# Estimating the costs of cognitive control: theoretical validation and potential pitfalls


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## Cognitive Control and Its Cost

**Cognitive control**: Ability to override habitual responses in order to successfully guide behavior in the service of current task goals

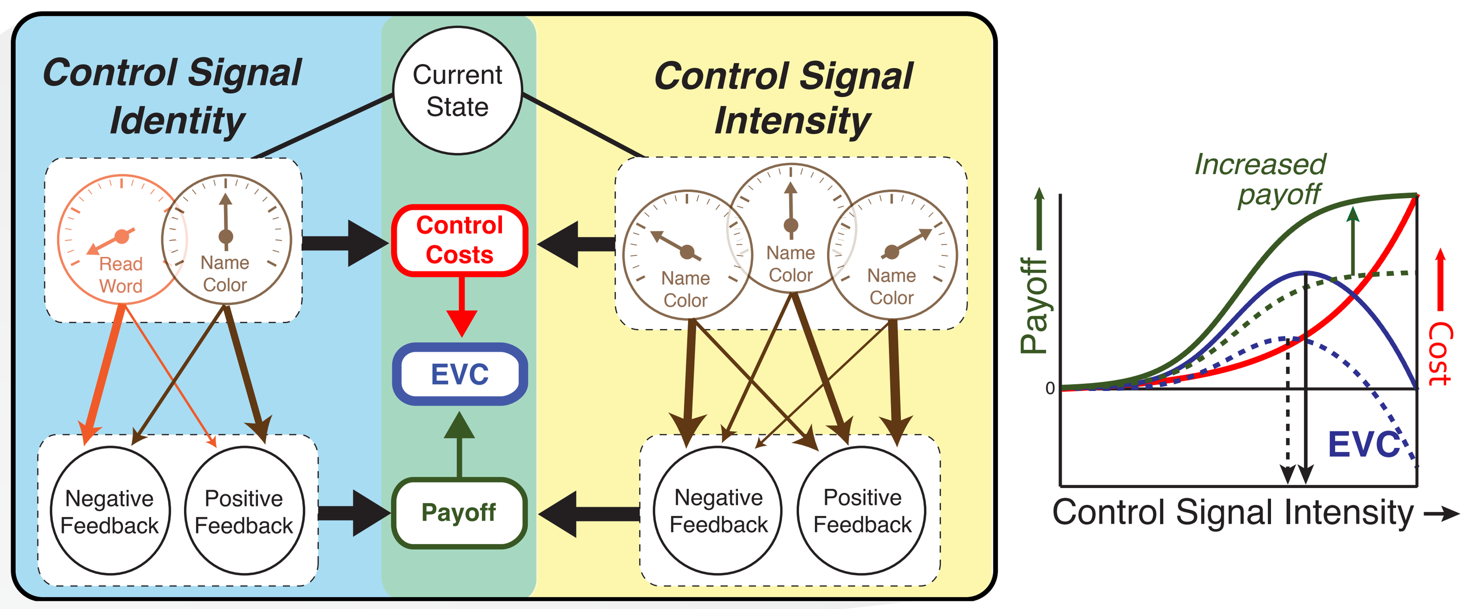
1» Exerting cognitive control is **costly** (Botvinick & Braver, 2015; Shenhav et al., 2017)

2» The cost of cognitive control imposes limitations on subjects task performance (Kool et al., 2010)

3» Individual differences in the cost of control explain behavior more generally in the real world and are linked to clinical symptoms (Westbrook, Kester & Braver, 2013; Gold et al., 2016)

**Expected Value of Control Theory (EVC)** by Shenhav, Botvinick & Cohen (2013)

4» We use EVC Theory as an economic model of decision making to inform the estimation of control costs in laboratory experiments




5» According to the EVC Theory identities and intensities of cognitive control signals are selected so as to maximize expected reward while discounting this reward by an intrinsic cost that attaches to increases in control allocation

## Estimating the Cost of Cognitive Control from Task Performance

### Task Environment

» Agents perform a control-demanding task (e.g. Stroop) with a fixed task difficulty under varying reward for correct response

» Before each trial/block the agent is provided with a reward cue

Experiment Conditions

$ RED • $$ RED • $$$ RED

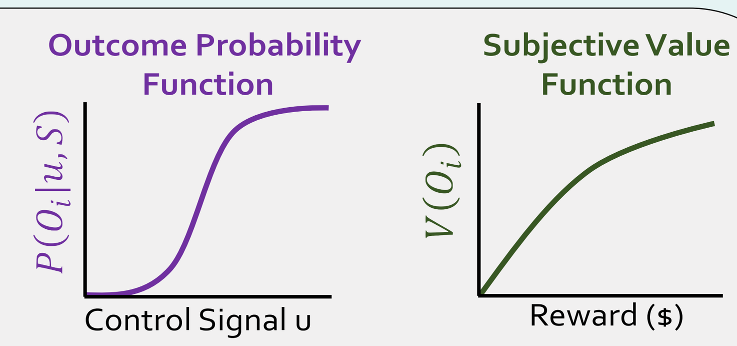
» Assess mean task accuracy for each reward condition

### Cognitive Control Allocation as Cost-Benefit Decision Making

**Agent Model**

2» Determine control signal intensity with maximum EVC

$$u^* = \underset{u}{\operatorname{argmax}}\, EVC(u,S)$$

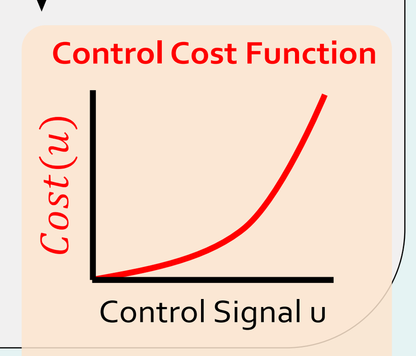



$$EVC(u,S) = \sum_i \left(P(O_i|u^*,S)V(O_i)\right) - Cost(u)$$

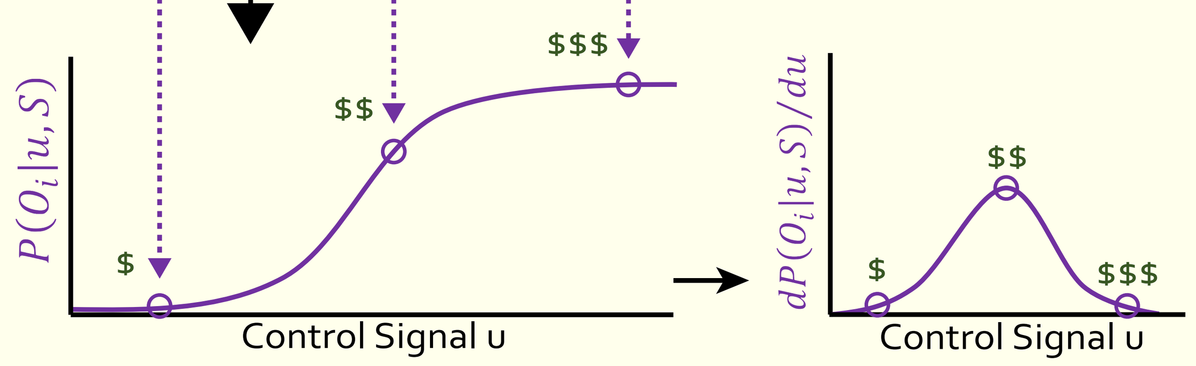
1» For every trial in a given task environment S, compute EVC for all possible control signal intensities

$P(O_i|u^*,S)$

3» Perform task with optimal control signal intensity

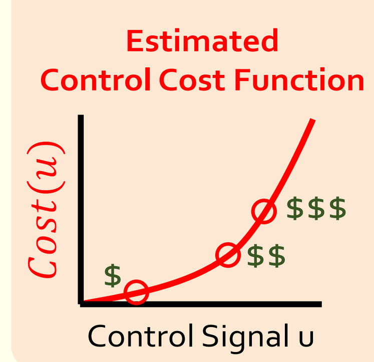



### Cognitive Control Cost Estimation




$$0 = \frac{dEVC}{du^*} = \frac{P(O_i|u^*,S)V(O_i)}{du^*} - \frac{dCost(u^*)}{du^*}$$

$$\frac{P(O_i|u,S)}{du}V(O_i) = \frac{dCost(u)}{du}$$




1» Map measured task accuracy to assumed outcome probability function

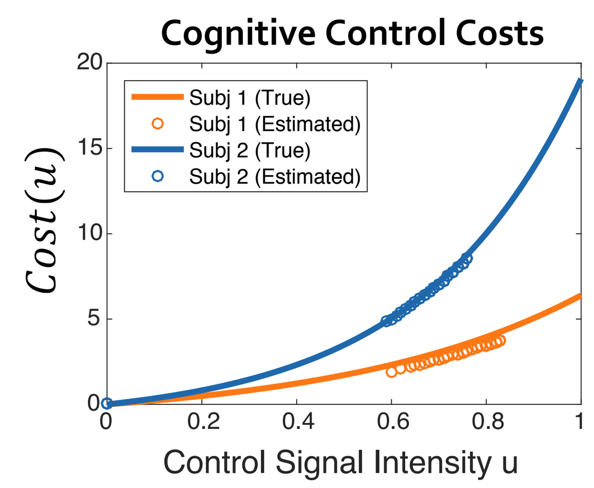
2» Compute derivative of measured task accuracies

3» Compute derivative of cost function

4» Integrate to recover control cost function

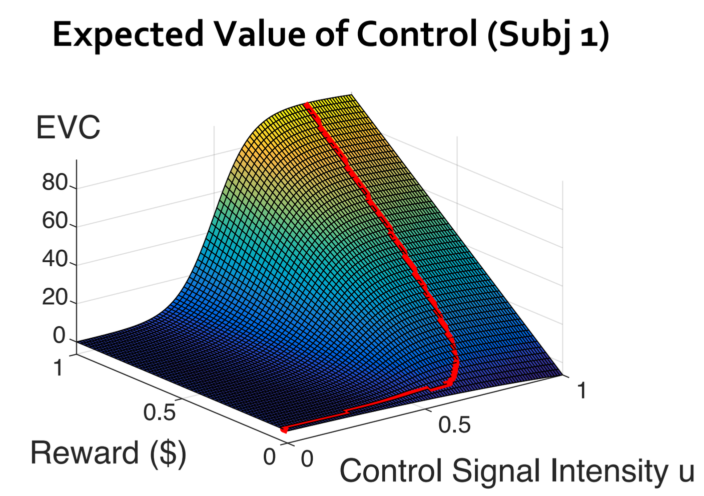
## Theoretical Validation and Potential Pitfalls

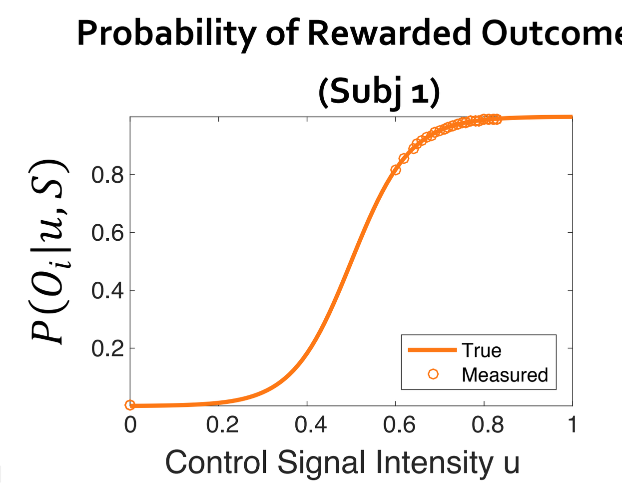
### Recovering the Cost of Cognitive Control Under Correct Assumptions




$$Cost(u) = e^{cu}$$

» Control cost parameters for subject 1 ($c_1 = 2$) and subject 2 ($c_2=3$) could be qualitatively and quantitatively recovered



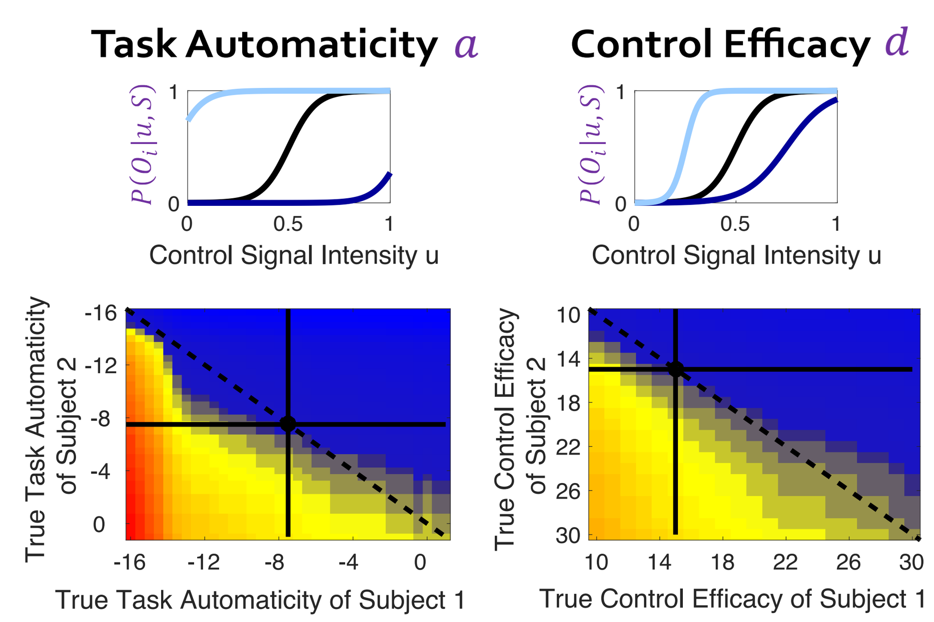

» The maximum EVC for both subjects can be found over a limited domain of (high) control signal intensities, thereby limiting the range of measured outcome probabilities as well as corresponding control costs

### Testing the Sensitivity of Cognitive Control Cost Estimation to Imperfect Assumptions

Our ability to recover the true cost of control is highly sensitive to the assumptions made about other motivational variables

**Outcome Probability Function**

$$P(O_i|u,S) = 1/(1+e^{-du-a})$$




» If subject 2 has a higher task automaticity or a higher control efficacy than subject 1 then control estimates falsely suggest a lower control cost for subject 2

» Control costs were estimated under the assumption that subjects share the same parameter values

» Sensitivity of control cost estimates was assessed by varying the true parameter values for each subject

*assumed parameter value*
*min. true parameter value*
*max. true parameter value*

» Subject 1 ($c_1$=2) has a lower true control cost than subject 2 ($c_2$=3)

» The heat map indicates the difference in subject's estimated control costs ($c_2$ - $c_1$)

*correctly estimated relationship between subject's control costs* ($c_1 < c_2$)

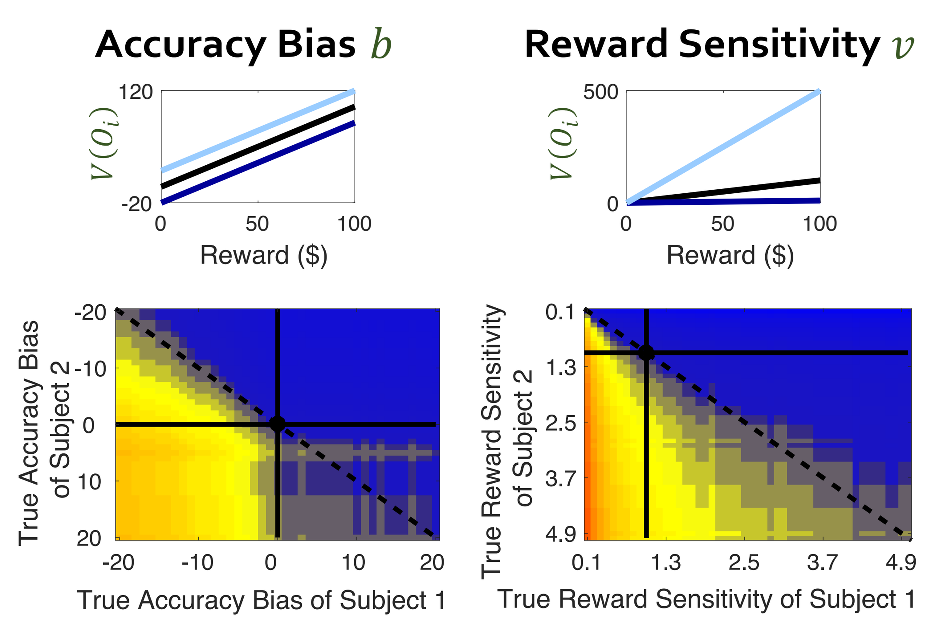
*true difference* ($c_2$ - $c_1$ = 1)

*incorrectly estimated relationship between subject's control costs* ($c_1 > c_2$)

*assumed parameter value*

**Subjective Value Function**

$$V(O_i) = vR(O_i) + b$$




» If subject 2 has a higher accuracy bias or a higher reward sensitivity than subject 1 then control estimates falsely suggest a lower control cost for subject 2

## Conclusions

» It is possible to recover the true relationship between subject's control costs if

(a) the derivative of the cost function for each individual is convex,
(b) the derivative of the value function is concave,
(c) the outcome probability function has a strictly positive derivative, and
(d) each participant is observed to expend a positive amount of effort in every reward condition.

» However, false assumptions about other motivational variables (e.g. reward sensitivity) can lead to quantitative and qualitative misestimations.

» Misestimation biases equally apply to imperfect estimates of motivational variables (e.g. choice behavior or neural correlates). However, independent behavioral assessments of other motivational variables can help to increase the robustness to false assumptions.

## References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