VG-2
The Radius of the Electron

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Deformations of the Vacuum by Electromagnetic Fields

A striking parallel exists between vacuum gauge potentials and deformations of an elastic continuum. Specifically, transverse and longitudinal deformations of the 3-Space theory can be simply replaced by time-like and space-like potentials of the 4-Space theory.

4-Space		**3-Space**
$A_v^\mu = A_e^\mu + A_\ell^\mu$	$\longleftrightarrow$	$\mathbf{u} = \mathbf{u}_t + \mathbf{u}_l$
$\partial_\mu A_e^\mu = 0$	$\longleftrightarrow$	$\boldsymbol{\nabla} \cdot \mathbf{u}_t = 0$
$\partial^\mu A_\ell^\nu - \partial^\nu A_\ell^\mu = 0$	$\longleftrightarrow$	$\boldsymbol{\nabla} \times \mathbf{u}_l = 0$
$A_\ell^\mu = \partial^\mu \varphi$	$\longleftrightarrow$	$\mathbf{u}_l = -\boldsymbol{\nabla}\varphi$

More Comparisons

A more complete list exhibiting this parallel includes the following:

4-Space		**3-Space**
$\Box^2 A^\mu_e = \dfrac{4\pi}{c} J^\mu_e$	$\longleftrightarrow$	$\Box^2_t \mathbf{u}_t = \tilde{\boldsymbol{f}}_t$
$\Box^2 A^\mu_\ell = \dfrac{4\pi}{c} J^\mu_\ell$	$\longleftrightarrow$	$\Box^2_l \mathbf{u}_l = \tilde{\boldsymbol{f}}_l$
$A^\mu_e = A^\mu_v + (A^\nu_v \mathcal{U}_\nu)\mathcal{U}^\mu$	$\longleftrightarrow$	$\mathbf{u}_t = \mathbf{u} - (\mathbf{u}\cdot\mathbf{k})\mathbf{k}$
$A^\mu_\ell = -(A^\nu_v \mathcal{U}_\nu)\mathcal{U}^\mu$	$\longleftrightarrow$	$\mathbf{u}_l = (\mathbf{u}\cdot\mathbf{k})\mathbf{k}$

Stresses and Strains on the Vacuum

The parallel between vacuum gauge electrodynamics and theory of continuous media can be used as a guide for the development of a ***vacuum Lagrangian*** having the ability to propagate the velocity fields of the electron. First define appropriate strain and stress tensors using vacuum gauge potentials:

$$\eta^{\mu\nu} \equiv \partial^{\mu} A^{\nu}_{v}$$ ← ***Vacuum Strain Tensor***

$$\Delta^{\mu\nu} = \eta^{\mu\nu} - \mathrm{g}^{\mu\nu}\eta$$ ← ***Vacuum Stress Tensor***

A scalar contraction of the vacuum strain tensor also produces a physical quantity called **vacuum dilatation** which is a measure of the extent to which the vacuum has been distorted. This is a radial distortion creating a small (theoretical) hole in the vacuum:

$$\eta \equiv g_{\mu\nu}\eta^{\mu\nu}$$ ← ***Vacuum Dilatation***

Vacuum Lagrangian

Vacuum stress and strain tensors can be combined to produce a vacuum Lagrangian. If an interaction term is included this Lagrangian can be written

$$\mathcal{L}_{vac} \equiv -\frac{1}{8\pi}\Delta^{\mu\nu}\eta_{\mu\nu} - \frac{1}{c}J_e^{*\nu}A_\nu$$

Equations of motion follow from the Euler-Lagrange equations producing

$$\partial_\mu \Delta^{\mu\nu} = \frac{4\pi}{c}J_e^{*\nu}$$

Aside from the presence of the causality sphere, this set of equations is identical to the Maxwell-Lorentz equations of the conventional theory---although arrived at from a completely different premise.

Vacuum Lagrangian

Unfortunately, the vacuum Lagrangian is not complete because there is still no way to propagate the velocity fields at light speed. For this, employ the symmetry operation on the vacuum tensor

$$\Delta^{\mu\nu} \longrightarrow \Delta^{\mu\nu} - 4\pi\sigma_e\, \mathrm{g}^{\mu\nu}$$

A little work then shows that the complete vacuum Lagrangian for the velocity theory is

$$\mathcal{L}_{vac} \equiv -\frac{1}{8\pi}\Delta^{\mu\nu}\eta_{\mu\nu} + \frac{1}{2}\sigma_e\,\eta - \frac{1}{c}J_e^{*\mu}A_\mu$$

It is easy to show that the presence of the linear term has no affect on the equations of motion. In other words, we are free to propagate field energy through the velocity fields without any consequences for the theory of electromagnetism.

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Stress Tensor

Calculating the symmetric stress tensor associated with the vacuum Lagrangian is very similar to its calculation from the conventional theory. It still needs to be symmetrized by eliminating a superfluous term having a zero divergence.

On the other hand the linear propagation term (which needs no symmetrization) added to the vacuum Lagrangian introduces new stresses representing the reaction of the vacuum to the creation of the charged particle. These stresses can be written

$$\Lambda^{\mu\nu} \equiv \frac{\sigma_e}{2} \Delta^{\dagger\,\mu\nu}$$

And the total stress tensor for the vacuum theory absent of particle accelerations can be written

$$\mathfrak{T}^{\mu\nu} \equiv \frac{1}{4\pi}\left[\tfrac{1}{2}\mathrm{g}^{\mu\nu}\eta^2 - \eta^{\mu\lambda}\eta^{\nu}{}_{\lambda}\right] + \Lambda^{\mu\nu}$$

Note that the term in brackets is the symmetric stress tensor, except written using vacuum gauge potentials.

Total Energy Tensor

A simple application of the vacuum theory is the calculation of the total energy tensor for the classical electron from the integral

$$\mathcal{E}^{\mu\nu}(\tau) = \int \mathfrak{T}^{\mu\nu} \cdot \vartheta(c\tau - r + r_e) d^3 r$$

Skipping details of the calculation, this integral can be written as the sum of two terms:

$$\mathcal{E}^{\mu\nu}_{part} = \begin{bmatrix} mc^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \mathcal{E}^{\mu\nu}_{vac} = \begin{bmatrix} -\frac{1}{2}\dot{\varrho}c\tau & 0 & 0 & 0 \\ 0 & \frac{1}{3}\dot{\varrho}c\tau & 0 & 0 \\ 0 & 0 & \frac{1}{3}\dot{\varrho}c\tau & 0 \\ 0 & 0 & 0 & \frac{1}{3}\dot{\varrho}c\tau \end{bmatrix}$$

The first term is simply the mass energy of the classical particle while the second term represents the total amount of vacuum stress radiated by the particle after proper time τ has elapsed.

Total Energy: Moving Frame

Calculation of the total energy tensor relative to a moving frame follows from a Lorentz transformation of the tensors on the previous slide. Time-like and space-like components of radiated stress transform as

$$\mathcal{E}'^{\mu\nu} = -\tfrac{1}{2}\dot{\varrho}c\tau\,\beta^{\mu}\beta^{\nu}$$

$$\mathcal{P}'^{\mu\nu} = \tfrac{1}{3}\dot{\varrho}\tau\cdot(\beta^{\mu}\beta^{\nu} - \mathrm{g}^{\mu\nu})$$

Add to this the particle term which formally requires its creation operator and the total moving frame energy tensor is

$$\mathcal{E}'^{\mu\nu}_{total} = mc^2\beta^{\mu}\beta^{\nu}\cdot\vartheta(\tau) - \tfrac{1}{2}\dot{\varrho}c\tau\,\beta^{\mu}\beta^{\nu} + \tfrac{1}{3}\dot{\varrho}c\tau\,(\beta^{\mu}\beta^{\nu} - \mathrm{g}^{\mu\nu})$$

It is easy to show that the radiated terms imply the scalar relation

$$\mathcal{E} = \tfrac{1}{2}\mathcal{P}c$$

which is frame independent.

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Particle Accelerations

Particle accelerations fit quite naturally into the vacuum theory and are handled by defining an acceleration strain tensor

$$\epsilon^{\mu\nu} \equiv A^{\mu} a_{\perp}^{\nu}$$

This is a bi-linear equation composed of vacuum gauge velocity potentials and the component of the four-acceleration perpendicular to the spacelike unit vector

$$a_{\perp}^{\nu} \equiv a^{\nu} + (a^{\lambda}\mathcal{U}_{\lambda})\mathcal{U}^{\nu}$$

The acceleration strain can be associated with its own acceleration current density

$$\partial_{\mu}\epsilon^{\mu\nu} = \frac{4\pi}{c} j_a^{\nu} \qquad\qquad j_a^{\nu} \equiv \frac{1}{4\pi c}\eta\, a_{\perp}^{\nu}$$

and the total vacuum strain during accelerated motions is given by

$$\eta^{\mu\nu} \longrightarrow \eta^{\mu\nu} - \epsilon^{\mu\nu}$$

Components of acceleration strain are small compared with velocity strain—Essentially a first order perturbation.

Total Stress Tensor

It is easy to show that velocity and acceleration strains of the vacuum theory still lead directly to the symmetric stress tensor of the electromagnetic theory. Specifically, this tensor may be written as

$$\Theta^{\mu\nu}_{vac} = \frac{1}{4\pi}\left[\tfrac{1}{2}\mathrm{g}^{\mu\nu}\eta^2 - \eta^{\mu\lambda}\eta^{\nu}{}_{\lambda} + \eta^{\mu\lambda}\epsilon^{\nu}{}_{\lambda} - \epsilon^{\mu\lambda}\epsilon^{\nu}{}_{\lambda} + \epsilon^{\mu\lambda}\eta^{\nu}{}_{\lambda}\right]$$

With the inclusion of the vacuum propagation term, the total stress tensor can also be written in the highly suggestive Einstein format

$$\mathcal{T}^{\mu\nu} = \tfrac{1}{4}\mathrm{g}^{\mu\nu}\mathcal{R} - \mathcal{R}^{\mu\nu} + \Lambda^{\mu\nu}$$

Of course these are not the field equations and they do not represent deformations of spacetime. Instead, they are deformations of something we are calling the ***vacuum,*** as defined by the vacuum gauge potentials.