Consociational Democracy and the Optimal Allocation of Power

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Abstract

Using a model with an overall population consisting of two different groups, I show that imposing optimal constraints on the set of implementable policies based on demographics and misalignment of preferences increases social welfare. I also argue that the more misaligned the preferences of two groups are, the more restrictive these optimal constraints should be. When using these optimal constraints or no constraints at all, allocating power based on the plurality rule is optimal. However, if restrictive sub-optimal constraints are utilized, then allocating power to a minority group becomes potentially optimal. Finally, I show that while overly-laxed sub-optimal constraints still increase welfare, overly-burdensome sub-optimal constraints do so if the two groups' preferences are sufficiently misaligned.

1 Introduction

An unfortunate feature of some ethnically diverse countries has been civil conflicts due to the clashing perspectives of all parties involved. In contrast, more fortunate multicultural countries never had their disagreements drift out of the political or judicial process. In either case, attempts at making

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diverse ethnic groups coexist with each other can often take the form of political and constitutional barriers designed to ensure one group has a limited ability to impose their will on other groups. These political systems containing these constraints are referred to as consociational democracies (see Lijphart, 1977).

These barriers can take the form of ethnic quotas for certain political positions. For example, article 67 of the Belgium Constitution requires that 25 out of 70 senators are to be elected by the Dutch electoral college who then appoint 6 other senators, 15 out of 70 by the French electoral college who then appoint 4 other senators, 10 by the Flemish Parliament, 10 by the Parliament of the French Community and 1 by the Parliament of the German-speaking community. Another example is the Irish North/South Inter-Parliamentary Association, where seats are equally allocated between members of the national parliament of the Republic of Ireland and the Northern Ireland Assembly.

When ethnic groups are loosely divided into distinct geographic regions, these barriers can also take the form of a very decentralized form of government, with each region having a strong local government which in turn has to deal with a relatively weak central government. One example is the Dayton Peace Agreements on Bosnia-Herzegovina, which initially restricts the role of the central government to central banking activities, foreign policies, law enforcement, air traffic control, communications and other areas to be determined while leaving substantial and numerous powers to the Republika Srpska and the Federation of Bosnia and Herzegovina, two entities largely based on distinct ethnicities. Another potential example would have been former Vice-President Joe Biden's proposal for the partitioning of Iraq into three distinct Kurdish, Sunni and Shiite independent provinces with a predetermined oil revenue-sharing scheme. In some instances, leaving a regional entity out of a country's constitution can leave the door open to granting that particular region extra powers not possessed by the rest of the country, which is exemplified by Quebec's non-ratification of the Canadian Constitution.

Regardless of the mechanism used, political and constitutional barriers are common tools used to promote the coexistence of two or more different ethnics groups within a single country. In this paper, I ask how restrictive these constitutional constraints should be and what the consequences of these constraints are on the optimal allocation of power within a country. To answer these questions, I develop a framework with a population made up two groups. Each citizen within a group has the same preference for the policy to be implemented conditional on the realization of the state of nature. The state of nature is ex ante unknown and all players have the same common prior. I abstract from an elaborate election process by simply assuming that with some exogenous probability q, either group can acquire the power to implement their preferred policies. The policy space is one dimensional and interval constraints of the policy space can be implemented on either end of the policy space by a benevolent social planner.

I start by showing what are the optimal constraints that should be imposed and argue that they should be more restrictive as the misalignment between the two groups' preferences increases. I also show that under these optimal constraints, a social planner who tries to maximize the expected social welfare would allocate power to the majority group, essentially arguing for the optimality of the plurality rule. However, if, for whatever reasons, sub-optimal thresholds are imposed, then deviations from the plurality rule can be optimal, meaning allocating power to a minority group can be optimal. This becomes a possibility if, for example, the misalignment between the two groups' preferences is sufficiently large and the minority group is more "constrained" than the majority group, which would imply that the minority group can do less harm to their counterpart relative to a scenario where the majority group is in power. Finally, I argue that while overly permissive sub-optimal constraints are always welfare-enhancing, overly restrictive sub-optimal constraints can be welfare-decreasing if both groups are sufficiently similar.

This paper is constructed as follows. Section 2 summarizes some of the papers related to this one. Section 3 describes the model and presents some of the basic results of this paper concerning the optimal constraints on the implementable policies. Section 4 describes how optimal and sub-optimal constraints affect the optimal allocation of power while section 5 concludes the paper.

2 Literature Review

This paper contributes to at least three different strands of literature. First, it is related to a group of papers that establishes a link between the presence of multiple ethnicities within a country to various economic and political performances (see Alesina and Ferrara (2005) for a literature review on this subject.) Amodio and Chiovelli (2016) and Esteban, Morelli, and Rohner (2015) argue that ethnic diversity is a key factor in predicting violence in resource-rich countries as well as countries experiencing a power-change like South Africa following the fall of the Apartheid regime. Alesina and Zhuravskaya (2011) and La Porta, Lopez-de Silanes, Shleifer, and Vishny (1999) associated ethnic diversity to poor government performance while Easterly and Levine (1997) reported a negative impact of ethnic diversity on economic growth. i Miquel (2007) also establishes a link between ethnic diversity and rent-extraction. Accompanying these results, I show that ethnic diversity can create a need for asymmetric constitutional constraints and potentially make deviations from the plurality rule optimal.

By studying the optimal constraints on the players' action space, this paper joins the likes of Athey, Atkeson, and Kehoe (2005) as well as Marino and Matsusaka (2005) and Malenko (2016), which argues respectively that a monetary policy maker and an investment manager should have full discretion up to some level beyond which restraining their strategies becomes optimal. In contrast, Halac and Yared (2014) shows that ex ante optimal level of discretion for public debt accumulation might not be sequentially optimal and could lead to an inefficient level of debt. In the context of a lobbying model, Cotton (2009) argues that a limit on political contribution can also be welfare-enhancing but only if this limit is not too large as to reduce the signaling abilities of the lobbies or not too small such that it no longer induces an honest behavior from the politician. When delegating multiple decisions to an agent, Frankel (2016) shows that placing a cap on the *total* number of actions instead of on each individual action is the optimal way of constraining this agent. Proposition 1 of this paper argues that an interval constraint of the original action space like in Amador and Bagwell (2013) and Gailmard (2009) increases expected social welfare.

In a broader sense, this optimal level of restriction of the politicians' action spaces joins a larger literature on the optimal organizational structure of government. Alesina and Tabellini (2007) uses the different career concerns of politicians and bureaucrats while Maskin and Tirole (2004) use the office-holding motives of politicians to construct the optimal delegation system. Persson, Roland, and Tabellini (1997) shows that creating a system of checks and balances through the diverging interests of the legislative and executive bodies and a delegation of agenda-setting powers to both of these bodies can be welfare-enhancing for voters. Focusing on a more direct form of constraints on power, Rogers and Vanberg (2007) show that unprincipled judicial review decreases the likelihood of minorities being oppressed in order to demonstrate the same thing. Arzaghi and Henderson (2005) and Boffa, Piolatto, and Ponzetto (2016) both argue that decentralized forms of governments are better suited to deal with largely heterogeneous preferences within a country. In contrast, Aghion, Alesina, and Trebbi (2004) argues that the discretion of a politician should increase with the heterogeneity of the population. While I do not focus specifically on judicial review or the centralization versus decentralization question, I argue for an intuition along the lines of Arzaghi and Henderson (2005) and Boffa et al. (2016) by arguing that larger constraints should be placed on centrally implementable policies when the misalignment of preferences between both groups increases.

3 Model

3.1 Preferences and Allocation of Power

This is a static model with a unit measure of citizens, each of whom belong to either group 1 or group 2, which represents a fraction π and $1-\pi$ of the overall population respectively. Each group differs solely based on their preferred policies. Like in Oates (1972) and Boffa et al. (2016), I assume that the citizens' preferences within each group are identical. Any citizen in group i would receive a payoff of $u(x_i, \mu, \theta) = -(\theta + x_i - \mu)^2$, where $\theta \in \mathcal{S} \equiv [\underline{\theta}, \bar{\theta}]$ is the state of nature and $x_i \in \mathcal{X} \equiv \{x_1, x_2\}$ is the preference parameter of the group to which the individual belongs, all of which following the implementation of a policy $\mu : \mathcal{S} \cup \mathcal{X} \to \mathcal{P}$, where $\mathcal{P} \equiv \mathbb{R}$ is the policy space[1].

For tractability purposes, I assume that $x_1 = 0$ and $x_2 = b > 0$. The parameter b will henceforth be referred to as the misalignment of preferences between the two groups. All citizens have the same common prior beliefs over θ, which I assume, for tractability purposes, is a uniform distribution function over $[\underline{\theta}, \bar{\theta}]$.

Both groups are represented by partisan politicians[2] (solely motivated by policy outcomes)[3] who select a policy $\mu \in \mathcal{P}$. I abstract from an elaborate electoral process by simply assuming that the authority to implement a policy is allocated to the partisan politician representing group 1 with probability q and to the one representing group 2 with probability (1-q). When no restrictions have been placed on the policies politicians can implement and the state of nature θ is known, the policies that solves either group's maximization problem is

$$\mu^*(\theta, x_i) \equiv \arg\max_{\mu \in \mathcal{P}} -(\theta + x_i - \mu)^2$$

[1]An implicit assumption here is that this policy is implemented uniformly to both groups. If members of each group are not geographically concentrated, this assumption is easily justifiable. But even in the case where both groups are separated geographically, some services are better suited to be provided uniformly, like the supply of money, the response to cyclical movements, the use of deficit spending, the redistribution of wealth, environmental protections, military protection, power and influence in international organizations as well as other public goods which might necessitate large economies of scope.

[2]See Dickens (2016) for empirical evidence that politicians favor their own ethnic group.

[3]See i Miquel (2007) for a paper with self-interested politicians and rent extraction with multiple ethnic groups.

and will henceforth be referred to as the *Full Discretion Policies* (FDP).

The timing of these interactions is as follows. First, each citizen observes its preference parameter and restrictions (if any) on the policies they may wish to implement. Later, the authority to implement a policy is allocated to one of the two groups. The state of nature is then revealed to everyone and the politician in power implements a policy of his choice. Note that the period preceding the allocation of authority is referred to as the ex ante stage and the period following it but preceding the revelation of the state of nature is referred to as the interim stage. Finally, the payoffs are distributed to all citizens.

3.2 Social Planner and Threshold Policies

From the assumption of $x_1 = 0$ and $x_2 = b > 0$, it can be observed that group 2 would prefer policies that are systematically "higher" than group 1. In order to model the political and constitutional barriers I mentioned in the introduction, I now introduce a social planner who can restrict the policies that can be implemented in the following way: she can set an upper bound on the policies that the politician representing group 2 is allowed to implement (group 2 prefers relatively higher policies) and a lower bound for the policies that the politician representing group 1 is allowed to implement (group 1 prefers relatively lower policies.)[4] Accordingly, the following policies

$$\tilde{\mu}(\theta, x_1) = \begin{cases} \mu^*(\theta_1, x_1) & \text{if } \theta \in [\underline{\theta}, \theta_1] \\ \mu^*(\theta, x_1), & \text{if } \theta \in (\theta_1, \bar{\theta}] \end{cases} \tag{1}$$

$$\tilde{\mu}(\theta, x_2) = \begin{cases} \mu^*(\theta, x_2), & \text{if } \theta \in [\underline{\theta}, \theta_2) \\ \mu^*(\theta_2, x_2) & \text{if } \theta \in [\theta_2, \bar{\theta}] \end{cases} \tag{2}$$

will henceforth be referred to as the *Threshold Policies*.

For group 1, the threshold policies consists of their FDP $\mu^*(\theta, x_1)$ for any state above θ_1. However, for any state under θ_1, group 1 would have zero discretion in its selection of a policy and would be obligated to implement $\mu^*(\theta_1, x_1)$. The threshold policies work in a similar fashion for group 2 but with an upper bound. For any state below θ_2, the threshold policies for group 2 consist of their full discretion policy $\mu^*(\theta, x_2)$ but for any state above θ_2,

[4]The notion that politicians representing different ethnic groups may potentially face different forms of checks and balances seems to match multiple real-world environments. For instance, uneven ethnic quotas like in Belgium's Senate or in Malaysia's civil service positions create uneven checks and balances on all ethnicities involved.

group 2 is forced to implement $\mu^*(\theta_2, x_2)$.[5] These threshold policies will be computed based on the two states θ_1 and θ_2 which will henceforth be referred to as the *threshold states*. These threshold states will be selected by the social planner whose objective is to maximizing the ex ante expected social welfare:

$$\max_{\theta_1,\theta_2} q[\pi \int_{\underline{\theta}}^{\bar{\theta}} \frac{-(\theta - \tilde{\mu}(\theta, x_1))^2}{(\bar{\theta} - \underline{\theta})} d\theta + (1-\pi) \int_{\underline{\theta}}^{\bar{\theta}} \frac{-(\theta + b - \tilde{\mu}(\theta, x_1))^2}{(\bar{\theta} - \underline{\theta})} d\theta]$$
$$(1-q)[\pi \int_{\underline{\theta}}^{\bar{\theta}} \frac{-(\theta - \tilde{\mu}(\theta, x_2))^2}{(\bar{\theta} - \underline{\theta})} d\theta + (1-\pi) \int_{\underline{\theta}}^{\bar{\theta}} \frac{-(\theta + b - \tilde{\mu}(\theta, x_2))^2}{(\bar{\theta} - \underline{\theta})} d\theta]. \quad (3)$$

For expositional purposes, I assume the *parameter restrictions* that $2b(1-\pi) \leq \bar{\theta} - \underline{\theta}$ and $2\pi b \leq \bar{\theta} - \underline{\theta}$. This ensures that the optimal threshold states are easier to study and excludes any corner solutions. Furthermore, I will now refer to the threshold policies which utilizes the optimal threshold states as determined by the maximization problem 3 as the Optimal Threshold Policies (OTP). Proposition 1 below shows what these optimal threshold states are and argues that implementing the OTP is welfare-enhancing.

Proposition 1. *The Optimality of OTP*
If a social planner uses the optimal threshold states $\hat{\theta}_1 = \underline{\theta} + 2b(1-\pi)$ and $\hat{\theta}_2 = \bar{\theta} - 2b\pi$, then the implementation of these threshold policies will be welfare-enhancing.

This and all other proofs are in the appendix. This result is predicated on the assumption that the implemented policy causes increasing marginal disutility to the group with no power as the distance between their preferred policy and the implemented policy increases while maintaining the same benefits for the group in power. This assumption is meant to embody the increasing likelihood of civil disorder when one group feels alienated by the political process. Simply put, when using the optimal threshold states, the damage caused by these constraints (the potential inability to implement one's preferred policy) are outweighed by the benefits (the potential reduction in harm suffered by the group with no power). This can be related to Dragu, Fan, and Kuklinski

[5] Checks and balances on one group are usually implemented by an opposite group. In this case, the opposition from group i implementing some policy will come from group $j \neq i$. However, given the clear preferences of both groups, group 1 would never implement policies that group 2 considers to be too high and group 2 would never implement policies that group 1 considers to be too low. Therefore, establishing an upper boundary on the policies that group 1 can implement and a lower boundary on the policies that group 2 can implement would either serve no purpose or be suboptimal.

(2014)'s result that checks and balances which favor "moderate" policies are efficient.

Corollary 1. *Comparative Statics with OTP*
i) $\frac{\partial \hat{\theta}_1}{\partial \pi} < 0$: The low threshold state becomes less constraining when the size of group 1 increases.
ii) -$\frac{\partial \hat{\theta}_2}{\partial \pi} > 0$: The high threshold state becomes less constraining when the size of group 2 increases.
iii) $\frac{\partial \hat{\theta}_1}{\partial q} = -\frac{\partial \hat{\theta}_2}{\partial q} = 0$: The threshold states are unaffected by allocation of power.
iv) $\frac{\partial \hat{\theta}_1}{\partial b} > 0; \frac{\partial \hat{\theta}_2}{\partial b} < 0$: Both threshold states becomes more constraining as the misalignment of preferences between the two groups increases.

Central to these results is that the threshold states are selected by a benevolent social planner whose sole purpose is to maximize the ex ante expected social welfare. Suppose group 1 or 2 become more powerful, meaning their probability of being in charge and implementing their preferred policies increases. Proposition 1 argues that this should have no impact on the optimal thresholds to be imposed on either group. The need to impose barriers on politicians arises out of a desire to restrict policies that impose greater harm than good on the overall population. If one group becomes more powerful within a context of threshold policies, a further restriction on either thresholds is of no use given the optimality of the existing constraints.

In contrast, a change in the relative size of either group will affect the optimal threshold states. Central to results 1-i-ii is that an increase in the relative size of one group should lead to a loosening of the restrictions imposed on this group. This is fairly intuitive since the costs on the ex ante expected social welfare associated with constraining group i grow larger as the size of group i increases and the benefits reaped by group $j \neq i$ grow smaller[6]. However, this result is also predicated on using the optimal threshold states. When sub-optimal threshold states are used, it will later be shown that these results can be potentially reversed.

The notion that a large misalignment of preferences should lead to tighter constraints on either group is an idea that has been implemented in multiple countries, including Austria and Bosnia and Herzegovina[7]. Following the Austrian Civil War and World War II, Austrian politicians started a tradition

[6] An example of this can be seen in Trebbi, Aghion, and Alesina (2008), where minorities in American cities are given more autonomy as their sizes grow larger.

[7] One can also think of the self-determination clause (235) of the current South African Constitution.

of a *grand coalition* where the cooperation of conservative Catholics and socialists became the standard for governance. As evidenced by the civil war preceding its grand coalition, the multiple groups living in Austria believed their interests to be so misaligned that armed conflict was thought to be a valid option of achieving one's goal. Therefore, as Engelmann (1962) puts it, "critics and objective observers agree with Austria's leading politicians in the assessment that the coalition was a response to the civil-war tension of the First Republic." Simply put, the sharp differences between the Austrian groups was thought to necessitate strong checks and balances through the use of a grand coalition.

Following the Bosnia War, several constitutional restrictions were placed to protect geographically dispersed minorities. For example, the constitution of the Herzegovina-Neretva Canton specifies an appeal process for policies that have been approved by the majority of the Assembly but rejected by representatives of only one ethnicity[8]. A more far-reaching example is the rotating state presidency, which requires that the presidency must consist of one Bosniak and one Croat elected from the Federation of Bosnia and Herzegovina and one Serb elected from the Republika Srpska. The presidency decisions are supposed to be approved unanimously by all three members. In the case where a decision was approved by only two members, the dissenting member can start a veto process with the support of two thirds of his respective legislative body[9].

4 Frictions in the Optimal Allocation of Authority

From a normative approach, an interesting question is the optimal allocation of authority if the social planner was allowed to set q herself. To answer this question, I will assume that the social planner now has two different roles. First, she computes the optimal threshold states based on the same criteria as before. Second, she chooses the optimal allocation of authority q^* by comparing the interim expected social welfares when group 1 is in power relative to when group 2 is in power. A remainder is in order that the optimal threshold states are independent of the allocation of power. As it turns out, the optimal allocation of authority with FDP will be identical to that of a setting with OTP.

[8]See Article 37 of the Constitution of the Herzegovina-Neretva Canton.

[9]These legislative bodies are the National Assembly of the Republika of Srpska for the Serb member, the Bosniak Delegates of the House of Peoples of the Federation for the Bosniak member and the Croat Delegates of that body for the Croat member.

Proposition 2. *Authority with FDP and OTP*

i) With Full Discretion Policies (FDP), the optimal allocation of authority is a perfect mapping from the sizes of each group π and $1-\pi$ to the probability q^*_{FDP}:

$$q^*_{FDP} \begin{cases} 0 & \text{if } \pi < \frac{1}{2} \\ \frac{1}{2} & \text{if } \pi = \frac{1}{2} \\ 1 & \text{if } \pi > \frac{1}{2}. \end{cases}$$

ii) With the Optimal Threshold Policies (OTP), the optimal allocation of authority is also a perfect mapping from the sizes of each group π and $1-\pi$ to the probability q^*_{OTP}:

$$q^*_{OTP} \begin{cases} 0 & \text{if } \pi < \frac{1}{2} \\ \frac{1}{2} & \text{if } \pi = \frac{1}{2} \\ 1 & \text{if } \pi > \frac{1}{2}. \end{cases}$$

In other words, when either FDP or OTP are implemented, the optimal allocation of authority is simply based on which group constitutes the majority of the overall population. However, as proposition 3 makes it clear, if sub-optimal threshold states (denoted by $\tilde{\theta}_1$ and $\tilde{\theta}_2$) are utilized[10], threshold policies actually introduce some frictions into the optimal allocation of authority. One can think of a second best setting where $\tilde{\theta}_1$ and $\tilde{\theta}_2$ are taken as given by the social planner who then allocate power in order to maximize interim expected social welfare. Interestingly, these Sub-Optimal Threshold Policies (SOTP) can actually make the empowerment of a minority group optimal.

Proposition 3. *Authority with SOTP*

i) If $b \geq \bar{b} \equiv \max\{\frac{\bar{\theta}^2(\bar{\theta}-3\tilde{\theta}_2)+(\tilde{\theta}_2)^2(3\bar{\theta}-\tilde{\theta}_2)}{\pi(\bar{\theta}-\tilde{\theta}_2)^2}, \frac{(\tilde{\theta}_1)^2(\tilde{\theta}_1-3\underline{\theta})+\underline{\theta}^2(3\tilde{\theta}_1-\underline{\theta})}{(1-\pi)(\tilde{\theta}_1-\underline{\theta})^2}\}$, then the more constrained group i is, the more likely it is that group i will be allocated the authority to implement their preferred policy.

ii) If $b \leq \underline{b} \equiv \min\{\frac{\bar{\theta}^2(\bar{\theta}-3\tilde{\theta}_2)+(\tilde{\theta}_2)^2(3\bar{\theta}-\tilde{\theta}_2)}{\pi(\bar{\theta}-\tilde{\theta}_2)^2}, \frac{(\tilde{\theta}_1)^2(\tilde{\theta}_1-3\underline{\theta})+\underline{\theta}^2(3\tilde{\theta}_1-\underline{\theta})}{(1-\pi)(\tilde{\theta}_1-\underline{\theta})^2}\}$, then the more constrained group i is, the less likely it is that group i will be allocated the authority to implement their preferred policy.

[10]For example, these can arise from the recommendations of an external mediator or a bargaining game resulting in any other threshold states than $\hat{\theta}_1 = \underline{\theta} + 2b(1-\pi)$ and $\hat{\theta}_2 = \bar{\theta} - 2b\pi$.

iii) Allocating authority to a minority group can be optimal in the presence of Sub-Optimal Threshold Policies (SOTP).

In the context of SOTP, the restrictions placed on either group are not necessarily equal and can either be a beneficial or a costly feature for the social planner's decision to allocate power. More specifically, if the misalignment of preferences is large enough, then the social planner seeks to limit the damage that a group in power can cause to their counterpart. In doing so, she views a delegation of power to a group that is relatively more constrained more favorably. In contrast, if the misalignment of preferences is sufficiently small, then the costs of these constraints start to outweigh their benefits such that the social planner would rather delegate power to a group that is relatively less constrained. These constraints can subsequently induce an imperfect mapping from the population's demographics to the optimal allocation of power, potentially making deviations from the plurality rule optimal. These deviation suggests that settings with elaborate but not necessarily optimal constitutional constraints based on ethnicity should also be associated with a more complicated electoral process based on their underlying demographics.

For example, in setting where the two groups' preferences are sufficiently misaligned, larger constraints on a minority group can actually make it optimal to allocate authority to this minority group. This possibility is a direct consequence of the SOTP because delegation to a minority group that is overly-burdened with constraints compared to a majority group that has overly-laxed constraints becomes more valuable given the minority group's relatively limited ability to impose harm on the other group. One can think of the Maronite Christian (minority group) Presidency in Lebanon given the military superiority of Shiite Muslim (majority group) militias. In contrast, in an environment where the interests of both groups are sufficiently aligned with one another, the opposite result hold: allocating power to a group that is over-burdened with constraints is detrimental to the interim expected social welfare since the costs associated with these extra constraints outweigh their benefits.

Proposition 4. *Deviations from the Optimal Threshold States*

i) Suppose sub-optimal threshold states are implemented which are more constraining than the optimal threshold states: $\tilde{\theta}_1 = \underline{\theta} + 2b(1-\pi) + \epsilon_1$ and $\tilde{\theta}_2 = \bar{\theta} - 2b\pi - \epsilon_2$ with $\epsilon_1 > 0$ and $\epsilon_2 > 0$. Then, the associated SOTP increase ex ante expected social welfare if $b \geq \max\{\frac{\epsilon_2}{\pi}, \frac{\epsilon_1}{(1-\pi)}\}$ and decrease it if $b \leq \min\{\frac{\epsilon_2}{\pi}, \frac{\epsilon_1}{(1-\pi)}\}$ with respect to FDP.

ii) Suppose sub-optimal threshold states are implemented which are less constraining than the optimal threshold states: $\tilde{\theta}_1 = \underline{\theta} + 2b(1-\pi) - \epsilon_1$ and $\tilde{\theta}_2 = \bar{\theta} - 2b\pi + \epsilon_2$ with $\epsilon_1 > 0$ and $\epsilon_2 > 0$. Then, the associated SOTP always increases ex ante expected social welfare with respect to FDP.

Proposition 4 is interesting because it argues that threshold policies are not necessarily welfare-enhancing if sub-optimal threshold states are utilized. In fact, if the threshold policies are overly constraining, then SOTP are welfare-enhancing only in intolerant environments (high b). Given the importance of constitutional checks and balances based on ethnicities in reducing the likelihood of armed conflicts like the Rwandan Genocide or the civil strife in Iraq following the Iraq War, maintaining the ex ante welfare-enhancing component of threshold policies can be crucial. This is important because it suggests when external mediators (who cannot compute $\bar{\theta}_1$ and $\bar{\theta}_2$) should err on the side of caution (overly-laxed constraints) and when overly-burdensome constraints can still be welfare-enhancing.

5 Conclusion

In this paper, I developed a simple theoretical framework to argue that optimal constraints on the policy space available to politicians is welfare-enhancing. More specifically, within an environment composed of two groups with different preferences, I argued that the optimal constraints to be imposed on politicians can vary from one group to another largely based on demographics: the larger a majority group is, the more discretion it should get. I also showed that the optimal constraints are independent of the allocation of power and that a larger misalignment of preferences should lead to more binding constraints.

When these optimal constraints are used or when politicians have full discretion, I show that allocating power based on the plurality rule is optimal. However, in the case of restrictive sub-optimal constraints, allocating power to a minority group can be optimal. For example, if the misalignment of preferences is so great that constraints become a desirable trait in the allocation of power, empowering a minority group that is sufficiently more constrained than the majority group is optimal. Finally, I show that while overly permissive sub-optimal constraints are always welfare-enhancing, overly restrictive sub-optimal constraints are only welfare-enhancing when the preferences of each group are sufficiently different.

6 Appendix

Proof of Proposition 1: First, it must be observed that the maximization problem 3 can be rewritten as

$$\begin{aligned}
&= \max_{\theta_1,\theta_2} q[\pi \int_{\underline{\theta}}^{\theta_1} \frac{-(\theta-\theta_1)^2}{(\bar{\theta}-\underline{\theta})} d\theta - (1-\pi)(\frac{\bar{\theta}-\theta_1}{\bar{\theta}-\underline{\theta}})b^2 \\
&\qquad + (1-\pi)\int_{\underline{\theta}}^{\theta_1} \frac{-(\theta+b-\theta_1)^2}{(\bar{\theta}-\underline{\theta})} d\theta] \\
&+ (1-q)[-\pi(\frac{\theta_2-\underline{\theta}}{\bar{\theta}-\underline{\theta}})b^2 + \pi\int_{\theta_2}^{\bar{\theta}} \frac{-(\theta-b-\theta_2)^2}{(\bar{\theta}-\underline{\theta})} + (1-\pi)\int_{\theta_2}^{\bar{\theta}} \frac{-(\theta-\theta_2)^2}{(\bar{\theta}-\underline{\theta})}]
\end{aligned} \tag{4}$$

$$\begin{aligned}
&= \max_{\theta_1,\theta_2} -\frac{q}{(\bar{\theta}-\underline{\theta})}[\int_{\underline{\theta}}^{\theta_1} (\theta^2 - 2\theta_1\theta + \theta_1^2)d\theta + (1-\pi)(\bar{\theta}-\theta_1)b^2 \\
&\qquad + (1-\pi)\int_{\underline{\theta}}^{\theta_1}(2b\theta - 2b\theta_1 + b^2)d\theta] \\
&- \frac{(1-q)}{(\bar{\theta}-\underline{\theta})}[\int_{\theta_2}^{\bar{\theta}}(\theta^2 - 2\theta_2\theta + \theta_2^2)d\theta + \pi(\theta_2-\underline{\theta})b^2 + \pi\int_{\theta_2}^{\bar{\theta}}(-2b\theta + b^2 + 2b\theta_2)d\theta]
\end{aligned}$$

$$\begin{aligned}
&= \max_{\theta_1,\theta_2} \frac{q}{(\bar{\theta}-\underline{\theta})}\{-(\frac{\theta_1^3}{3}-\frac{\underline{\theta}^3}{3})+2\theta_1(\frac{\theta_1^2}{2}-\frac{\underline{\theta}^2}{2})-\theta_1^2(\theta_1-\underline{\theta})-(1-\pi)\bar{\theta}b^2+(1-\pi)\theta_1 b^2 \\
&\qquad + (1-\pi)[-2b(\frac{\theta_1^2}{2}-\frac{\underline{\theta}^2}{2}) + 2b\theta_1(\theta_1-\underline{\theta}) - b^2(\theta_1-\underline{\theta})]\} \\
&\quad + \frac{(1-q)}{(\bar{\theta}-\underline{\theta})}[-(\frac{\bar{\theta}^3}{3}-\frac{\theta_2^3}{3}) + 2\theta_2(\frac{\bar{\theta}^2}{2}-\frac{\theta_2^2}{2}) - \theta_2^2(\bar{\theta}-\theta_2) - \pi b^2\theta_2 + \pi b^2\underline{\theta} \\
&\qquad + \pi 2b(\frac{\bar{\theta}^2}{2}-\frac{\theta_2^2}{2}) - \pi b^2(\bar{\theta}-\theta_2) - \pi 2b\theta_2(\bar{\theta}-\theta_2)]
\end{aligned}$$

$$\begin{aligned}
&= \max_{\theta_1,\theta_2} \frac{q}{(\bar{\theta}-\underline{\theta})}[\frac{-\theta_1^3}{3}+\frac{\underline{\theta}^3}{3}-\theta_1\underline{\theta}^2+\theta_1^2\underline{\theta}-(1-\pi)b^2\bar{\theta}+(1-\pi)b\theta_1^2-(1-\pi)2b\theta_1\underline{\theta} \\
&\qquad + (1-\pi)b^2\underline{\theta} + (1-\pi)b\underline{\theta}^2] \\
&+ \frac{(1-q)}{(\bar{\theta}-\underline{\theta})}[\frac{-\bar{\theta}^3}{3} + \frac{\theta_2^3}{3} + \theta_2\bar{\theta}^2 - \theta_2^2\bar{\theta} + \pi b^2\underline{\theta} - \pi b^2\bar{\theta} - \pi 2b\theta_2\bar{\theta} + \pi b\theta_2^2 + \pi b\bar{\theta}^2]
\end{aligned} \tag{5}$$

which yields the following first order conditions:

$$\text{FOC}(\theta_1) : -\hat{\theta}_1^2 - \underline{\theta}^2 + 2\hat{\theta}_1\underline{\theta} + (1-\pi)2b\hat{\theta}_1 - (1-\pi)2b\underline{\theta} = 0 \tag{6}$$

$$\text{FOC}(\theta_2) : \hat{\theta}_2^2 + \bar{\theta}^2 - 2\hat{\theta}_2\bar{\theta} - \pi 2b\bar{\theta} + 2\pi b\hat{\theta}_2 = 0. \tag{7}$$

Focusing on the first order condition 6, I isolate $\hat{\theta}_1$ and get:

$$\hat{\theta}_1^2 - \hat{\theta}_1[2\underline{\theta} + (1-\pi)2b] + \underline{\theta}^2 + (1-\pi)2b\underline{\theta} = 0.$$

The quadratic formula results in $\hat{\theta}_1 = \underline{\theta} + (1-\pi)b \pm (1-\pi)b$. This yields two potential solutions: $\hat{\theta}_1 = \underline{\theta}$ or $\hat{\theta}_1 = \underline{\theta} + 2(1-\pi)b$. I must now show that for any θ_2, the optimal threshold state for group 1 is $\hat{\theta}_1 = \underline{\theta} + 2(1-\pi)b$ by comparing the expected utility for both potential $\hat{\theta}_1$. Denote by $E[U(\theta_1 = x)]$ the ex ante expected social welfare when one of the threshold state is $\theta_1 = x$. I must show that

$$E[U(\theta_1 = \underline{\theta})] \leq E[U(\theta_1 = \underline{\theta} + 2(1-\pi)b)].$$

Using the expression for ex ante expected social welfare in 5, the above inequality is equivalent to

$$\begin{aligned} -(1-\pi)b^2(\bar{\theta}-\underline{\theta}) \leq \frac{-[\underline{\theta}+2b(1-\pi)]^3}{3} + \frac{\underline{\theta}^3}{3} - \underline{\theta}^2[\underline{\theta}+2b(1-\pi)] + [\underline{\theta}+2b(1-\pi)]^2\underline{\theta} \\ -(1-\pi)b^2\bar{\theta} + (1-\pi)b[\underline{\theta}+2b(1-\pi)]^2 - (1-\pi)2b\underline{\theta}[\underline{\theta}+2b(1-\pi)] + (1-\pi)b^2\underline{\theta} \\ +(1-\pi)b\underline{\theta}^2 \end{aligned} \tag{8}$$

$$\begin{aligned} \Leftrightarrow -(1-\pi)b^2(\bar{\theta}-\underline{\theta}) \leq \frac{-\underline{\theta}^3}{3} - \frac{2b(1-\pi)\underline{\theta}^2}{3} - \frac{4b(1-\pi)\underline{\theta}^2}{3} - \frac{8b^2(1-\pi)^2\underline{\theta}}{3} \\ -\frac{4b^2(1-\pi)^2\underline{\theta}}{3} - \frac{8b^3(1-\pi)^3}{3} + \frac{\underline{\theta}^3}{3} - \underline{\theta}^3 - 2b(1-\pi)\underline{\theta}^2 + \underline{\theta}^3 + 4b(1-\pi)\underline{\theta}^2 \\ +4b^2(1-\pi)^2\underline{\theta} - (1-\pi)b^2\bar{\theta} + (1-\pi)b\underline{\theta}^2 + 4b^2(1-\pi)^2\underline{\theta} + 4b^3(1-\pi)^3 \\ -(1-\pi)2b\underline{\theta}^2 - 4b^2(1-\pi)^2\underline{\theta} + (1-\pi)b^2\underline{\theta} + (1-\pi)b\underline{\theta}^2 \end{aligned}$$

$$\Leftrightarrow 0 \leq \frac{4b^3(1-\pi)^3}{3}$$

which clearly holds.

Focusing now on the first order condition 7, I isolate $\hat{\theta}_2$ an get:

$$\theta_2^2 + \theta_2(2\pi b - 2\bar{\theta}) + \bar{\theta}^2 - 2\pi b\bar{\theta} = 0.$$

The quadratic formula results in $\hat{\theta}_2 = \bar{\theta} - \pi b \pm \pi b$. This yields two potential solutions: $\hat{\theta}_2 = \bar{\theta}$ or $\hat{\theta}_2 = \bar{\theta} - 2\pi b$. Denote by $E[U(\theta_2 = x)]$ the ex ante expected social welfare when one of the threshold state is $\theta_2 = x$. I must show that

$$E[U(\theta_2 = \bar{\theta})] \leq E[U(\theta_2 = \bar{\theta} - 2\pi b)]$$

Once again, using the expression for the ex ante expected social welfare in 5, the above inequality is equivalent to

$$\Leftrightarrow \pi b^2 \underline{\theta} - \pi b^2 \bar{\theta} - 2\pi b\bar{\theta}^2 + \pi b\bar{\theta}^2 + \pi b\bar{\theta}^2 \leq \frac{-\bar{\theta}^3}{3} + \frac{(\bar{\theta} - 2\pi b)^3}{3}$$
$$+\bar{\theta}^2(\bar{\theta} - 2\pi b) - \bar{\theta}(\bar{\theta} - 2\pi b) + \pi b^2 \underline{\theta} - \pi b^2 \bar{\theta} - 2\pi b\bar{\theta}(\bar{\theta} - 2\pi b) + \pi b(\bar{\theta} - 2\pi b)^2 + \pi b\bar{\theta}^2 \tag{9}$$

$$\Leftrightarrow 0 \leq \frac{-\bar{\theta}^3}{3} + \frac{\bar{\theta}^3}{3} - \frac{4\pi b\bar{\theta}^2}{3} + \frac{4\pi^2 b^2 \bar{\theta}}{3} - \frac{2\pi b\bar{\theta}^2}{3} + \frac{8\pi^2 b^2 \bar{\theta}}{3} - \frac{8\pi^3 b^3}{3} + \bar{\theta}^3 - 2\pi b\bar{\theta}^2$$
$$- \bar{\theta}^3 + 4\pi b\bar{\theta}^2 - 4\pi^2 b^2 \bar{\theta} - 2\pi b\bar{\theta}^2 + 4\pi^2 b^2 \bar{\theta} + \pi b\bar{\theta}^2 - 4\pi^2 b^2 \bar{\theta} + 4\pi^3 b^3 + \pi b\bar{\theta}^2$$

$$\Leftrightarrow 0 \leq \frac{4\pi^3 b^3}{3}$$

which clearly holds.

Coincidently, two conditions which are together sufficient for the OPT to increase ex ante expected social welfare with respect to FDP are for both 8 (multiplied by q on both sides) and 9 (multiplied by $(1 - q)$ on both sides) to hold. Since these respective multiplications do not change the fact that 8 and 9 hold, the proposition is proven.

QED

Proof of Proposition 2: First, I show that the optimal allocation of authority with FDP is a perfect mapping from the population's demographic to q^*_{FD} (based on the plurality rule). I simply compare the interim expected social welfare when group 1 is in power (left-hand side of 10) to the interim expected social welfare when group 2 is in power (right-hand side of 10):

$$-(1-\pi)b^2 \overset{?}{\leq} -\pi b^2 \tag{10}$$

$$\Leftrightarrow \frac{1}{2} \overset{?}{\leq} \pi$$

which clearly indicates that allocating power to whichever group constitutes the majority is optimal.

As for the Optimal Threshold Policies, I simply do the same comparison (the left-hand side of 11 is the interim expected utility when group 1 is in power and the right-hand side of 11 is the interim expected utility when group 2 is in power):

$$\begin{aligned}
&\frac{-[\underline{\theta}+2b(1-\pi)]^3}{3} + \frac{\underline{\theta}^3}{3} - \underline{\theta}^2[\underline{\theta}+2b(1-\pi)] + \underline{\theta}[\underline{\theta}+2b(1-\pi)]^2 - (1-\pi)b^2\bar{\theta} \\
&+(1-\pi)b[\underline{\theta}+2b(1-\pi)]^2 - (1-\pi)2b\underline{\theta}[\underline{\theta}+2b(1-\pi)] + (1-\pi)b^2\underline{\theta} + (1-\pi)b\underline{\theta}^2 \\
&\text{vs } \frac{-\bar{\theta}^3}{3} + \frac{(\bar{\theta}-2b\pi)^3}{3} + \bar{\theta}^2(\bar{\theta}-2b\pi) - \bar{\theta}(\bar{\theta}-2b\pi)^2 + \pi b^2\underline{\theta} - \pi b^2\bar{\theta} \\
&\qquad - 2\pi b\bar{\theta}(\bar{\theta}-2b\pi) + \pi b(\bar{\theta}-2b\pi)^2 + \pi b\bar{\theta}^2
\end{aligned} \tag{11}$$

which simplifies into

$$\Leftrightarrow \frac{4b(1-\pi)^3}{3} - (1-\pi)(\bar{\theta}-\underline{\theta}) \text{ vs } \frac{4b\pi^3}{3} - \pi(\bar{\theta}-\underline{\theta}). \tag{12}$$

First, it must be observed that for $\pi = \frac{1}{2}$, the expression in 11 simplifies to 0 vs 0. Based on this observation, a necessary and sufficient condition for the allocation of power to the majority group to be optimal is

$$\left.\frac{d[\frac{-4b(1-\pi)^3}{3} + (1-\pi)(\bar{\theta}-\underline{\theta}) + \frac{4b\pi^3}{3} - \pi(\bar{\theta}-\underline{\theta})]}{d\pi}\right|_{\pi=\frac{1}{2}} \overset{?}{\leq} 0$$

$$\Leftrightarrow \left.[4b(1-\pi)^2 - (\bar{\theta}-\underline{\theta}) + 4b\pi^2 - (\bar{\theta}-\underline{\theta})]\right|_{\pi=\frac{1}{2}} \overset{?}{\leq} 0$$

$$\Leftrightarrow b \overset{?}{\leq} (\bar{\theta}-\underline{\theta}) \tag{13}$$

which holds given the parameter restriction $2\pi b \leq (\bar{\theta}-\underline{\theta})$ since this restriction, when evaluated at $\pi = \frac{1}{2}$, implies that inequality 13 holds and will continue to hold for any $\pi > \frac{1}{2}$. The proposition is therefore proven since it is also optimal to allocate authority to the majority group when using OTP.

QED

Proof of Proposition 3: It can be seen from 4 that $q^*_{SOTP} = 1$ if

$$
\begin{aligned}
&-\pi \int_{\underline{\theta}}^{\tilde{\theta}_1} \frac{-(\theta-\tilde{\theta}_1)^2}{(\bar{\theta}-\underline{\theta})} d\theta - (1-\pi)\frac{\bar{\theta}-\tilde{\theta}_1}{\bar{\theta}-\underline{\theta}} b^2 - (1-\pi)\int_{\underline{\theta}}^{\tilde{\theta}_1} \frac{(\theta+b-\tilde{\theta}_1)^2}{\bar{\theta}-\underline{\theta}} d\theta \\
&\quad \geq -\pi \frac{(\tilde{\theta}_2-\underline{\theta})}{(\bar{\theta}-\underline{\theta})} b^2 - \pi \int_{\tilde{\theta}_2}^{\bar{\theta}} \frac{(\theta-\tilde{\theta}_2-b)^2}{(\bar{\theta}-\underline{\theta})} d\theta - (1-\pi)\int_{\tilde{\theta}_2}^{\bar{\theta}} \frac{(\theta-\tilde{\theta}_2)^2}{(\bar{\theta}-\underline{\theta})} d\theta.
\end{aligned}
$$

$$
\begin{aligned}
&\Leftrightarrow -\int_{\underline{\theta}}^{\tilde{\theta}_1} \frac{(\theta-\tilde{\theta}_1)^2}{(\bar{\theta}-\underline{\theta})} d\theta - (1-\pi)\frac{(\bar{\theta}-\tilde{\theta}_1)}{(\bar{\theta}-\underline{\theta})} b^2 \\
&\qquad - (1-\pi)\int_{\underline{\theta}}^{\tilde{\theta}_1} \frac{(2b\theta-2b\tilde{\theta}_1)}{(\bar{\theta}-\underline{\theta})} d\theta - (1-\pi)\frac{(\tilde{\theta}_1-\underline{\theta})}{(\bar{\theta}-\underline{\theta})} b^2 \\
&\geq -\int_{\tilde{\theta}_2}^{\bar{\theta}} \frac{(\theta-\tilde{\theta}_2)^2}{(\bar{\theta}-\underline{\theta})} d\theta - \pi\frac{(\tilde{\theta}_2-\underline{\theta})}{(\bar{\theta}-\underline{\theta})} b^2 - \pi \int_{\tilde{\theta}_2}^{\bar{\theta}} \frac{(2b\tilde{\theta}_2-2b\theta)}{(\bar{\theta}-\underline{\theta})} d\theta - \pi b^2 \frac{(\bar{\theta}-\tilde{\theta}_2)}{(\bar{\theta}-\underline{\theta})}
\end{aligned}
$$

$$
\begin{aligned}
&\Leftrightarrow \pi + \underbrace{\frac{\int_{\tilde{\theta}_2}^{\bar{\theta}} (\theta-\tilde{\theta}_2)[(\theta-\tilde{\theta}_2)-2b\pi] d\theta}{(\bar{\theta}-\underline{\theta})b^2}}_{\text{A}} \\
&\qquad\qquad \geq (1-\pi) + \underbrace{\frac{\int_{\underline{\theta}}^{\tilde{\theta}_1} (\tilde{\theta}_1-\theta)[(\tilde{\theta}_1-\theta)-2b(1-\pi)] d\theta}{(\bar{\theta}-\underline{\theta})b^2}}_{\text{B}}. \qquad (14)
\end{aligned}
$$

It can then be observed that the terms A and B represent the frictions in the allocation of authority introduced by the SOTP. Furthermore, if $b \geq \max\{\frac{\bar{\theta}^2(\bar{\theta}-3\tilde{\theta}_2)+(\tilde{\theta}_2)^2(3\bar{\theta}-\tilde{\theta}_2)}{\pi(\bar{\theta}-\tilde{\theta}_2)^2}, \frac{(\tilde{\theta}_1)^2(\tilde{\theta}_1-3\underline{\theta})+\underline{\theta}^2(3\tilde{\theta}_1-\underline{\theta})}{(1-\pi)(\tilde{\theta}_1-\underline{\theta})^2}\}$, then both terms A and B are positive and the constraints imposed on a group actually increase the likelihood of that group being given authority to implement their policies. If $b \leq \min\{\frac{\bar{\theta}^2(\bar{\theta}-3\tilde{\theta}_2)+(\tilde{\theta}_2)^2(3\bar{\theta}-\tilde{\theta}_2)}{\pi(\bar{\theta}-\tilde{\theta}_2)^2}, \frac{(\tilde{\theta}_1)^2(\tilde{\theta}_1-3\underline{\theta})+\underline{\theta}^2(3\tilde{\theta}_1-\underline{\theta})}{(1-\pi)(\tilde{\theta}_1-\underline{\theta})^2}\}$, then both terms A and B are negative and the opposite result holds.

Proposition 3-iii then follows from inequality 14. For example, suppose $\pi < \frac{1}{2}$ and group 1 is the minority group. If the term A in inequality 14 is sufficiently greater than the term B, meaning group 1 is essentially more constrained than group 2, and if $b \geq \max\{\frac{\bar{\theta}^2(\bar{\theta}-3\tilde{\theta}_2)+(\tilde{\theta}_2)^2(3\bar{\theta}-\tilde{\theta}_2)}{\pi(\bar{\theta}-\tilde{\theta}_2)^2}, \frac{(\tilde{\theta}_1)^2(\tilde{\theta}_1-3\underline{\theta})+\underline{\theta}^2(3\tilde{\theta}_1-\underline{\theta})}{(1-\pi)(\tilde{\theta}_1-\underline{\theta})^2}\}$, then allocating power to the minority group is optimal.

QED

Proof of Proposition 4: Using the expression in 5, I compare the ex ante expected social welfare under FD and SOTP with threshold states $\tilde{\theta}_1 = \underline{\theta} + 2(1-\pi)b + \epsilon_1$ and $\tilde{\theta}_2 = \bar{\theta} - 2b\pi - \epsilon_2$:

$$
\begin{aligned}
&- q(1-\pi)b^2(\bar{\theta}-\underline{\theta}) - (1-q)\pi b^2(\bar{\theta}-\underline{\theta}) \le q\{\frac{-[\underline{\theta} + 2b(1-\pi)+\epsilon_1]^3}{3} + \frac{\underline{\theta}^3}{3} \\
&\quad - \underline{\theta}^2[\underline{\theta} + 2b(1-\pi)+\epsilon_1] + \underline{\theta}[\underline{\theta}+2b(1-\pi)+\epsilon_1]^2 - (1-\pi)b^2\bar{\theta} \\
&+(1-\pi)b[\underline{\theta}+2b(1-\pi)+\epsilon_1]^2 - (1-\pi)2b\underline{\theta}[\underline{\theta}+2b(1-\pi)+\epsilon_1] + (1-\pi)b^2\underline{\theta} + (1-\pi)b\underline{\theta}^2\} \\
&\quad + (1-q)[-\frac{\bar{\theta}^3}{3} + \frac{(\bar{\theta}-2\pi b-\epsilon_2)^3}{3} + \bar{\theta}^2(\bar{\theta}-2\pi b-\epsilon_2) - \bar{\theta}(\bar{\theta}-2\pi b-\epsilon_2)^2 \\
&\quad\quad + \pi b^2\underline{\theta} - \pi b^2\bar{\theta} - 2\pi b\bar{\theta}(\bar{\theta}-2\pi b-\epsilon_2) + \pi b(\bar{\theta}-2\pi b-\epsilon_2)^2 + \pi b\bar{\theta}^2]
\end{aligned}
$$

which simplifies to

$$
\begin{aligned}
\Leftrightarrow -q(1-\pi)b^2(\bar{\theta}-\underline{\theta}) - (1-q)\pi b^2(\bar{\theta}-\underline{\theta}) \le q[&\frac{4b^3(1-\pi)^3}{3} - b(1-\pi)\epsilon_1^2 - \frac{\epsilon_1^3}{3} \\
&- (1-\pi)b^2(\bar{\theta}-\underline{\theta})] + (1-q)[\frac{4b^3\pi^3}{3} - \pi b\epsilon_2^2 - \frac{\epsilon_2^3}{3} - \pi b^2(\bar{\theta}-\underline{\theta})]
\end{aligned}
$$

$$
\Leftrightarrow q[b(1-\pi)\epsilon_1^2 + \frac{\epsilon_1^3}{3}] + (1-q)(\pi b\epsilon_2^2 + \frac{\epsilon_2^3}{3}) \le \frac{q4b^3(1-\pi)^3}{3} + \frac{(1-q)4b^3\pi^3}{3}. \quad (15)
$$

Two sufficient conditions for 15 to hold is:

$$
\text{a) } b(1-\pi)\epsilon_1^2 + \frac{\epsilon_1^3}{3} \le \frac{4b^3(1-\pi)^3}{3}
$$

$$
\text{b) } b\pi\epsilon_2^2 + \frac{\epsilon_2^3}{3} \le \frac{4b^3\pi^3}{3}.
$$

I start with a) by solving for the cubic equation $[\frac{4(1-\pi)^3}{3}]b^3 - [(1-\pi)\epsilon_1^2]b - \frac{\epsilon_1^3}{3} = 0$. The solution comes from:

$$
\begin{aligned}
b = (-(\frac{-\epsilon_1^3}{3})(\frac{3}{8(1-\pi)^3}) + \{[-(\frac{-\epsilon_1^3}{3})(\frac{3}{8(1-\pi)^3})]^2 \\
+ [-(1-\pi)\epsilon_1^2(\frac{3}{12(1-\pi)^3})]^3\}^{\frac{1}{2}})^{\frac{1}{3}} + \\
(-(\frac{-\epsilon_1^3}{3})(\frac{3}{8(1-\pi)^3}) - \{[-(\frac{-\epsilon_1^3}{3})(\frac{3}{8(1-\pi)^3})]^2 + [-(1-\pi)\epsilon_1^2(\frac{3}{12(1-\pi)^3})]^3\}^{\frac{1}{2}})^{\frac{1}{3}}
\end{aligned}
$$

$$\Leftrightarrow b = \{\frac{\epsilon_1^3}{8(1-\pi)^3} + [\frac{\epsilon_1^6}{64(1-\pi)^6} - \frac{\epsilon_1^6}{64(1-\pi)^6}]^{\frac{1}{2}}\}^{\frac{1}{3}} + \{\frac{\epsilon_1^3}{8(1-\pi)^3} - [\frac{\epsilon_1^6}{64(1-\pi)^6} - \frac{\epsilon_1^6}{64(1-\pi)^6}]^{\frac{1}{2}}\}^{\frac{1}{3}}$$

$$\Leftrightarrow b = \frac{\epsilon_1}{(1-\pi)}.$$

For b), I solve for $(\frac{4\pi^3}{3})b^3 - \pi\epsilon_2^2 b - \frac{\epsilon_2^3}{3} = 0$ and the solution comes from:

$$b = (-(\frac{-\epsilon_2^3}{3})(\frac{3}{8\pi^3}) + \{[-(\frac{-\epsilon_2^3}{3})(\frac{3}{8\pi^3})]^2 + [-\pi\epsilon_2^2(\frac{3}{12\pi^3})]^3\}^{\frac{1}{2}})^{\frac{1}{3}} + (-(\frac{-\epsilon_2^3}{3})(\frac{3}{8\pi^3}) - \{[-(\frac{-\epsilon_2^3}{3})(\frac{3}{8\pi^3})]^2 + [-\pi\epsilon_2^2(\frac{3}{12\pi^3})]^3\}^{\frac{1}{2}})^{\frac{1}{3}}$$

$$\Leftrightarrow b = [\frac{\epsilon_2^3}{8\pi^3} + (\frac{\epsilon_2^6}{64\pi^6} - \frac{\epsilon_2^6}{64\pi^6})^{\frac{1}{2}}]^{\frac{1}{3}} + [\frac{\epsilon_2^3}{8\pi^3} - (\frac{\epsilon_2^6}{64\pi^6} - \frac{\epsilon_2^6}{64\pi^6})^{\frac{1}{2}}]^{\frac{1}{3}}$$

$$\Leftrightarrow b = \frac{\epsilon_2}{\pi}.$$

So if $b \geq \max\{\frac{\epsilon_2}{\pi}, \frac{\epsilon_1}{(1-\pi)}\}$ hold, then the SOTP associated with $\tilde{\theta}_1 = \underline{\theta} + 2(1-\pi)b + \epsilon_1$ and $\tilde{\theta}_2 = \bar{\theta} - 2b\pi - \epsilon_2$ with $\epsilon_1 > 0$ an $\epsilon_2 > 0$ increases the ex ante expected social welfare with respect to FD. If $b \geq \min\{\frac{\epsilon_2}{\pi}, \frac{\epsilon_1}{(1-\pi)}\}$ holds, then the opposite result hold.

Suppose instead of increasing the constraints on both groups I loosened them by using the sub-optimal threshold states $\tilde{\theta}_1 = \underline{\theta} + 2b(1-\pi) + \epsilon_3$ and $\tilde{\theta}_2 = \bar{\theta} - 2b\pi - \epsilon_4$ with $\epsilon_3 < 0$ and $\epsilon_4 < 0$. Repeating the entire previous analysis would show that SOTP using as threshold states $\tilde{\theta}_1 = \underline{\theta} + 2b(1-\pi) + \epsilon_3$ and $\tilde{\theta}_2 = \bar{\theta} - 2b\pi - \epsilon_4$ with $\epsilon_3 < 0$ and $\epsilon_4 < 0$ increase ex ante expected social welfare if $b \geq \max\{\frac{\epsilon_4}{\pi}, \frac{\epsilon_3}{(1-\pi)}\}$, which always hold since $b > 0$. The proposition is therefore proven.

QED

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