Overview of:

Vacuum
Gauge
Electrodynamics

Introduction

The ability of elementary particles to be created or destroyed is fundamental to a modern understanding of what an elementary particle is. In fact, the Quantum Field Theory of electrons and positrons uses creation and annihilation operators

$$b_{\boldsymbol{p}}^{s\dagger} \quad b_{\boldsymbol{p}}^{s} \quad d_{\boldsymbol{p}}^{s\dagger} \quad d_{\boldsymbol{p}}^{s}$$

to create and destroy Dirac particles out of the vacuum.

This fact alone poses an insurmountable difficulty for anyone trying to solve the problem of stability of the classical electron beginning with a Coulomb field given by

$$\mathbf{E} = \frac{e}{r^2}\hat{\boldsymbol{r}}$$

This function is inconsistent with the philosophy of Quantum Field Theory and represents a gross violation of causality, implying that the electron has lived an eternal existence. In contrast, an electron created recently in a laboratory should rightfully possess a Coulomb field constrained by the causality principle.

Creating a Classical Electron

The consequences of strictly enforcing causality in the electromagnetic field is the subject of **vacuum gauge electrodynamics**. Instead of a traditional Coulomb field, the rest frame electric field vector is written

$$\mathbf{E}(\boldsymbol{r}, t) = \frac{e}{r^2} \cdot \vartheta(ct - r)\hat{\boldsymbol{r}}$$

In this equation the function ϑ is an expanding light sphere (causality step) which limits the causal reach of the Coulomb field.

Coulombs' law must now be re-interpreted as a field which carries information from the source into the surrounding vacuum. Energy conservation is violated and the (generalized) Coulomb fields of the electron become momentum flux fields.

Note that ϑ can be shown to be Lorentz invariant. When placed next to the Coulomb field it operates on the field as a classical creation operator similar to its quantum mechanical counterpart. It creates an electron by creating its electric field vector.

VG

Vacuum Gauge Condition

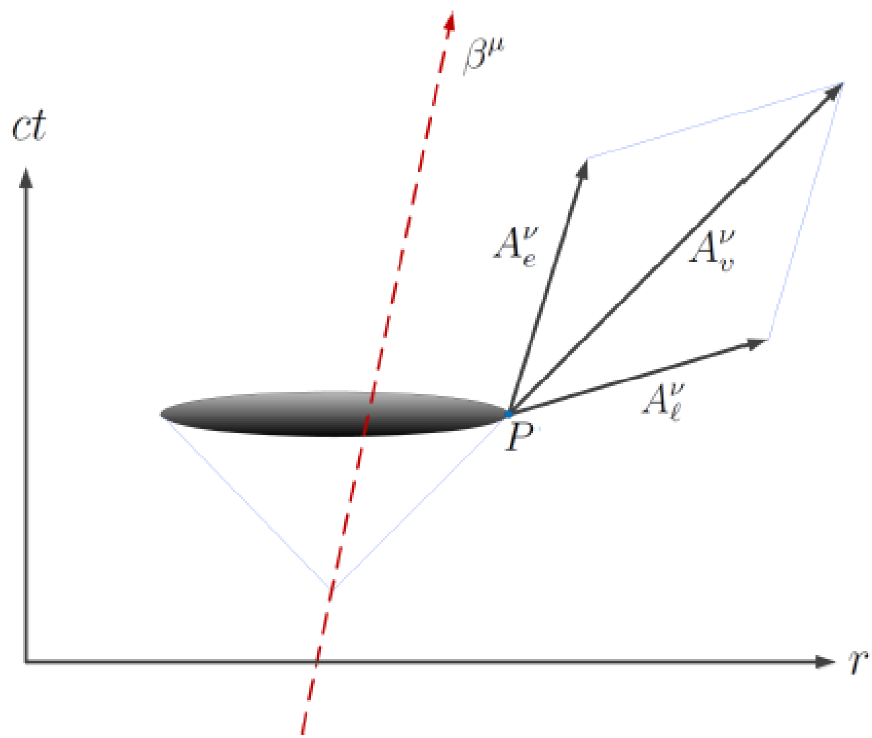
Enforcing causality in the Coulomb field makes a description of the electron in the Lorentz gauge highly untenable. Instead, you will find very quickly that causality compels you to work in the **vacuum gauge** given by the covariant gauge condition

$$\partial_\nu A_v^\nu \equiv \sqrt{\mathrm{E}^2 - \mathrm{B}^2}$$

For motion relative to an arbitrary observer, appropriate scalar and vector velocity potentials for the causal theory will then be

$$A_v = \frac{eR}{\rho^2} \cdot \vartheta \qquad \boldsymbol{A}_v = \frac{e\boldsymbol{R}}{\rho^2} \cdot \vartheta$$

These are zero norm null potentials---also constrained by causality---and just right for propagating information into outer space at the speed of light.



De-Coupling in the Vacuum Gauge

By far, the most conspicuous property of vacuum gauge electrodynamics is the natural separation of particle velocity and accelerations into independent theories. For an arbitrary electron the total potentials consist of a velocity potential and an acceleration potential which appears only when the particle accelerates

$$A^\nu = A^\nu_v + A^\nu_a$$

Each independent set of potentials also satisfies its own set of equations

$$\Box^2 A^\nu_v - \partial^\nu \partial_\mu A^\mu_v = \frac{4\pi}{c} J^\nu_e$$

$$\Box^2 A^\nu_a - \partial^\nu \partial_\mu A^\mu_a = \frac{4\pi}{c} J^\nu_a$$

An explicit form of the total potentials is

$$A^\nu(\boldsymbol{r}, t) = \frac{e(1 - a^\lambda R_\lambda)}{\rho^2} R^\nu \cdot \vartheta$$

Vacuum Waves

Energy propagating through the velocity fields of the classical particle may be determined by writing the (velocity) field strength tensor exclusively in terms of time-like and space-like portions of vacuum gauge potentials

$$\pi^{\mu\nu} = \frac{1}{a_e}\left[A_\ell^\mu,\ A_e^\nu\right]$$

Electric and magnetic flux densities in this equation are proportional to the conventional fields and given by

$$\boldsymbol{\pi}_{\mathrm{E}} = \sigma_e \mathbf{E} \qquad \boldsymbol{\pi}_{\mathrm{B}} = \sigma_e \mathbf{B}$$

Fourier modes of time-like and space-like potentials are

$$A_{e:\omega}^\nu = \sqrt{\frac{2}{\pi}} \cdot A_e^\nu \cdot \frac{\sin \omega(\tau - \rho/c + \tau_e)}{\omega}$$
$$A_{\ell:\omega}^\nu = \sqrt{\frac{2}{\pi}} \cdot A_\ell^\nu \cdot \frac{\sin \omega(\tau - \rho/c + \tau_e)}{\omega}$$

and may be inserted above allowing the velocity fields to propagate as conventional classical waves over all frequencies.

Lagrangian Formulation

The vacuum gauge electron comes complete with its own specialized Lagrangian formulation which can be easily derived from the conventional electromagnetic Lagrangian

$$\mathcal{L}_{vac} \equiv -\frac{1}{8\pi}\Delta^{\mu\nu}\eta_{\mu\nu} + \frac{1}{2}\sigma_e\,\eta - \frac{1}{c}J_e^{*\mu}A_\mu$$

The associated stress tensor has a similarity to the Einstein field equations with the final term added to propagate the velocity fields:

$$\mathfrak{T}^{\mu\nu} = \tfrac{1}{4}\mathrm{g}^{\mu\nu}\mathcal{R} - \mathcal{R}^{\mu\nu} + \Lambda^{\mu\nu}$$

In the rest frame this tensor can be integrated over all accessible space. It will enforce particle stability and produce two tensors

$$\mathcal{E}^{\mu\nu}_{part} = \begin{bmatrix} mc^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \mathcal{E}^{\mu\nu}_{vac} = \begin{bmatrix} -\frac{1}{2}\dot{\varrho}c\tau & 0 & 0 & 0 \\ 0 & \frac{1}{3}\dot{\varrho}c\tau & 0 & 0 \\ 0 & 0 & \frac{1}{3}\dot{\varrho}c\tau & 0 \\ 0 & 0 & 0 & \frac{1}{3}\dot{\varrho}c\tau \end{bmatrix}$$

The first describes a particle of mass m, while the second is associated with its radiated field energy.

Energy-Momentum Four-Vector

In the vacuum gauge, the energy-momentum four-vector of the electron cannot be written as

$$\mathcal{E}^{\mu} = mc^2 \beta^{\mu}$$

Instead, causality forces the modification:

$$\mathcal{E}^{\mu}_{total}(\tau) = mc^2 \beta^{\mu} \cdot \vartheta(\tau) - \tfrac{1}{2}\dot{\varrho} c \tau \beta^{\mu}$$

In this equation the function $\vartheta(\tau)$ indicates that a moving electron is created at proper time zero. In addition, the last term represents the four-vector radiation emitted by the particle after it was born. Contracting with the four-velocity then determines the particle Hamiltonian:

$$\mathcal{H}(\tau) = mc^2 \cdot \vartheta(\tau) - \tfrac{1}{2}\dot{\varrho} c \tau$$

For a particle which has been both created and destroyed, one may slightly re-define creation and annihilation operators and write

$$\mathcal{H}(\tau) = mc^2 \cdot \vartheta^{\dagger} \cdot \vartheta - \tfrac{1}{2}\dot{\varrho} c\, \tau$$

Vacuum Gauge Electrodynamics

Vacuum gauge potentials give the electron the ability to propagate a new form of energy. In addition to this, if you read details of the theory you will also:

- Determine that this energy is massless and related to its momentum by the simple formula

$$\mathcal{E} = \frac{1}{2}\mathcal{P}c$$

- Solve the fundamental problem of particle stability
- Remove the problem of infinite self-energy
- Generate sensible solutions to the problem of the radiation reaction for all possible motions without any of the difficulties associated with the conventional theory
- Derive the Dirac Lagrangian and its associated energy-momentum Stress Tensor.

VG