VG-3
Vacuum Dilatation
Functions

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Charge Density and Vacuum Dilatation

The structure of the vacuum gauge electron is readily available in an arbitrary reference frame beginning with the vacuum gauge velocity potentials

$$A_v = \frac{eR}{\rho^2} \cdot \vartheta \qquad \boldsymbol{A}_v = \frac{e\boldsymbol{R}}{\rho^2} \cdot \vartheta$$

These can be separated into scalar and vector fields defining the charge density and a set of vacuum dilatation functions

$$\boldsymbol{A}(\boldsymbol{r}, t) = 4\pi \cdot \sigma_e(\theta, \phi)\,\mathbf{u}(R) \qquad A(\boldsymbol{r}, t) = 4\pi \cdot \sigma_e(\theta, \phi)\,\mathrm{u}(R)$$

Explicit forms are given by

$$\sigma_e(\theta, \phi) \equiv \frac{\sigma_e}{\gamma^2(1 - \hat{\boldsymbol{n}} \cdot \beta)^2} \qquad \mathbf{u}(R) \equiv \frac{{r_e}^2}{R}\,\hat{\boldsymbol{n}}$$

Note that charge density is Doppler shifted when viewed in a moving frame. However the particle radius is a fundamental constant associated with the dilatation---a small orifice in the vacuum with a radius equal to the classical electron radius.

R-Space

Without any reference to the covariant theory, it is possible to construct a theory of the dilatation functions. For this we work in R-space which derives from a simple transformation of components of the null vector into spherical-polar coordinates

$$\begin{aligned} R_x &= R\sin\theta\cos\phi \\ R_y &= R\sin\theta\sin\phi \\ R_z &= R\cos\theta \end{aligned}$$

$$\nabla_R \equiv \hat{\boldsymbol{n}}\frac{\partial}{\partial R} + \hat{\boldsymbol{\theta}}\frac{1}{R}\frac{\partial}{\partial\theta} + \hat{\boldsymbol{\phi}}\frac{1}{R\sin\theta}\frac{\partial}{\partial\phi}$$

R-space operations on the vector dilatation can be written

$$\nabla_R \mathbf{u} \equiv \boldsymbol{\eta}$$

$$\eta = \nabla_R \cdot \mathbf{u} = \frac{r_e^2}{R^2}$$

Likewise, and R-space version of the vacuum tensor is

$$\boldsymbol{\Delta} \equiv \tfrac{1}{2}\left(-\boldsymbol{\eta} + \mathbf{1}\eta\right)$$

R-space Lagrangian

The free term covariant vacuum Lagrangian is

$$\mathcal{L} = -\frac{1}{8\pi}\left[\partial^{\mu}A^{\nu}\partial_{\mu}A_{\nu} - (\partial_{\nu}A^{\nu})^{2}\right]$$

As with the potentials, the electron charge density may be removed from the theory rendering an R-space Lagrangian given by

$$L = \frac{1}{2}\left[\boldsymbol{\nabla}_{R}\mathrm{u}\cdot\boldsymbol{\nabla}_{R}\mathrm{u} - \boldsymbol{\nabla}_{R}\mathbf{u} : \boldsymbol{\nabla}_{R}\mathbf{u} + (\boldsymbol{\nabla}_{R}\cdot\mathbf{u})^{2}\right]$$

This Lagrangian can be written more precisely by defining a scalar strain and including a point interaction term:

$$\epsilon = 4\pi r_{e}^{2}\delta^{3}(\boldsymbol{R}) \qquad\qquad \boldsymbol{\xi} \equiv \boldsymbol{\nabla}_{R}\mathrm{u}$$

Then

$$\boxed{L = \tfrac{1}{2}\boldsymbol{\xi}\cdot\boldsymbol{\xi} + \boldsymbol{\Delta} : \boldsymbol{\eta} - \epsilon\mathrm{u}}$$

R-space Stress Tensor

The associated R-space stress tensor is follows as

$$\mathbf{T} = \tfrac{1}{2}\eta\boldsymbol{\eta} + \boldsymbol{\Delta}$$

Instead of a covariant theory which propagates vacuum energy, R-space is a geometrical theory which only knows how to propagate volume. For the classical electron, the amount of volume radiated per unit time in each of three spatial directions is determined from

$$\boldsymbol{\mathcal{S}} = \int_{r_e}^{ct+r_e}\int_{\Omega} \boldsymbol{\Delta}\cdot\vartheta d^3R = \mathbf{1}\,\mathcal{V}\frac{c\tau}{r_e}$$

The flux of volume dispensed through the particle radius is then

$$\mathbf{1} : \boldsymbol{\mathcal{S}} = 4\pi r_e^2 c\tau$$

and this determines a volume emission rate:

$$\frac{dV}{d\tau} = 7.48\times10^{-21}\,\mathrm{m}^3/\mathrm{s}$$

Fourier Modes of Dilatation

When placed alongside the causality step, dilatation functions are a sum over Fourier modes:

$$\mathbf{u}(R,t) = \mathrm{Im}\left[\frac{2}{\pi}\int_0^\infty \mathbf{u}_\omega(R)e^{i\omega t}\,d\omega\right]$$

$$\mathbf{u}_\omega(R) = \tilde{a}_\omega \frac{e^{-i\omega R/c}}{\omega R}\hat{\boldsymbol{n}} \quad \text{where} \quad \tilde{a}_\omega \equiv r_e^2 e^{i\omega\tau_e}$$

Each individual Fourier mode my also be evaluated at the particle radius giving the simple expression

$$\mathbf{u}_\omega(R,t)\Big|_{r_e} = \frac{1}{\omega}\left[r_e \sin\omega t\,\hat{\boldsymbol{n}}\right]$$

According to this formula longitudinal modes radiated by the electron are generated by longitudinal oscillations of the radius vector.

Fourier Mode Lagrangian

Fourier modes of the scalar and vector dilatation can also be determined by an R-space Fourier mode Lagrangian having components

$$L_k^{(s)} \equiv \frac{1}{2}\left[\boldsymbol{\nabla}_R \mathrm{u}_k \cdot \boldsymbol{\nabla}_R \mathrm{u}_k^* - k^2 \mathrm{u}_k \mathrm{u}_k^*\right] - \epsilon_k \mathrm{u}_k^* - \epsilon_k \mathrm{u}_k$$

$$L_k^{(\boldsymbol{v})} \equiv -\frac{1}{2}[\boldsymbol{\nabla}_R \mathbf{u}_k : \boldsymbol{\nabla}_R \mathbf{u}_k^* - \boldsymbol{\nabla}_R \cdot \mathbf{u}_k \cdot \boldsymbol{\nabla}_R \cdot \mathbf{u}_k^*]$$

The source current for the scalar field is given by

$$\epsilon_k(\boldsymbol{R}) = \frac{4\pi \tilde{a}_k}{k}\,\delta^3(\boldsymbol{R})$$

Equations of motion for each Lagrangian along with the associated stress tensor are

$$(\boldsymbol{\nabla}_R^2 + k^2)\,\mathrm{u}_k = -\epsilon_k$$

$$(\boldsymbol{\nabla}_R^2 + k^2)\,\mathbf{u}_k = -\mathbf{J}_k$$

$$\mathbf{T}_k = \frac{1}{2k^2 r_e^2}\eta\,\boldsymbol{\eta} + \boldsymbol{\Delta}$$