhoods as the universe expands.