VG-1
The Maxwell Limit

Causal Theory of the Electron

Vacuum Gauge Electrodynamics requires the field strength tensor to be written

$$F^{\mu\nu} = F_M^{\mu\nu} \cdot \vartheta$$

In this equation $F_M^{\mu\nu}$ is the well known Maxwell-Lorentz field while the theta-function is a radial step which depends on the retarded time and propagates from the source at the speed of light:

$$\vartheta(ct_r/\gamma) \equiv \begin{cases} 1 & ct_r \geq 0 \\ 0 & ct_r < 0 \end{cases}$$

Introducing causality requires the charge of the electron to take on a slightly new meaning. Applying the divergence operator to the (velocity) field strength tensor determines

$$\partial_\mu F^{\mu\nu} \equiv \frac{4\pi}{c} J_e^{*\nu}$$

Charge in a Causal Theory

where:

$$J_e^{*\nu} \equiv J_e^\nu \cdot \vartheta + J_N^\nu \qquad \text{and} \qquad J_N^\nu \equiv \frac{c}{4\pi} F_M^{\mu\nu} \cdot \partial_\mu \vartheta$$

The equation on the left indicates that the charge of the electron is actually composed of two pieces. The first piece is a point charge density with a finite life history as dictated by the radial step. The second piece may be referred to as the ***null current.***

The null current is a radial delta-current of charge –e defined on the very edge of the expanding radial step and moving at the speed of light. This is an important part of the total electric current in the causal theory, and its purpose is to “hide” the point charge from any observer separated from the charge by a space-like interval.

The Maxwell Limit

It is easy to show that the divergence of the total causal charge density is zero. Write

$$\partial_\nu J_e^{*\nu} = \partial_\nu J_e^\nu \cdot \vartheta + J_e^\nu \cdot \partial_\nu \vartheta + \partial_\nu J_N^\nu$$

The first term on the right is zero and the second two terms cancel. Note that we tried to violate conservation of charge by creating an electric field but causality wouldn't allow it.

Now define the ***Maxwell Limit*** by allowing the causal charge to exist for longer and longer times.

$$\lim_{ct \to \infty} J_e^{*\nu} = J_e^\nu$$

In the limit that time goes to infinity, the null current evaporates along with the causality step. What remains is the conventional theory of the classical electron---an irrational theory in which the electron lives an eternal existence.

Vacuum Gauge Condition

The causal charge density is a more complex object than the traditional point particle current density. This fact alone has serious consequences if one decides to write the field strength tensor in terms of a set of potentials

$$F_M^{\mu\nu} \cdot \vartheta = \partial^\mu A^\nu - \partial^\nu A^\mu$$

Like the field strength tensor, the potentials themselves must also be constrained by causality and this destroys the ability to simply pick a gauge condition—like the Lorentz gauge condition. However, there is one simple possibility which rescues the causal theory from any hardship:

$$\partial_\nu A_v^\nu = \frac{e}{\rho^2} \cdot \vartheta$$

This is the covariant ***vacuum gauge condition*** and its meaning and purpose can only be ascertained by allowing it to re-define classical electron theory.

Vacuum Gauge Condition

The vacuum gauge condition has many important properties. First observe that it can also be written

$$|\partial_\nu A_v^\nu| \equiv \sqrt{\mathrm{E}^2 - \mathrm{B}^2}$$

This represents an intimate connection between the vacuum gauge condition and the conventional electromangetic Lagrangian.

For electromagnetic waves the right side of the vacuum gauge condition vanishes producing the Lorentz gauge condition.

More generally---and if we let the causality sphere propagate to infinity---then Lorentz gauge potentials and vacuum gauge potentials can be connected by a gauge transformation

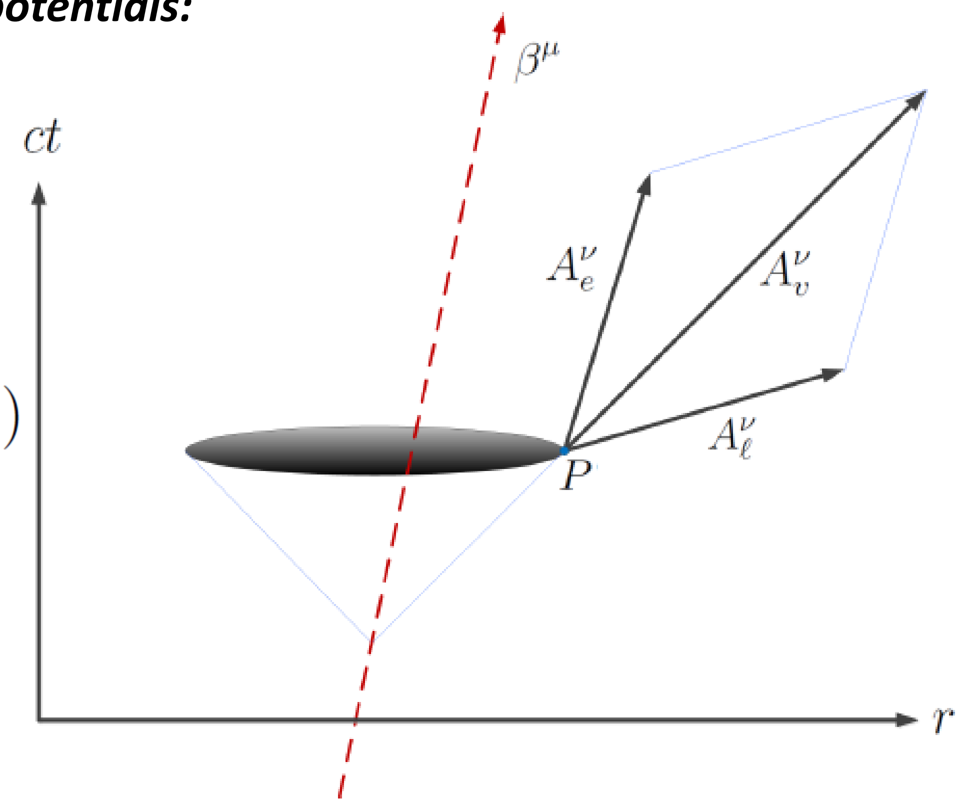
$$A^\nu = A_e^\nu + \partial^\nu \varphi \qquad\qquad \varphi = -e \ln \rho$$

Where A_e^ν are the Lorentz gauge potentials.

Vacuum Gauge Velocity Potentials

Velocity Potentials determined by the vacuum gauge condition may be referred to as ***vacuum gauge potentials:***

$$A_v^\mu(\boldsymbol{r}, t) = \frac{eR^\mu}{\rho^2} \cdot \vartheta(ct_r/\gamma)$$



The picture at right shows how these potentials arise by rotating the time-like Lienard-Wiechert potentials onto the light cone using the space-like gauge field.

De-Coupling in the Vacuum Gauge

Of the many interesting features of vacuum gauge electrodynamics, the most important is the ability to treat velocities and accelerations of the particle as independent theories.

When a vacuum gauge electron accelerates, a small acceleration current density is produced of the form

$$J_a^\nu \equiv -\frac{ec}{2\pi\rho^4}(a^\lambda R_\lambda)R^\nu$$

The presence of this current can be associated with a small correction to the velocity potentials so that the total potentials during accelerated motions are

$$A^\nu = A_v^\nu + A_a^\nu$$

and where the acceleration potentials take the form

$$A_a^\nu(\boldsymbol{r}, t) = -\frac{e}{\rho^2}(a^\lambda R_\lambda)R^\nu \cdot \vartheta$$

De-Coupling in the Vacuum Gauge

Meanwhile the velocity and acceleration fields obey the equations

$$\partial_\mu F_v^{\mu\nu} \equiv \frac{4\pi}{c} J_e^{*\nu} \qquad\qquad \partial_\mu F_a^{\mu\nu} = \frac{4\pi}{c} J_a^{\nu}$$

and in terms of the potentials, these two equations read

$$\Box^2 A_v^\nu - \partial^\nu \partial_\mu A_v^\mu = \frac{4\pi}{c} J_e^{*\nu}$$

$$\Box^2 A_a^\nu - \partial^\nu \partial_\mu A_a^\mu = \frac{4\pi}{c} J_a^{\nu}$$

The velocity and acceleration potentials can then be combined as

$$A^\nu(\boldsymbol{r}, t) = \frac{e(1 - a^\lambda R_\lambda)}{\rho^2} R^\nu \cdot \vartheta$$

Covariant Integrals

The covariant integral of the vacuum gauge condition is of primary importance to vacuum gauge theory. It can be shown that this integral is proportional to the interval

$$x^\mu x_\mu = \frac{1}{2\pi e}\int_{\mathcal{V}} \partial_\nu A^\nu_v d^4x$$

where the finite four-volume is that of the light cone constrained by causality. This integral can also be calculated through an application of Gauss' law over the associated 3D hyper-surfaces.