Tournaments, Limited Liability and the Optimal Organizational Structure *

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Abstract

This paper develops a theoretical framework that incorporates the deleterious effects of tournaments on cooperation in order to generate a more complete theory of the optimal organizational structure. I study how well information will be generated by the agents and utilized by the decision maker(s) in either a centralized or a decentralized setting. I show that tournaments can achieve what I refer to as the Constrained First Best outcome and will negatively impact cooperation only in a decentralized setting. This paper also suggests that a limited liability constraint will increase the complementarity between decentralization and the agents' productivity. Furthermore, I argue that periods of economic contraction (expansion) will favor a centralized (decentralized) setting and I uncover a fundamental trade-off between communication noise, which depresses centralized profits, and production noise, which depresses decentralized profits. Finally, I show that managers might find it optimal to communicate less with highly motivated employees than with regular employees.

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1 Introduction

1.1 Motivation

In his seminal work *The Visible Hand*, Chandler (1977) argued that basing the appointment of high-level managers on "training, experience and performance rather than on family relationship" was a crucial component in the development of the modern firm[1]. Since then, this predominance of promotion tournaments to fill a firm's managerial positions has been well documented. In a database that followed 600 firms for over 8 years, Bognanno (2001) observed that roughly 80% of executives were promoted from within instead of filling these positions with outside hires. Similarly, using a data set of a large firm employing over 100,000 people, Lazear (1992) reported that 99.7% of workers were in positions that provided opportunities for promotions.

As Lazear (1989) argued and Drago and Garvey (1998) and Cowgill (2015) documented, tournaments for promotions can negatively impact cooperation among employees. If these effects are more severe in one organizational structure relative to another, then their absence becomes problematic and can lead to biased results. One form these negative effects can take is to cut the flow of information inside a firm. As many papers on the optimal organizational structure tend to focus on information acquisition (see Aghion and Tirole, 1997; Gibbons, Holden, and Powell, 2012; Angelucci, 2015) and exchange (see Dessein, 2002; Dessein, Garicano, and Gertner, 2010; Mookherjee and Tsumagari, 2014), evaluating the influence of promotion tournaments on these two issues could be a crucial step towards a more complete theory of the optimal organizational structure.

More broadly speaking, compensation in general is related to the optimality of an organizational structure. However, the relationship between constraints on how employees can be compensated, which can arise from collective bargaining agreements, governmental regulations or labor-market competition, and the optimal allocation of authority within a firm is poorly understood. This relationship could contribute to a better understanding of the differences in management styles across industries and countries and can also be beneficial in understanding how mergers and acquisitions in foreign markets with different labor laws should be managed.

To incorporate these features, I develop a framework with one principal and two agents where two production decisions must be made. The two agents acquire information concerning the state of nature at a cost and may communicate this information through a nonfalsifiable message to the

[1] Alfred P. Chandler, *The Visible Hand: The Managerial Revolution in American Business*, (Massachusetts: Belknap Press of Harvard University Press, 1977), 8-9.

player(s) with the relevant authority. The decision maker(s) then makes the production decision(s) where this information is a valuable input.

Under centralization, the authority over both production decisions is retained by the principal while it is delegated to the agents under decentralization (each agent takes one production decision). In a centralized setting, agents communicate with the principal, which I refer to as vertical communication. In a decentralized setting, communication is horizontal- that is, agents communicate with each other. All forms of communication are subject to some noise.

Agents are motivated to acquire this costly information by the prizes of a tournament. A centralized tournament will be different from a decentralized tournament since agents will have different responsibilities in both structures. Subsequently, this leads to two sets of incentives which yields two different effort levels. These contrasting efforts as well as the communication patterns and the way information is utilized will be the main components that the principal will have to take into account when determining the optimal organizational structure.

One of the communication patterns within a decentralized setting is that an agent will be unwilling to share his information since the receiver of his message will be his tournament rival. This withholding of information represents the negative impact of tournaments on cooperation among employees which is specific to decentralization and explains why tournaments can only achieve what I call the Constrained First Best. In one of the main results of this paper, I show that in high productivity settings, imposing a lower boundary on the agents' compensation increases the complementarity between decentralization and the agents' productivity and might lead to a decrease in the spread of prizes (the difference between the winner's prize and the loser's prize) only in a centralized setting. I also argue that firms are more likely to centralize their decision-making process during recessions. These results are helpful in understanding how external factors affect the optimal allocation of authority within a firm. Additionally, I uncover a fundamental trade-off between two different forms of volatility. I argue that a decrease in communication noise, which represents the various barriers communication faces within a firm, favors centralization while a decrease in the production noise, which represents the importance of randomness in the production process, favors decentralization.

I eventually relax the assumption that communication noise between the agents and the principal is exogenous by allowing the principal to choose it at some cost. When the agents are very eager to work, I show that the principal is less likely to exert a lot of resources communicating with his agents than with regular agents. This increase in communication noise adds another channel through which decentralization is better suited to deal with highly motivated and productive workers.

I first review the literature in section 1.2 and then introduce the model and some basic results in section 2. The main results of this the paper are presented in sections 3 and 4 and the paper is concluded in section 5.

1.2 Literature Review

This paper argues that the optimality of either organizational structure is based on a comparison of endogenous communication patterns (how players communicate), the ways information is utilized to generate revenue and the incentives agents have to exert effort within each setting. Most papers in the literature include at most two of these elements: Zabojnik (2002), Che and Kartik (2009) and Dessein et al. (2010) include endogenous communication patterns and incentive provisions in their models, Alonso et al. (2008) and Rantakari (2008) include utilization of information and endogenous communication patterns in their framework and Friebel and Raith (2010) include utilization of information and incentive provisions in their paper. Rantakari (2013) is, to my knowledge, the only other paper to include all three elements. My paper mainly differs from it by assuming the misalignment of the agents' interests is exogenous and not chosen by the principal. Two other important differences are that I assume some overlap between the information acquired by both agents and that information is nonfalsifiable. These assumptions allow me to construct a framework that highlights the effects of promotions on the optimal allocation of authority within a firm.

The number of papers that incorporate the misalignment of interests between agents competing in a tournament into a theory of the optimal organizational structure have so far been relatively limited. The existence of a prize that can only be handed out to one agent (like promotions for employees or the right to market their prototype for business units) creates institutional constraints which in turn creates an incentive misalignment between the agents. While a few papers do include various forms of competitive pay in their model (see Marino and Zabojnik, 2004; Rantakari, 2012), only a handful incorporate the negative impacts of competitive pay into their models (see Ozbas, 2005; Inderst and Klein, 2007; Friebel and Raith, 2010). Perhaps the closest paper to this one is Friebel and Raith (2010). In their framework, competing for a prize cannot have a negative impact on horizontal communication since the latter cannot provide any benefits by assumption. In contrast, I assume both horizontal and vertical communications are beneficial, which creates a framework that allows for the negative effects of tournaments to differ from one organizational structure to another.

The number of papers attempting to link a firm's optimal organizational structure to a limited liability constraint or to periods of economic expansion/contraction is even more limited. Zabojnik (2002) develops a framework where an agent who disagrees with his manager concerning the optimal project will be more difficult to motivate in a centralized setting and argues that imposing a minimum salary for the agent should have a decentralizing effect. I show that even when the agents and the principal agree concerning the optimal production decisions, this complementarity between a limited liability constraint and decentralization still holds. As for the effects of periods of economic expansion or contraction, the evidence has so far been mixed. Dowell, Shackell, and Stuart (2011) argues that concentrated power for the CEO during times of financial distress increases the likelihood of a firm's survival while Han, Nanda, and Silveri (2016) argues otherwise.

Various other subjects can be linked to the centralization versus decentralization question. Alonso, Dessein, and Matouschek (2008), Rantakari (2008) and Choe and Ishiguro (2011) model centralization as a structure that optimizes the benefits from coordination and report that even when these benefits are very important, various forms of decentralization can still dominate centralization. To relate the question of delegation to competition, Alonso, Dessein, and Matouschek (2015) develop a framework where revenues are based on uncertain consumer demands, traditionally left out of the optimal organizational structure literature. They find that if the price sensitivities of consumers increase (due to increased competition), centralization should dominate decentralization[2].

2 Preliminaries

2.1 Model Framework

Preferences.- This is a model with three risk-neutral players: one principal and two agents. The principal wants to maximize his expected profits subject to the agents' participation constraint and a limited liability constraint. The

[2]Many other papers relate the question of the optimal organizational structure to different and interesting phenomenons. For example, this question is associated with the intensity of human/physical capital by Rajan and Zingales (2001), with career concerns by Hirata (2014) and Swank and Visser (2015), with rational inattention by Dessein, Galeotti, and Santos (2016), with the informativeness of a price mechanism by Gibbons, Holden, and Powell (2012), with organizational language by Cremer, Garicano, and Prat (2007) with dynamic and timing issues by Li, Matouschek, and Powell (2015) and Grenadier, Malenko, and Malenko (2016) and is studied through a mechanism design approach by Bester (2009) and Mookherjee and Tsumagari (2014).

agents maximize their expected utilities consisting of their expected compensation minus the cost of their efforts.

Information Acquisition.- Agent i exerts effort $e_i \in [0,1]$ at cost $c(e_i) = \theta \frac{e_i^2}{2}$ in order to learn the state of nature $\omega \in \Omega$, where $\Omega \subset \mathbb{R}$ is the space from which the state of nature is drawn. Henceforth, $0 < \theta < \infty$ will be referred to as the cost parameter of the agents. Agent i learns ω with probability e_i and learns nothing with probability $(1 - e_i)$.

Production Technology and Beliefs.- The production output $u(a, \omega, \epsilon) = u_1(\omega, a_1) + \epsilon_1 + u_2(\omega, a_2) + \epsilon_2$ depends on the two production decisions $a = (a_1, a_2)$. Each production decision $a_i \in A$ is taken by the player with the relevant authority, where $A \subset \mathbb{R}$ is the action space. The production output also depends on the state of nature ω and on the error terms ϵ_1 and ϵ_2. For tractability purposes, I assume $\epsilon_2 - \epsilon_1 \sim U[-L, L]$, with $0 < L < \infty$. All three players have the same prior beliefs over ω denoted by $F(\omega)$.

Each output $u_i(\omega, a_i)$ can only take two possible values. When uninformed about the state of nature, the decision maker takes what he believes (based on his priors) to be the optimal action $\hat{a}_i$ resulting in an output of $\hat{u}_i = u_i(\omega, \hat{a}_i)$. When the decision maker knows what the state of nature is, he can compute the optimal action $\bar{a}_i(\omega)$ which results in an output of $\bar{u}_i = u_i(\omega, \bar{a}_i(\omega))$.[3]

The possibility that one production decision is relatively more important than the other adds little intuition to the paper, so I assume that $\hat{u}_1(\omega, \hat{a}_1) = \hat{u}_2(\omega, \hat{a}_2) = \hat{u}$ and $\bar{u}_1(\omega, \bar{a}_1(\omega)) = \bar{u}_2(\omega, \bar{a}_2(\omega)) = \bar{u}$ with $\hat{u} < \bar{u}$. I will refer to the difference between $\bar{u}$ and $\hat{u}$ as the value of information, denoted by Δu, which can also be thought of as the productivity of the agents. To simplify the exposition of this model, I assume $E(\epsilon_1) = E(\epsilon_2)$ and without loss of generality, I assume $E(\epsilon_1) = E(\epsilon_2) = 0$.

Organizational Structure.- The model has two possible organizational structures. A centralized setting allocates the authority of the production decisions (a_1, a_2) to the principal who requests information from the agents, meaning communication is vertical. In contrast, a decentralized setting allocates the authority of the production decisions to the agents. Each agent will be in charge of one dimension of the production decision $a = (a_1, a_2)$. Since there is no need for communication with the principal, agents only exchange information with each other if they choose to. This form of communication is referred to as horizontal.

Communication.- When an agent learns ω, he may decide to share this

[3]The probability that $\hat{a}_i = \bar{a}_i(\omega)$ is infinitely small, so I simplify the framework by assuming an output function where the agent is either right and gets $\bar{u}_i(\omega, \bar{a}_i)$ or wrong and gets $\hat{u}_i(\omega, \hat{a}_i)$ with certainty.

information with the (other) decision maker. Like in Dessein and Santos (2006) and Che and Kartik (2009), the messages sent by the agents are assumed to be nonfalsifiable. This is simpler to work with and cheap talk would not add any major insights. Denote by $m_i = \{0, 1\}$ the decision to communicate, with $m_i = 0$ meaning agent i did not communicate ω and $m_i = 1$ the opposite. Examples of information exchanges are sharing details about clients, the behavior of competitors, production conditions, investment opportunities, tips for solving production problems, the costs of introducing new features, etc.

All forms of communication are subject to errors. This may be due to multiple things, like the receiver having a limited amount of time for understanding the messages sent by the agent(s), having limited knowledge of the production process, language barriers, faulty communication equipment, turbulent environments or needlessly long business reports. This communication noise will be modeled in the following way. In a centralized (decentralized) setting, with probability q, all messages sent by both agents are fully received by the principal (by the other agents). With probability 1 - q, none of the messages sent by the agents reach their receiver(s).

Contracting.- With the exception of the First Best and Constrained First Best analyses, I assume that the agents' efforts are unobservable and only relative performance measures are observable and contractible. Therefore, some form of relative incentive pay or tournament must be used to motivate the agents to exert efforts. While the principal can observe the total output $u(a, \omega, \epsilon)$, he can only observe an ordinal ranking between the sums of $u_1(a_1, \omega) + \epsilon_1$ and $u_2(a_2, \omega) + \epsilon_2$. Since agent i is in charge of a_i in a decentralized setting, the principal will use the ordinal ranking between $u_1(a_1, \omega) + \epsilon_1$ and $u_2(a_2, \omega) + \epsilon_2$ to determine the winner of a decentralized tournament. In a centralized setting, the principal can only deduce which agent communicated the most informative message and use this deduction to determine the winner of the centralized tournament. Given these assumptions, the agents are motivated to learn the state of nature by the prospect of winning either tournament and receiving the winner's prize W^U (the other agent is compensated with the loser's prize W^L.)

Parameter Restriction.- In order to ensure that the errors terms always have some role to play in determining the winner of the decentralized tournament, I assume that $\Delta u \leq L$.

Outline.- I first proceed by solving the First Best outcome, where efforts are observable and contractible and the principal can force the agents to reveal their information. I then solve for the Constrained First Best outcome where efforts are still observable and contractible but the agents no longer exchange information with each other. Afterwards, I simultaneously introduce unobservable efforts and the contracting friction that only relative performance measures are contractible. This is to study the use of rank-order tournaments as a payment mechanism as well as a motivational tool. I then introduce one last additional friction, a limited liability constraint for the agents. Finally, I allow the principal to optimally choose the communication noise between himself and the agents.

2.2 First Best Outcome

As a benchmark, I assume a social planner exists and can dictate to the agents what efforts they must exert. The social planner can also force the agents to disclose any acquired information. For tractability purposes, I assume that the reservation utility of both agents is zero. Since the social planner wants to maximize the expected surplus, he would solve the following two problems in a decentralized and centralized setting respectively:

$$\max_{e} \quad (e^2)2\Delta u + [2e(1-e)](\Delta u + q\Delta u) + 2\hat{u} - 2[\frac{\theta}{2}(e^2)]$$

and

$$\max_{e} \quad 2q\Delta u[e^2 + 2e(1-e)] + 2\hat{u} - 2[\frac{\theta}{2}(e^2)].$$

Since the agents have an identical cost parameter θ, the social planner induces an identical level of effort e for both agents. If both agents learn the state of nature, which happens with probability e^2, the payoffs in a decentralized setting is simply $2\hat{u} + 2\Delta u$ while the payoffs in a centralized setting is $2\hat{u} + 2q\Delta u$ (q is the probability that the agents' information was successfully communicated to the principal.) If only one agent learns the state of nature, which happens with probability $2e(1-e)$, the payoffs in a decentralized setting are $2\hat{u} + \Delta u + q\Delta u$, where the $q\Delta u$ term represents the expected output of an uninformed agent (with probability q, the informed agent will successfully share the realized state of nature ω with the uninformed agent). In a centralized setting, the payoff would once again be $2\hat{u} + 2q\Delta u$ since the principal only needs one message for both production decisions. Since the reservation utility of both agents is zero, the social planner makes the agents' participation constraint binding and can replace whatever payment destined to the agents by both of the agents' cost of effort.

Lemma 1: In the First Best (FB) outcome, decentralization is strictly preferred to centralization.

Since there is no moral hazard in this setup, the only thing that matters for the optimal organizational structure is the way information is utilized in either setting. In contrast to a centralized setting, when communication fails due to noise, the decision makers in a decentralized setting (the agents) potentially still have some information to use due to their own information acquisition efforts. This implies that decentralization will be more profitable (in expectation) than centralization.

2.3 Constrained First Best Outcome

As it will later be made clear by proposition 2, the principal is never able to achieve the First Best outcome through the use of tournaments. However, he will be able to achieve the outcome of a slightly different setting that I call the Constrained First Best (CFB). To create this setting, the friction I introduce is a withholding of information at the horizontal communication stage. This is to emulate one of the upcoming consequences of tournaments, which is a negative effect on cooperation (this takes the form of no information sharing between the agents). The social planner can still dictate the agents' efforts.

The participation constraints bind and each agent receives an expected utility of zero. Therefore, in a decentralized setting, given the lack of information sharing, the social planner's problem is to maximize the expected surplus

$$\max_{e} 2\hat{u} + (2\Delta u)e - 2(\frac{\theta e^2}{2})$$

Given the constraint that $e \in [0,1]$ and the linearity of the revenues with respect to the agents' efforts, this problem results in solutions which are summarized in Lemma 2.

Lemma 2: In the Constrained First Best (CFB) setting, decentralization yields the following results:

$$\text{i) } e_D^{CFB} = \begin{cases} \frac{\Delta u}{\theta} & \text{if } \Delta u \leq \theta \\ 1 & \text{if } \Delta u > \theta \end{cases}$$

$$\text{ii) } E(\Pi_D^{CFB}) = \begin{cases} \frac{\Delta u^2}{\theta} + 2\hat{u} & \text{if } \Delta u \leq \theta \\ 2\Delta u + 2\hat{u} - \theta & \text{if } \Delta u > \theta. \end{cases}$$

In a centralized setting, since the probability of the principal receiving at least one informative message can be rewritten as $e^2 + 2e(1-e) = 2e - e^2$, the social planner's problem is

$$\max_e 2q\Delta u(2e - e^2) + 2\hat{u} - 2(\frac{\theta e^2}{2})$$

subject to each agent receiving an expected utility of zero.

Lemma 3: In the CFB setting, centralization yields:

$$\text{i) } e_C^{CFB} = \frac{2q\Delta u}{2q\Delta u + \theta}$$

$$\text{ii) } E(\Pi_C^{CFB}) = \frac{4q^2\Delta u^2}{2q\Delta u + \theta} + 2\hat{u}.$$

The introduction of the withholding of information at the horizontal communication stage is crucial for one simple reason: it highlights that decentralization no longer strictly dominates centralization when agents do not share information with each other. In fact, Proposition 1 shows how the opposite can occur.

Proposition 1: The optimal organizational structure in the Constrained First Best setting.

i) When $\Delta u \leq \theta$, if and only if $q \geq q^* \equiv \frac{\Delta u + \sqrt{\Delta u^2 + 4\theta^2}}{4\theta}$ will centralization be preferred to decentralization.

ii) When $\Delta u \geq \theta$, if and only if $q \geq q^{**} \equiv \frac{2\Delta u - \theta + \sqrt{(\theta + 2\Delta u)^2 - 4\theta^2}}{4\Delta u}$ will centralization be preferred to decentralization

Proposition 1 establishes that a necessary condition for centralization to actually dominate decentralization is a sufficiently low level of communication noise. Since agents do not communicate with each other, communication noise has an actual impact only on centralization. Proposition 1 is in stark contrast to the First Best setting, where decentralization is always preferred to centralization. It can then be deduced that the withholding of information at the horizontal communication stage and sufficiently low levels of communication noise are together necessary and sufficient conditions for centralization to dominate decentralization when efforts are contractible.

3 The Optimal Organizational Structure

I now introduce two additional frictions and discard the social planner from the model. I simultaneously introduce unobservable efforts and the contracting friction that only relative performance measures are contractible, which means some form of tournament will have to be used in order to motivate and compensate the agents.

In a decentralized setting, agent i is responsible for output $u_i(a_1, \omega)$. Since the principal can observe the ordinal ranking between the sums of $u_1(a_1, \omega) + \epsilon_1$ and $u_2(a_2, \omega) + \epsilon_2$, the winner of the decentralized tournament will be the agent who is responsible for the relatively higher output.

In a centralized setting, the agents are no longer responsible for the production decisions. Therefore, another form of relative performance measure must be used. Given the agents' responsibilities, the winner of the centralized tournament will be determined by who communicated the most information to the principal. For example, assuming communication was successful, if the principal receives ω by agent 1 and nothing by agent 2, then the winner of the tournament will be agent 1 and vice-versa. If both agents send the same amount of information, the winner is determined by the flip of a coin. If messages were not successfully received by the principal due to communication noise, the winner is also determined by the flip of a coin.

The principal must choose the optimal prizes W_D^U, W_D^L, W_C^U and W_C^L subject to the agents' participation constraints and, in subsection 3.4, a limited liability constraint. To simplify the notation, denote by $e^g = (e_1^g, e_2^g)$ and $m^g = (m_1^g, m_2^g)$ the strategies adopted by the agents in an organizational structure $g \in \{C, D\}$. Given the tournament setting, agent i's payoff function is

$$Q_i^g(e^g, m^g)W_g^U + [1 - Q_i^g(e^g, m^g)]W_g^L - \theta\frac{(e_i^g)^2}{2}$$

and the principal's expected profits are

$$\sum_{i=1}^{2}\{B_i^g(e^g, m^g, q)\bar{u}_i + [1 - B_i^g(e^g, m^g, q)]\hat{u}_i\} - W_g^U - W_g^L$$

where the notation $Q_i^g(e^g, m^g)$ represents the probability of agent i winning the tournament in g and $B_i^g(e^g, m^g, q)$ represents the probability that the decision maker responsible for action a_i in g will learn ω.

The timing of the game is as follows. First, the principal sets the prizes for both the centralized and decentralized settings. Second, the principal chooses between centralization or decentralization. Third, agents observe the organizational structure and simultaneously choose the level of effort to exert in order to learn the state of nature. Fourth, the state of nature is potentially revealed to the agents. The fifth step is the communication stage. In a decentralized setting, each agent decides whether or not to disclose their information to the other agent. In a centralized setting, agents communicate with the principal. Then, production decisions are made by the players with the relevant authority. Finally, the production outputs are realized, the winner of the tournament is determined and the prizes are handed out.

3.1 Perfect Bayesian Equilibrium

Denote by G the decision of the principal to choose between centralization and decentralization with $G \in \{D, C\}$, by ω^* the realized state of nature and by $F_i^*(\omega)$ the updated belief for player $i \in \{1, 2, P\}$. Furthermore, the type of a player is defined by their private information, which is $t \in \mathcal{T} \equiv \{\text{Uninformed}\} \cup \Omega$. In other words, a player can either be uninformed or know what the state of nature is and thereby have a type defined by $\omega \in \Omega$.

In a decentralized setting, a strategy for agent i of type t is a set of probability distributions $\sigma_i^D(\cdot|t)$ over his production decision a_i, his communication decision m_i^D and his effort level e_i^D. In a centralized setting, a strategy for agent i of type t is a set of probability distributions $\sigma_i^C(\cdot|t)$ over his communication decision m_i^D and his effort level e_i^C. The principal (of type t) strategy consists of his decision to choose between centralization and decentralization $G \in \{D, C\}$ as well as both sets of prizes, W_D^U, W_D^L and W_C^U, W_C^L, and a set of probability distributions $\sigma_P(\cdot|t)$ over the production decisions $a = (a_1, a_2)$ in the case of centralization.

Given the prior beliefs $F(\omega)$, a Perfect Bayesian Equilibrium will consist of the strategy profile $\sigma^* = (\sigma_1^{*D}, \sigma_2^{*D}, \sigma_1^{*C}, \sigma_2^{*C}, \sigma_P^*, G^*, W_D^{*U}, W_D^{*L}, W_C^{*U}, W_C^{*L})$ and all three players' posterior beliefs $F_i^*(\omega)$ for $i \in \{1, 2, P\}$ such that:

-$\forall$ t, $\sigma_i^{*g}(\cdot|t)$ maximizes $E(V_i^g)$ $\forall$ $i \in \{1, 2\}$ and $g \in \{D, C\}$,
-$\forall$ t, $\sigma_P^*(\cdot|t)$ maximizes $E(\Pi^C)$,
-W_g^{*U} and W_g^{*L} maximizes $E(\Pi^g)$ $\forall$ g $\in \{D, C\}$,
-$G^* = D$ if $E(\Pi^D) > E(\Pi^C)$ or $G^* = C$ otherwise,
-the beliefs $F(\omega)$ are updated using Bayes' rule

3.2 The Agents' Optimal Strategies

I first solve for the agents' optimal strategies in both settings. Denote the difference between the winner's and the loser's prizes in organizational structure g by $\Delta W^g = W_g^U - W_g^L$, which the agents take as given. The optimal strategies taken by the agents are summarized in Lemma 4.

Lemma 4:

$$e^*_D = \begin{cases} (\frac{\Delta u}{L})(\frac{\Delta W^D}{2\theta}) & \text{if } \Delta u \Delta W^D \leq 2L\theta \\ 1 & \text{if } \Delta u \Delta W^D > 2L\theta \end{cases} \qquad m^*_D = 0$$

$$e^*_C = (q)(\frac{\Delta W^C}{2\theta}) \qquad m^*_C = 1.$$

The proof of Lemma 4 has been placed in the Appendix 6.3. The two effort functions are composed of a standard component and a structure-specific component. The standard component is made up of a ratio of the difference in the prizes over the cost of information acquisition. In essence, the less costly (low θ) and the more beneficial (high ΔW^g) it is to acquire information, the more efforts the agents are willing to exert in setting g. The structure-specific component is made up of either the communication noise (q) or the ratio of the value of information (Δu) over the production noise (L). Both of these values reflect the importance of the agents' efforts in determining the winner of their corresponding tournament relative to blind luck. Audas, Barmby, and Treble (2004) provide empirical support on how increased certainty in the promotion process induces the agents to work harder.

Clearly, an agent finds it optimal not to disclose their private information to the other agent ($m^*_D = 0$). This is fairly intuitive since, in the horizontal communication stage, the receiver of the agent's message is his direct rival. By sharing the state of nature ω with his rival, an agent would be directly reducing his chances of winning the decentralized tournament and the subsequent winner's prize. This is in direct contrast with the vertical communication stage for the exact opposite reason: withholding any information from the principal would simply reduce the agent's chances of winning the centralized tournament, so $m^*_C = 1$ is the optimal strategy. This suggests that centralized firms might be better suited in using tournaments than decentralized firms, which complements a result by Rantakari (2008) stating that centralization is optimal when the agents' interests are sufficiently misaligned.

Multiple examples exist of how internal competition can lead employees and business units to withhold valuable information from their colleagues and counterparts. Using a data set consisting of 121 development teams and 41 subsidiaries of a large high-technology company, Hansen, Mors, and Løvås (2005) reported that subsidiaries were less likely to spend resources sharing knowledge with their counterparts whom they perceived as rivals. He, Baruch, and Lin (2014) conducted a survey in IT firms in Taiwan and reported that employees were less likely to share their job experiences with their coworkers when they felt intense interpersonal competition. Additionally, Cowgill (2015) reported that workers were less likely to share knowledge about their project with workers who were competing for the same promotions[4].

3.3 The Consequences of Tournaments

By incorporating the agents' optimal strategies, I can now start solving for the principal's expected profit functions, which are the ultimate determinant of the optimal organizational structure. Since $B_1^C(e_C^*, m_C^*, q) = B_2^C(e_C^*, m_C^*, q) = q[2e_C^* - (e_C^*)^2]$, the centralized problem without a limited liability constraint is

$$\max_{W_C^U, W_C^L} q\{2[\frac{q(W_C^U - W_C^L)}{2\theta}] - [\frac{q(W_C^U - W_C^L)}{2\theta}]^2\}2\Delta u + 2\hat{u} - W_C^U - W_C^L$$

$$\text{subject to } \frac{1}{2}W_C^U + \frac{1}{2}W_C^L - \frac{\theta}{2}[\frac{q(W_C^U - W_C^L)}{2\theta}]^2 \geq 0. \qquad \text{PC}$$

In contrast, it can be seen that $B_1^D(e_D^*, m_D^*, q) = B_2^D(e_D^*, m_D^*, q) = e_D^*$, which means the decentralized problem without a limited liability constraint is

$$\max_{W_D^U, W_D^L} 2\Delta u[\frac{\Delta u(W_D^U - W_D^L)}{2L\theta}] + 2\hat{u} - W_D^U - W_D^L$$

$$\text{subject to } \frac{1}{2}W_D^U + \frac{1}{2}W_D^L - \frac{\theta}{2}[\frac{\Delta u(W_D^U - W_D^L)}{2L\theta}]^2 \geq 0. \qquad \text{PC}$$

The participation constraints (PC) require that both agents get an expected utility, which is the expected prize minus the cost of effort, equal to at least 0. Given the symmetric strategies of the agents, the expected probability of winning the tournament of either organizational structure is one half.

Proposition 2: In a tournament setting with no limited liability constraint, the expected profits in both centralization and decentralization are the same as in the Constrained First Best setting.

Lazear and Rosen (1981) showed that tournaments can achieve the First Best under risk neutrality and in the absence of a common shock to the outputs. In contrast, proposition 2 shows that while tournaments might not be

[4]See also Johnson, Hollenbeck, Humphrey, Ilgen, Jundt, and Meyer (2006) for an experiment and Birkinshaw (2001) for some case studies related to this result.

able to achieve the First Best outcome even under the same two assumptions as in Lazear and Rosen (1981), it will be able to achieve the Constrained First Best outcome. This difference is due to this model's inclusion of the negative consequences of tournaments on cooperation. Proposition 2 follows from the principal's freedom to set arbitrary prizes unhindered by liquidity or limited liability constraints. This freedom allows the principal to extract all of the surplus which means his maximization problem is equivalent to maximizing the total surplus under the Constrained First Best setting.

3.4 The Consequences of a Limited Liability Constraint

The notion that the principal can set negative prizes is a very strong assumption. By introducing a limited liability constraint, I force the principal to set (weakly) positive prizes. Given the predominance of regulations and laws imposing some bound on contractual penalties, the limited liability constraint is an important assumption in reflecting the features of the modern firm.

The centralized problem with a limited liability constraint is the same as in section 3.3 but with an added constraint of $W_C^L \geq 0$. The steps leading to the solutions have been placed in appendix 6.5. The only possible solutions are $W_C^L = 0$ and

$$W_C^U = \begin{cases} 0 & \text{if } \Delta u \leq \frac{\theta}{2q^2} \\ \frac{\theta(2q^2\Delta u - \theta)}{q^3 \Delta u} & \text{if } \Delta u \geq \frac{\theta}{2q^2}. \end{cases}$$

In both cases, the participation constraint (PC) is satisfied and $e_C^* < 1$. This yields a centralized profit of

$$\Pi_{LL}^C = \begin{cases} 2\hat{u} \equiv \Pi_0^C & \text{if } \Delta u \leq \frac{\theta}{2q^2} \\ 2\hat{u} + \frac{(2q^2\Delta u - \theta)^2}{2q^3 \Delta u} \equiv \Pi_1^C & \text{if } \Delta u \geq \frac{\theta}{2q^2}. \end{cases}$$

Similarly, the decentralized problem with a limited liability constraint is the same as in section 3.3 but with an added constraint of $W_D^L \geq 0$. Once again, the steps leading to the solutions have been placed in appendix 6.6.

If $\theta \geq L$, then the solutions are $W_D^U = W_D^L = 0$ with $\Pi_{LL}^D = 2\hat{u}$. If $\theta \leq L$, then the solutions are $W_D^L = 0$ and

$$W_D^U = \begin{cases} 0 & \text{if } \Delta u \leq \sqrt{L\theta} \\ \frac{2L\theta}{\Delta u} & \text{if } \Delta u \in [\sqrt{L\theta}, L]. \end{cases}$$

which results in

$$\Pi^D_{LL} = \begin{cases} 2\hat{u} \equiv \Pi^D_0 & \text{if } \Delta u \leq \sqrt{L\theta} \\ 2\hat{u} + 2\Delta u - \frac{2L\theta}{\Delta u} \equiv \Pi^D_1 & \text{if } \Delta u \in [\sqrt{L\theta}, L]. \end{cases}$$

Proposition 3: The optimal organizational structure.

i) If the communication noise is above a certain threshold ($q \leq \sqrt{\frac{\theta}{2\Delta u}}$), then decentralization (weakly) dominates centralization.

ii) If the production noise is sufficiently high ($L > L^* \equiv \min\{\frac{\Delta u^2}{\theta}, \frac{\Delta u^2}{\theta}(1-q) + \frac{\Delta u}{q} - \frac{\theta}{4q^3}\}$), then centralization (weakly) dominates decentralization.

iii) If $\Delta u > \Delta u^* \equiv \max\{\frac{-\theta}{2q(1-q)} + \sqrt{\frac{\theta^2 + 4L\theta q^3(1-q)}{4q^3(1-q)^2}}, \sqrt{L\theta}\}$, then decentralization dominates centralization.

Too much communication noise once again leads to decentralization being the optimal organizational structure. However, the main takeaway from Proposition 3 is that not only do both organizational structures differ in their communication patterns but also in how they utilize information and in how they motivate agents to exert effort[5]. The withholding of information is the main disadvantage in the decentralized communication patterns whereas noisy communication is centralization's disadvantage. In Lemma 4, it has already been shown how each setting motivates the agents while Lemma 5 below formalizes how both settings differ in how they utilize the agents' efforts.

Lemma 5:

-The marginal returns of e^*_C on the expected centralized revenues $z^C \equiv 2q\Delta u[2e^*_C - (e^*_C)^2]$ are always positive but are decreasing as e^*_C becomes larger.

-The marginal returns of e^*_C on the expected decentralized revenues $z^D \equiv 2\Delta u e^*_D$ are always positive and constant.

A remainder is in order that the value of information Δu can also be though of as the productivity of the agents. Result 3-iii argues that, when agents are sufficiently productive, decentralization is the optimal organizational structure.

[5]This is similar to the point made by Rantakari (2013): centralized and decentralized structures will differ not only in the quality of information they generate but also in the value of this information, which he defines as how well each structure uses this information.

Based on the decreasing marginal returns of centralized effort and the constant marginal returns of decentralized efforts (see Lemma 5).

These decreasing marginal returns arise in a centralized setting for the following reason. When the principal receives two fully informative messages, one of them is redundant and the effort that generated it is wasted. As the agents work harder, the likelihood of this redundancy increases, which explains the decreasing marginal returns in a centralized setting. This increasing likelihood of wasting an effort is absent in a decentralized setting: an agent will never waste his own effort.

Additionally, when only one agent learns ω, centralization can generate more revenue than decentralization because, if communication was successful, the principal fully utilizes this information by taking the appropriate production decisions (the agent who discovered ω will withhold this information from the agent who did not, thereby leading to a low output in one dimension). Therefore, as the value of information Δu increases, the agents exert more effort in both settings which in turn favors decentralization, implying there exists some threshold Δu^* such that decentralization is more profitable than centralization for any $\Delta u > \Delta u^*$.

Finally, it can be seen that more production noise discourages agents from exerting effort since they become more likely to be judged based on blind luck rather than their work. Result 3-ii states that if the production noise is sufficiently high, then the agents exert such little effort in a decentralized setting that centralization (weakly) dominates decentralization. These results as well as the boundaries in results 3-i:ii are illustrated in Figure 1.

Denote by Π^C_{noLL} and the Π^D_{noLL} the centralized and the decentralized profits when the principal is unhindered by a limited liability constraint (like in subsection 3.3). Furthermore, denote by R the agents' reservation utility. I continue to assume that it is zero but this notation is useful for result 4-iii.

Proposition 4 - Comparative statics.

i) Decreasing the communication noise (increasing q) favors centralization.

ii) Increasing the production noise (L) favors centralization.

iii) Assuming that agents are sufficiently productive ($\Delta u \geq \max\{\frac{\theta}{2q^2}, \sqrt{L\theta}\}$) such that the principal induces them to exert strictly positive levels of effort, a simultaneous increase in the productivity of the agents Δu and the agents' reservation utility (starting from $R = 0$) favors decentralization while a simultaneous decrease favors centralization.

The intuition for result 4-i is fairly straightforward. A reduction in com-

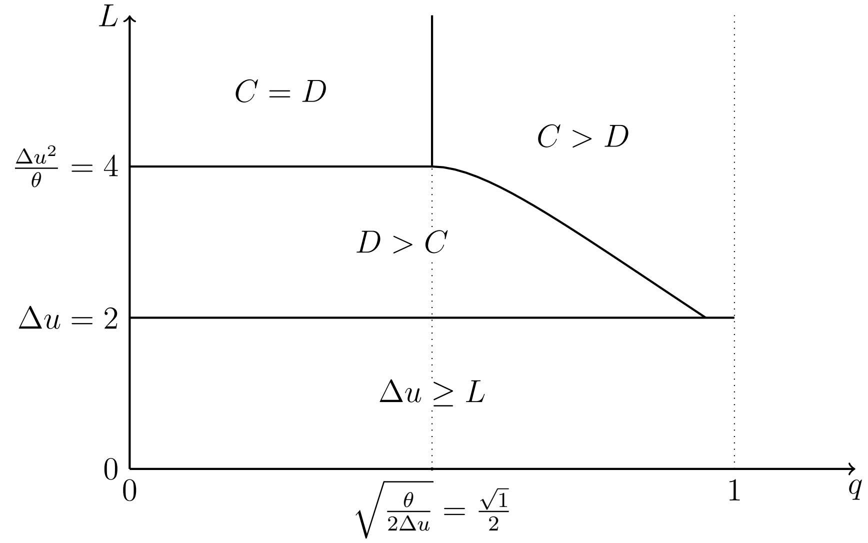

Figure 1 - Characterization of the Parameter Space for $\theta = 1, \Delta u = 2$

munication noise (an increase in q) motivates agents to exert additional effort since they are more likely to be judged based on the content of their message rather than blind luck. It also increases the efficiency of the decision making process by increasing the odds of the decision maker becoming informed while having no impact on decentralized profits since agents do not communicate with each other. A reduction in communication noise therefore creates a centralizing force, which has been empirically documented by Bloom, Garicano, Sadun, and Van Reenen (2014) and Lai (2011).

As previously mentioned, an increase in the production noise L increases the importance of the error term in the expectation of the probability of winning the decentralized tournament. Subsequently, the benefits of acquiring information are diminished, leading the agents to reduce their efforts. This is in contrast to a centralized setting where the production noise has no impact on the agents' efforts or the principal's profits. The agent simply attempts to acquire information and, if successful, pass on that information to the principal. This is consistent with empirical results reported by Ho, Dessein, Ghosh, and Lafontaine (2016) and Meagher and Wait (2013), who find that delegating decisions to employees is less likely to occur in rapidly-changing, difficult to predict and uncertain environments. The first two parts of Proposition 4 embody the fundamental trade-off in this model between the two forms of volatility: a large production noise L and a small communication noise (high q) favors centralization and vice-versa.

Result 4-iii is interesting because a simultaneous increase (decrease) in Δu

and R can be interpreted as the consequences of a period of expansion (contraction): the agents are more (less) productive and their wages tend to go up (down). On one hand, a variation of the agents' reservation utility has no impact on the decision to centralize or decentralize. This happens because the limited liability constraint is more binding than the agents' participation constraints which remains slack in either setting (when $\Delta u \geq \max\{\frac{\theta}{2q^2}, \sqrt{L\theta}\}$). On the other hand, an increase (decrease) in the agents' productivity Δu will favor decentralization (centralization), which is intuitively related to result 3-iii. There also exists emerging empirical support for the idea that firms are more likely to adopt a more centralized organizational structure during recessions. Kunisch, Schimmer, and Müller-Stewens (2012) conducted a survey of 761 of the largest publicly listed companies in Western Europe and North America and reported that these companies reacted to the 2007-10 economic crisis by tightening their corporate headquarter's control over important strategic and functional decisions. Similarly, Bakonyi and Muraközy (2016) studied the reactions of 14,000 manufacturing Europeans firms to the Great Recession and reported that negative shocks to employment and product demand are significantly correlated with an increased use of centralization.

Proposition 5 - The impacts of a limited liability constraint.

i) In centralization, the introduction of the limited liability constraint causes the principal to reduce the spread of prizes.

ii) In decentralization, the introduction of the limited liability constraint causes the principal to decrease the spread of prizes if $\Delta u \leq \min\{L, \sqrt{L\theta}\}$ or to maintain it if $\Delta u \in [\sqrt{L\theta}, L]$.

iii) When $\Delta u \geq \sqrt{L\theta}$, the introduction of the limited liability constraint increases the complementarity between decentralization and productive employees ($\frac{\partial(\Pi^D_{LL} - \Pi^C_{LL})}{\partial \Delta u} \geq \frac{\partial(\Pi^D_{noLL} - \Pi^C_{noLL})}{\partial \Delta u}$).

Various governmental regulations or collective bargaining agreements can impose a limit on how firms can compensate their employees. Results 5-i and 5-ii highlight how the impacts of these minimum boundaries on the variance of pay within a firm will differ according to the firm's organizational structure. When a binding limited liability constraint is introduced, it makes it more expensive for the principal to motivate his agents through ΔW^D (which is equal to W^U_D because $W^L_D = 0$) and he is then faced with a choice: increase, decrease or maintain the winner's prize.

In a sufficiently unproductive decentralized setting ($\Delta u \leq \min\{\sqrt{L\theta}, L\}$), the principal finds it optimal to decrease the winner's prize to the bare minimum, $W_D^U = 0$, because the marginal costs of W_D^U always outweigh the marginal benefits. For the inverse reason, when the productivity of the agents is sufficiently high ($\Delta u \in [\sqrt{L\theta}, L]$), then the principal finds it optimal to set the winner's prize to its maximum (the upper bound on $W_D^U = \Delta W^D$ is determined by the limit $e_D^* \leq 1$). In a centralized setting, the decreasing marginal returns of e_C^* due to the increasing likelihood of an effort being redundant are the reasons why it is never optimal for the principal to increase the spread of prizes. Therefore, these results suggest that a limited liability constraint will reduce the variability of pay within a firm only in a centralized setting (while having no such impact within a decentralized firm).

A remainder is in order that the agents' productivity is linked to the agents' motivation to exert effort only in a decentralized setting: the higher Δu is, the more likely it is that the agents will be compensated based on their work rather than blind luck. This motivation is absent in a centralized setting where the agents' chances of winning W_C^U are unaffected by their productivity. It must then be observed that, due to the constant marginal returns of e_D^*, the optimal decentralized prizes yield "corner" solutions: the principal either induces the agents to exert the maximum level of effort or no effort at all. If the agents are already exerting the maximum level of effort, then the principal can profit from the increased motivation induced by an increase in Δu by reducing the winner's prize while simultaneously keeping the agents' effort at its maximum level. However, this is only possible when the agents' participation constraints are slack. Since the limited liability constraint is more binding than the participation constraint when $\Delta u \geq \sqrt{L\theta}$, this additional channel through which an increase in Δu increases profits is not only absent in a centralized setting but also in a decentralized setting without a limited liability constraint.

Result 5-iii is helpful in establishing a link between laws dictating a minimum boundary on what employees can be paid in the case of a relatively poor performance and the optimal organizational structure of a firm. If employees are sufficiently productive (high Δu), these regulations are potentially a decentralizing force. Since a possible interpretation of a limited liability constraint is that the probability of a job termination is lower, result 5-iii can then be related to the "star" and "commitment" models reported by Baron and Hannan (2002), where firms who provide more employment security tend to rely more on employee autonomy and self-management in the high technology start-ups of Silicon Valley. It essentially provides a theoretical argument for the observed complementarity between job security and decentralization in high productivity firms. Further evidence of this complementarity has been documented by Brown, Reich, and Stern (1993), Ichniowski, Shaw, and

Prennushi (1997) and Ichniowski and Shaw (1999). Finally, result 5-iii is helpful in understanding how differences in labor laws can affect the management of a newly acquired firm in a foreign market: if this foreign market provides more (less) job security to their employees, then the purchasing firm should consider implementing a more (less) decentralized organizational structure than its current one.

4 Endogenous Communication Noise

All communication noise, including the communication noise between the principal and the agents, have been assumed to be exogenous so far. However, in many situations, a manager has some control over the communication noise between his employees and himself. For instance, the principal can choose to spend more time and efforts into understanding the agents' messages, like in Dessein et al. (2016) or Dessein and Santos (2006), through exhaustive meetings and oversight. A company can choose to adapt a communication enhancing technology at some cost, which allows for an easier transfer of data on sales, forecasts of market conditions, etc.

For expositional purposes, I will use the simplifying assumption for this section that the prizes are exogenously determined, so both the principal and the agents take them as given. Furthermore, since communication noise between the agents remains untouched, I must assume a separate level of noise for vertical and horizontal communications. Let q denote the communication noise between the principal and the agents and q^D the communication noise between the agents. The timing of the game would have to be slightly modified to incorporate this new decision. After his decision to centralize but before the agents take their decisions to exert effort, the principal would choose q at some cost $s(q) = (\theta^P)\frac{q^2}{2}$ which would be observed by the agents. If the principal chooses decentralization, then the timing remains the same as before. This would lead to the following problem for the principal in a centralized setting

$$\max_q 2q\Delta u\{2e^*_C(q) - [e^*_C(q)]^2\} + 2\hat{u} - W^U - W^L - \frac{(\theta^P)q^2}{2}$$

resulting in a first order condition of

$$2\Delta u\{2e^*_C(q) - [e^*_C(q)]^2\} + 2q\Delta u\{2\frac{\partial e^*_C(q)}{\partial q} - \frac{\partial[e^*_C(q)^2]}{\partial q}\} - (\theta^p)q = 0 \quad (1)$$

The optimal strategy of the principal and the main results of this section are summarized in Proposition 6. Most importantly, result 6-iii suggests that endogenizing the communication noise creates an additional channel through which decentralization can dominate centralization when agents are highly motivated, which complements result 3-iii.

Proposition 6

i) When $q \leq \frac{2\theta}{\Delta W}$,

$$q^* = \begin{cases} 0 & \text{if } \frac{\theta}{\Delta W} \geq 4(\frac{\Delta u}{\theta^P}) \\ \frac{\theta}{\Delta W}(\frac{8}{3}) - \frac{\theta^2}{\Delta W^2}\frac{2\theta^P}{3\Delta u} & \text{if } \frac{\theta}{\Delta W}(\frac{8}{3}) - \frac{\theta^2}{\Delta W^2}\frac{2\theta^P}{3\Delta u} \in [0,1] \\ 1 & \text{if } 3\Delta W^2\Delta u + 2\theta^2\theta^P \leq 8\theta\Delta W\Delta u. \end{cases}$$

When $q > \frac{2\theta}{\Delta u}$, $q^* = \frac{2\Delta u}{\theta^P}$

ii) $\frac{\partial q^*}{\partial \theta^P} \leq 0$; $\frac{\partial q^*}{\partial \Delta u} \geq 0$

iii) There exists a $\frac{\tilde{\theta}}{\Delta \tilde{W}} \equiv \frac{2\Delta u}{\theta^P}$ such that:
$\frac{\partial q^*}{\partial(\frac{\theta}{\Delta W})} \leq 0$ for all $\frac{\tilde{\theta}}{\Delta \tilde{W}} \leq \frac{\theta}{\Delta W}$
$\frac{\partial q^*}{\partial(\frac{\theta}{\Delta W})} \geq 0$ for all $\frac{\tilde{\theta}}{\Delta \tilde{W}} \geq \frac{\theta}{\Delta W}$

The result that $\frac{\partial q^*}{\partial \theta^P} < 0$ is fairly straightforward: the harder it is for the principal to decrease the communication noise between himself and the agents, the more costly increasing q will become, resulting in an lesser q^*. Since a growing communication noise still favors decentralization, it is then obvious that a higher θ^P (a less competent principal) will strictly favor decentralization through a reduced q^*. As for $\frac{\partial q^*}{\partial \Delta u} > 0$, this comparative static is also straightforward: the more valuable the information is, the more resources the principal is willing to expand to ensure its appropriate use. This suggests that the more valuable a business report will be, the more time a manager will spend trying to decipher it.

In order to understand result 6-iii, a more thorough understanding of q^* is needed. I refer to the first term on the left of equation 1 as the communication benefit (the principal has a higher chance of receiving the agents' messages) and the second term as the productivity benefit (the agents are motivated to exert additional effort). While the communication benefit is always increasing in e_C^*, the productivity benefit may actually decrease when e_C^* is too high. This happens due the increased likelihood of one of the agents' messages being redundant (see Lemma 5). When further combined with the

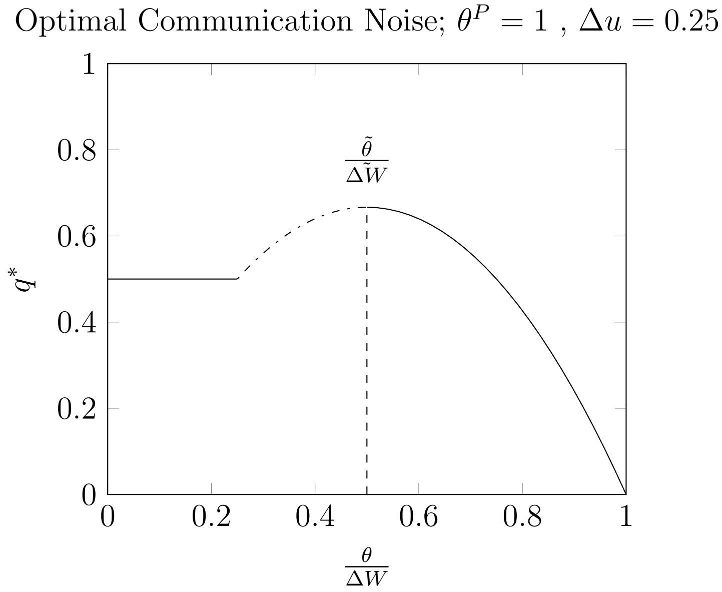

Figure 2

convex cost $s(q)$, this combination implies that the principal may actually find it optimal to reduce q^* when hard-working agents exert additional efforts.

Think of the ratio $\frac{\theta}{\Delta W}$ as the agents' unwillingness to work. An intuitive result would have been that the principal is always willing to spend more resources to match his agents' increasing willingness to work. This happens when $\frac{\theta}{\Delta W} \leq \frac{\tilde{\theta}}{\Delta \tilde{W}}$ (see the curve in figure 2 to the right of $\frac{\tilde{\theta}}{\Delta \tilde{W}}$). However, the above decomposition exposes a counterintuitive result. When the agents willingness to work is high enough (small $\frac{\theta}{\Delta W}$), the decreasing marginal returns of the productivity benefits and the costs of such a low level of communication noise (the high q that should be associated with such a large e_C^*) start to dominate the communication benefits. This creates a setting where the principal finds it optimal to spend less time/resources into communicating with highly motivated agents. This phenomenon is illustrated in figure 2 by the dashed part of the curve (the horizontal line represents an environment where $e_C^* = 1$). By creating a channel through which decentralization is more likely to dominate centralization when agents are highly motivated, result 6-iii can be seen as complementing result 3-iii: when agents are highly productive or very eager to work, decentralization is more likely to be preferred to centralization.

5 Conclusion

This paper studied the influence of tournaments and limited liability constraints on the optimal organizational structure. I showed that tournaments will harm cooperation between agents in a decentralized setting by inhibiting horizontal communication. Furthermore, I identified a fundamental trade-off between production noise, which harms decentralization, and communication noise, which harms centralization. I also provided a link between the optimal organizational structure and periods of economic expansion/contraction and argued that centralization is more likely to be optimal during recessions. I then showed that the introduction of a limited liability constraint will increase the complementarity between decentralization and the agents' productivity as well as reduce (maintain) the spread of prizes in a centralized (decentralized) setting. I finally showed that the principal's incentives to work very closely with his employee will be a function of their willingness to work, a function which will actually induce the principal to communicate less with highly motivated employees than with regular workers.

6 Appendix

6.1 Proofs of Lemmas 1-3

Proof of Lemma 1: In a decentralized setting, the maximization problem of the principal is

$$\max_e 2\Delta u(e^2) + [2e(1-e)](1+q)\Delta u + 2\hat{u} - \theta e^2$$

$$\text{FOC(e) } 4\Delta ue + (2-4e)(1+q)\Delta u - 2\theta e = 0$$

$$\Leftrightarrow 2\Delta u(1+q) = e(4q\Delta u + 2\theta)$$

$$\Leftrightarrow e = \frac{\Delta u(1+q)}{2q\Delta u + \theta}.$$

Putting this effort function into the expected decentralized profits, the principal would get:

$$\Pi_{FB}^{D} = \frac{\Delta u^2(1+q)^2}{(2q\Delta u+\theta)^2}(2\Delta u) + [\frac{2\Delta u(1+q)}{2q\Delta u+\theta} - \frac{2\Delta u^2(1+q)^2}{(2q\Delta u+\theta)^2}](1+q)\Delta u + 2\hat{u} - \theta\frac{\Delta u^2(1+q)^2}{(2q\Delta u+\theta)^2}.$$

$$\Leftrightarrow \Pi_{FB}^{D} = 2\hat{u} + \frac{2\Delta u^3(1+q)^2 + 4q\Delta u^3(1+q)^2 + 2\Delta u^2\theta(1+q)^2 - 2\Delta u^3(1+q)^3 - \Delta u^2\theta(1+q)^2}{(2q\Delta u+\theta)^2}$$

$$\Leftrightarrow \Pi_{FB}^{D} = \frac{2q\Delta u^3(1+q)^2 + \Delta u^2\theta(1+q)^2}{(2q\Delta u+\theta)^2} + 2\hat{u}$$

$$\Leftrightarrow \Pi_{FB}^{D} = \frac{\Delta u^2(1+q)^2}{2q\Delta u+\theta} + 2\hat{u}.$$

In a centralized setting, the maximization problem of the principal is

$$\max_{e} 2q\Delta u(2e - e^2) + 2\hat{u} - \theta e^2$$

$$\text{FOC(e) } 2q\Delta u(2-2e) = 2\theta e$$

$$\Leftrightarrow e = \frac{2q\Delta u}{2q\Delta u+\theta}.$$

Putting this effort function into the expected centralized profits, the principal would get:

$$\Pi_{FB}^{C} = 2q\Delta u[\frac{4q\Delta u}{2q\Delta u+\theta} - \frac{4q^2\Delta u^2}{(\theta+2q\Delta u)^2}] + 2\hat{u} - \frac{4q^2\Delta u^2\theta}{(\theta+2q\Delta u)^2}$$

$$\Leftrightarrow \Pi_{FB}^{C} = \frac{8q^2\Delta u^2\theta + 8q^3\Delta u^3 - 4q^2\Delta u^2\theta}{(2q\Delta u+\theta)^2} + 2\hat{u}$$

$$\Leftrightarrow \Pi_{FB}^{C} = \frac{4q^2\Delta u^2}{\theta+2q\Delta u} + 2\hat{u}.$$

By comparing Π_{FB}^{C} and Π_{FB}^{D}, it can be seen that

$$\Pi_{FB}^{C} \leq \Pi_{FB}^{D}$$

$$4q^2 \leq (1+q)^2$$

$$0 \leq 1 + 2q - 3q^2$$

which holds for all $q \in [0,1]$.

QED

Lemmas 2 through 3 are trivial optimization problems.

6.2 Proof of Proposition 1

Proof of Proposition 1: First, it must be observed that

$$\frac{4q^2\Delta u^2}{2q\Delta u + \theta} \geq \frac{\Delta u^2}{\theta}$$

$$\Leftrightarrow 4q^2\theta - 2q\Delta u - \theta \geq 0$$

$$\Leftrightarrow q^2 - (\frac{\Delta u}{2\theta})q - \frac{1}{4} \geq 0 \tag{2}$$

is a sufficient condition for centralization to dominate decentralization when $\Delta u \leq \theta$. Inequality 2 would bind if

$$q = \frac{1}{2}(\frac{\Delta u}{2\theta} \pm \sqrt{\frac{\Delta u^2}{4\theta^2} + 1})$$

$$\Leftrightarrow q = \frac{\Delta u}{4\theta} \pm \frac{\sqrt{\Delta u^2 + 4\theta^2}}{4\theta}.$$

Since $\frac{\Delta u}{4\theta} - \frac{\sqrt{\Delta u^2+4\theta^2}}{4\theta}$ is negative, I take $q^* \equiv \frac{\Delta u}{4\theta} + \frac{\sqrt{\Delta u^2+4\theta^2}}{4\theta}$. For any $q > q^*$, centralization would dominate decentralization for all $\Delta u \leq \theta$. This proves part i).

When $\Delta u \geq \theta$, the necessary and sufficient condition for centralization to dominate decentralization is

$$2\Delta u - \theta \leq \frac{4q^2\Delta u^2}{2q\Delta u + \theta}$$

$$\Leftrightarrow 0 \leq q^2 + q(\frac{2\Delta u\theta - 4\Delta u^2}{2\Delta u}) + (\frac{\theta^2 - 2\Delta u\theta}{4\Delta u^2}).$$

Solving the quadratic formula results in two possible values:

$$\frac{2\Delta u - \theta \pm \sqrt{(\theta + 2\Delta u)^2 - 4\theta^2}}{4\Delta u}.$$

The (-) value turns out to be negative:

$$\frac{2\Delta u - \theta - \sqrt{(\theta + 2\Delta u)^2 - 4\theta^2}}{4\Delta u} \overset{?}{>} 0$$

$$\Leftrightarrow 2\Delta u - \theta \overset{?}{>} \sqrt{(\theta + 2\Delta u)^2 - 4\theta^2}$$

$$\Leftrightarrow 4\Delta u^2 - 4\Delta u\theta + \theta^2 \overset{?}{>} \theta^2 + 4\Delta u\theta + 4\Delta u^2 - 4\theta^2$$

$$\Leftrightarrow \theta \overset{?}{>} 2\Delta u$$

which never holds when $\Delta u > \theta$. The (+) is the only remaining feasible value, so

$$q^{**} \equiv \frac{2\Delta u - \theta + \sqrt{(\theta + 2\Delta u)^2 - 4\theta^2}}{4\Delta u}.$$

Therefore, when $\Delta u \geq \theta$, for any $q \leq q^{**}$, decentralization dominates centralization and the proposition is proven.

QED

6.3 Proof of Lemma 4

In a decentralized organizational structure, agents communicate with each other and agent i has the authority over a_i, the output decision in dimension i. During the decentralized communication stage, if agent i learns what ω is, agent i has the choice between sharing this information with agent -i or keeping it to himself. In decentralization, there is no communication between the agents and the principal. In essence, the notation $m_i = 1$ indicates that agent i shared ω with the other agent and $m_i = 0$ otherwise.

In a decentralized setting, agent i is the winner if the output for which he is responsible is larger than the other one: $u_i(a_i, \omega) + \epsilon_i > u_{-i}(a_{-i}, \omega) + \epsilon_{-i}$. The agent solves the following problem in a decentralized setting

$$\max_{e_i \in [0,1], m_i \in \{0,1\}} E(V_i^D) = E[Q_i^D(e^D, m^D)]W_D^U + \{1 - E[Q_i^D(e^D, m^D)]\}W_D^L - \theta\frac{e_i^2}{2}$$

where $E[Q_i^D(e^D, m^D)]$ is the expected probability of winning the decentralized tournament and W_D^U and W_D^L are the prizes awarded to the winner and loser respectively. To simplify the notation, I will use $Q_i^D(e, m)$ instead of $Q_i^D(e^D, m^D)$. The probability of agent i winning is:

$$Q_i^D(e, m) = Prob[u_i(\omega, a_i) + \epsilon_i > u_{-i}(\omega, a_{-i}) + \epsilon_{-i}]$$

$$\Leftrightarrow Q_i^D(e, m) = Prob[u_i(\omega, a_i) - u_{-i}(\omega, a_{-i}) > \epsilon_{-i} - \epsilon_i].$$

Since $\epsilon_{-i} - \epsilon_i \sim U[-L, L]$, it can be seen that:

$$\Leftrightarrow Q_i^D(e, m) = \begin{cases} 0 & \text{if } u_i(\omega, a_i) - u_{-i}(\omega, a_{-i}) + L \leq 0 \\ \frac{u_i(\omega,a_i) - u_{-i}(\omega,a_{-i}) + L}{2L} & \text{if } -L \leq u_i(\omega, a_i) - u_{-i}(\omega, a_{-i}) \leq L \\ 1 & \text{if } u_i(\omega, a_i) - u_{-i}(\omega, a_{-i}) \geq L. \end{cases}$$

For expositional purposes, it will henceforth be implicitly assumed that probabilities are automatically set to 1 when the numerator exceeds the denominator and to 0 when the number is negative. A remainder is in order that $B_i^D(e, m, q)$ denotes the probability that agent i has learn ω given the effort and message vectors. This can be done through the information acquisition process or by receiving a message from the other agent.

The expected probability of agent i winning is therefore:

$$\begin{aligned} E(Q_i^D(e, m)) = {} & B_i^D(e, m, q)B_{-i}^D(e, m, q)(\frac{\bar{u} - \bar{u} + L}{2L}) \\ & + B_i^D(e, m, q)[1 - B_{-i}^D(e, m, q)](\frac{\bar{u} - \hat{u} + L}{2L}) \\ & + [1 - B_i^D(e, m, q)]B_{-i}^D(e, m, q)(\frac{\hat{u} - \bar{u} + L}{2L}) \\ & [1 - B_i^D(e, m, q)][1 - B_{-i}^D(e, m, q)](\frac{\hat{u} - \hat{u} + L}{2L}) \end{aligned}$$

and becomes, after some simplification,

$$\Leftrightarrow E[Q_i^D(e,m)] = \frac{[B_i^D(e,m,q) - B_{-i}^D(e,m,q)](\bar{u} - \hat{u}) + L}{2L}.$$

Since $B_i^D(e,m,q) = e_i + qm_{-i}[(1-e_i)e_{-i}]$, $B_i^D(e,m,q) - B_{-i}^D(e,m,q)$ can be rewritten as

$$B_i^D(e,m) - B_{-i}^D(e,m) = e_i - e_{-i} - q[m_i e_i(1-e_{-i}) + m_{-i}e_{-i}(1-e_i)]$$

which implies

$$E[Q_i^D(e,m)] = \frac{[e_i - e_{-i} - qm_i e_i(1-e_{-i}) + qm_{-i}e_{-i}(1-e_i)](\bar{u} - \hat{u}) + L}{2L}.$$

The agent's problem becomes

$$\max_{e_i\in[0,1],m_i\in\{0,1\}} \{\frac{[e_i - e_{-i} - qm_i e_i(1-e_{-i}) + qm_{-i}e_{-i}(1-e_i)]\Delta u + L}{2L}\}\Delta W^D + W_D^L - \theta\frac{e_i^2}{2}$$

where $\Delta W^D = W_D^U - W_D^L$ and $\Delta u = \bar{u} - \hat{u}$. Focusing on a symmetric equilibrium, the first order conditions to this problem are

$$\frac{\Delta W^D \Delta u}{2L\theta} = e_i = e_D^* \qquad m_i^* = m_D^* = 0.$$

The double derivative of $E(V_i^D)$ with respect to e_i is clearly negative so e_D^* is a global maximum[6]. Additionally, if $\Delta W^D \Delta u > 2L\theta$, then the agent simply chooses $e_D^* = 1$.

In a centralized organizational structure, agents communicate with the principal who subsequently makes both production decisions. Therefore, agents are no longer in competition for a prize with the receiver of their message and will share all information acquired in order to enhance their chances of winning the tournament. The communication decision of the agents in a centralized setting is trivial so I set $m_i^C = 1$ for both agents before

[6]In the cases where $L \to 0$ or $\theta \to 0$, the restriction $e_D^* \leq 1$ prevents the introduction of some nonconvexity in the maximand that would have disrupted the pure strategy solution (see Nalebuff and Stiglitz, 1983).

solving for the agents' effort function. Communication with the principal is subject to errors. With probability q, all messages sent by both agents are fully received by the principal. With probability 1 - q, none of the messages sent by the agents are received by the principal.

As it was explained in the main text, the centralized tournament rewards the agent who provided information about the state of nature ω. Assuming messages were received by the principal, if he receives ω by agent 1 and nothing by agent 2, then the winner of the tournament will be agent 1 and vice-versa. If both agents send the same amount of information, the winner is determined by the flip of a coin. If messages were not successfully received by the principal, the winner is also determined by the flip of a coin. The probabilities of both agents acquiring the same amount of information, of agent i acquiring more information than the other and of agent i acquiring less information are respectively

$$\bar{P}^C(e) = (1 - e_i)(1 - e_{-i}) + e_i e_{-i} = 1 - e_i - e_{-i} + 2e_i e_{-i}$$

$$P_i^C(e) = e_i(1 - e_{-i})$$

$$[1 - P_i^C(e) - \bar{P}^C(e)] = e_{-i}(1 - e_i).$$

The probability of agent i winning the tournament adds up to:

$$Q_i^C(e) = q(P_i^C + \frac{\bar{P}^C}{2}) + \frac{(1-q)}{2} = q[e_i - e_i e_{-i} + \frac{1}{2} - \frac{e_{-i}}{2} - \frac{e_i}{2} + e_i e_{-i}] + \frac{(1-q)}{2}$$

$$\Leftrightarrow Q_i^C(e) = q(\frac{e_i}{2} - \frac{e_{-i}}{2}) + \frac{1}{2}$$

Given all this, agent i's problem can be written as :

$$\max_{e_i \in [0,1]} \quad E(V_i^C) = Q_i^C(e)\Delta W^C + W_C^L - \theta \frac{e_i^2}{2}.$$

Solving this problem gives us the optimal effort function of both agents:

$$e_i = \frac{q\Delta W^C}{2\theta} = e_C^*.$$

QED

6.4 Proof of Proposition 2

In the centralized problem, it is obvious that the participation constraint should clearly hold with equality. Substituting it into the principal's objective function yields

$$\max_{\Delta W^C, W_C^L} \frac{2q^2\Delta u}{\theta}(\Delta W^C - \frac{q(\Delta W^C)^2}{4\theta}) + 2\hat{u} - \frac{q^2(\Delta W^C)^2}{4\theta}$$

with a first order condition of

$$\frac{2q^2\Delta u}{\theta}(1 - \frac{q\Delta W^C}{2\theta}) - \frac{q^2\Delta W^C}{2\theta} = 0. \qquad \text{FOC}(\Delta W^C)$$

$$\Leftrightarrow \Delta W^C = \frac{4\Delta u\theta}{2q\Delta u + \theta} \tag{3}$$

I can rewrite the participation constraint (which holds with equality) as

$$2W_C^L + \Delta W^C = \frac{q^2(\Delta W^C)^2}{4\theta}. \tag{4}$$

By plugging in 3 into 4, I would get

$$W_C^L = \frac{2\Delta u^2\theta q^2 - 4q\Delta u^2\theta - 2\Delta u\theta^2}{(2q\Delta u + \theta)^2} \tag{5}$$

which is clearly negative. This results in

$$W_C^U = \frac{2q^2\Delta u^2\theta + 2\Delta u\theta^2 + 4q\Delta u^2\theta}{(2q\Delta u + \theta)^2}.$$

These are the solutions for the centralized problem without limited liability constraint for any parameters which results in

$$\Pi_{noLL}^C = \frac{4q^2\Delta u^2}{2q\Delta u + \theta}.$$

In the decentralized problem, it is obvious that the participation constraint should hold with equality. After plugging the binding PC into the principal's objective function, the problem becomes

$$\max_{\Delta W^D, W_D^L} \frac{\Delta u^2 \Delta W^D}{L\theta} + 2\hat{u} - \frac{\Delta u^2(\Delta W^D)^2}{4L^2\theta}$$

which yields a first order condition of

$$\frac{\Delta u^2}{L\theta} = \frac{\Delta u^2 \Delta W^D}{2L^2\theta} \qquad \text{FOC}(\Delta W^D)$$

$$\Leftrightarrow \Delta W^D = 2L$$

By plugging $\Delta W^D = 2L$ into the binding participation constraint, I get

$$2W_D^L + 2L = \theta \frac{\Delta u^2}{4L^2\theta^2} 4L^2$$

$$\Leftrightarrow W_D^L = \frac{\Delta u^2 - 2L\theta}{2\theta}$$

which results in

$$W_D^U = \frac{\Delta u^2 + 2L\theta}{2\theta}.$$

This results in a profit function of

$$\Pi^D = \frac{\Delta u^2}{\theta} + 2\hat{u}$$

If $\Delta u > \theta$, the principal is faced with a situation where $\frac{\Delta u \Delta W^D}{2L\theta} > 1$ but $e_D^* = 1$. When $\Delta u > \theta$, it therefore becomes optimal for the principal to change the spread of prizes to $\Delta W^D = \frac{2L\theta}{\Delta u}$ so that $e_D^* = 1$ and maintain a binding participation constraint

$$\frac{1}{2}(W_D^U + W_D^L) = \frac{\theta}{2}(\frac{\Delta u \Delta W^D}{2L\theta})^2$$

$$\Leftrightarrow 2W^L + \Delta W^D = \theta(\frac{\Delta u \Delta W^D}{2L\theta})^2$$

$$\Leftrightarrow W_D^L = \frac{\theta(\Delta u - 2L)}{2\Delta u}$$

which implies $W_D^U = \frac{\theta(\Delta u + 2L)}{2\Delta u}$. The profits become

$$\Pi^D = 2\Delta u + 2\hat{u} - \theta$$

which is larger than $2\Delta u + 2\hat{u} - \frac{\Delta u^2}{\theta}$ when $\Delta u > \theta$. This results in a decentralized profit function without a limited liability constraint of

$$\Pi^D_{no\ LL} = \begin{cases} \frac{\Delta u^2}{\theta} + 2\hat{u} & \text{if } \Delta u \leq \theta \\ 2\Delta u + 2\hat{u} - \theta & \text{if } \Delta u \geq \theta. \end{cases}$$

A straightforward comparison of the profit functions in the Constrained First Best setting and the profit functions just computed prove proposition 2.

QED

6.5 Centralized Lagrangian with Limited Liability Constraint

Step 1: To be able to use the Kuhn-Tucker conditions, I must simply show that these three conditions hold.

Step 1-a: First, I must show that $f(W) = 2q\Delta u[\frac{q\Delta W^C}{\theta} - \frac{q^2(\Delta W^C)^2}{4\theta^2}] + 2\hat{u} - W_C^U - W_C^L$ is concave. For W_C^U, I have to show:

$$\frac{2q^2\Delta u}{\theta}[\lambda W_C^U + (1-\lambda)\hat{W}_C^U - W_C^L - \frac{q[\lambda W_C^U + (1-\lambda)\hat{W}_C^U - W_C^L]^2}{4\theta}]$$
$$+ 2\hat{u} - \lambda W_C^U - (1-\lambda)\hat{W}_C^U - W_C^L$$
$$\overset{?}{\geq} \lambda(\frac{2q^2\Delta u}{\theta})[W_C^U - W_C^L - \frac{q(W_C^U - W_C^L)^2}{4\theta}] + \lambda(2\hat{u} - W_C^U - W_C^L)$$
$$+ (1-\lambda)(\frac{2q^2\Delta u}{\theta})[\hat{W}_C^U - W_C^L - \frac{q(\hat{W}_C^U - W_C^L)^2}{4\theta}] + (1-\lambda)(2\hat{u} - \hat{W}_C^U - W_C^L)$$

$$\Leftrightarrow -[\lambda^2(W_C^U)^2 + \lambda(1-\lambda)W_C^U\hat{W}_C^U - \lambda W_C^U W_C^L + \lambda(1-\lambda)W_C^U\hat{W}_C^U + (1-\lambda)^2(\hat{W}_C^U)^2$$
$$- (1-\lambda)\hat{W}_C^U W_C^L - \lambda W_C^U W_C^L - (1-\lambda)\hat{W}_C^U W_C^L + (W_C^L)^2] \overset{?}{\geq}$$
$$-[\lambda(W_C^U)^2 - 2\lambda W_C^U W_C^L + \lambda(W_C^L)^2] - [(1-\lambda)(\hat{W}_C^U)^2 - 2(1-\lambda)\lambda\hat{W}_C^U W_C^L + (1-\lambda)(W_C^L)^2]$$

$$\Leftrightarrow \lambda(W_C^U)^2(\lambda - 1) + 2\lambda(1-\lambda)W_C^U\hat{W}_C^U + (1-\lambda)(\hat{W}_C^U)^2(-\lambda) \overset{?}{\leq} 0$$

$$\Leftrightarrow 0 \overset{?}{\leq} (\hat{W}_C^U - W_C^U)^2$$

which always hold. As for W_C^L, I have to show

$$\frac{2q^2\Delta u}{\theta}[W_C^U - \lambda W_C^L - (1-\lambda)\hat{W}_C^L - \frac{q[W_C^U - \lambda W_C^L - (1-\lambda)\hat{W}_C^L]^2}{4\theta}]$$
$$+ 2\hat{u} - W_C^U - \lambda W_C^L - (1-\lambda)\hat{W}_C^L$$
$$\overset{?}{\geq} \lambda(\frac{2q^2\Delta u}{\theta})[W_C^U - W_C^L - \frac{q(W_C^U - W_C^L)^2}{4\theta}] + \lambda(2\hat{u} - W_C^U - W_C^L)$$
$$+ (1-\lambda)(\frac{2q^2\Delta u}{\theta})[W_C^U - \hat{W}_C^L - \frac{q(W_C^U - \hat{W}_C^L)^2}{4\theta}] + (1-\lambda)(2\hat{u} - W_C^U - \hat{W}_C^L)$$

$$\Leftrightarrow -[(W_C^U)^2 - \lambda W_C^U W_C^L - (1-\lambda)W_C^U\hat{W}_C^L - \lambda W_C^U W_C^L + \lambda^2(W_C^L)^2$$
$$\lambda(1-\lambda)W_C^L\hat{W}_C^L - (1-\lambda)W_C^U\hat{W}_C^L + \lambda(1-\lambda)W_C^L\hat{W}_C^L + (1-\lambda)^2(\hat{W}_C^L)^2]$$
$$\overset{?}{\geq} -\lambda[(W_C^U)^2 - 2W_C^U W_C^L + (W_C^L)^2] - (1-\lambda)[(W_C^U)^2 - 2W_C^U\hat{W}_C^L + (\hat{W}_C^L)^2]$$

which simplifies into

$$\Leftrightarrow 0 \overset{?}{\leq} (W_C^L - \hat{W}_C^L)^2$$

which always hold. This means $2q\Delta u[\frac{q\Delta W^C}{\theta} - \frac{q^2(\Delta W^C)^2}{4\theta^2}] + 2\hat{u} - W_C^U - W_C^L$ is concave in both of its arguments.

Step 1-b:

Now, I define the PC constraint as $g(W) = \frac{q^2(W_C^U - W_C^L)^2}{4\theta} - W_C^U - W_C^L$ with $W = (W_C^L, W_C^U)$. I now show that g(W) is convex in W_C^U:

$$\frac{q^2[\lambda W_C^U + (1-\lambda)\hat{W}_C^U - W_C^L]^2}{4\theta} - \lambda W_C^U - (1-\lambda)\hat{W}_C^U - W_C^L \overset{?}{\leq}$$
$$\frac{\lambda q^2(W_C^U - W_C^L)^2}{4\theta} - \lambda W_C^U - \lambda W_C^L + \frac{(1-\lambda)q^2(\hat{W}_C^U - W_C^L)^2}{4\theta} - (1-\lambda)\hat{W}_C^U - (1-\lambda)W_C^L$$

$$\Leftrightarrow \lambda^2(W_C^U)^2 + \lambda(1-\lambda)W_C^U\hat{W}_C^U - \lambda W_C^U W_C^L + \lambda(1-\lambda)W_C^U\hat{W}_C^U + (1-\lambda)^2(\hat{W}_C^U)^2$$
$$-(1-\lambda)\hat{W}_C^U W_C^L - \lambda W_C^U W_C^L - (1-\lambda)\hat{W}_C^U W_C^L + (W_C^L)^2 \overset{?}{\leq} \lambda(W_C^U)^2 - 2\lambda W_C^U W_C^L$$
$$+ \lambda(W_C^L)^2 + (1-\lambda)(\hat{W}_C^U)^2 - 2(1-\lambda)\hat{W}_C^U W_C^L + (1-\lambda)(W_C^L)^2$$

$$\Leftrightarrow 0 \overset{?}{\leq} (W_C^U - \hat{W}_C^U)^2$$

which always holds. I now have to show that g(W) is convex in W_C^L:

$$\frac{q^2[W_C^U - \lambda W_C^L - (1-\lambda)(\hat{W}_C^L)]^2}{4\theta} - W_C^U - \lambda W_C^L - (1-\lambda)\hat{W}_C^L \overset{?}{\leq} \frac{\lambda q^2 (W_C^U - W_C^L)^2}{4\theta}$$
$$- \lambda W_C^U - \lambda W_C^L + \frac{(1-\lambda)q^2(W_C^U - \hat{W}_C^L)^2}{4\theta} - (1-\lambda)W_C^U - (1-\lambda)\hat{W}_C^L$$

$$\Leftrightarrow (W_C^U)^2 - \lambda W_C^U W_C^L - (1-\lambda)W_C^U \hat{W}_C^L - \lambda W_C^U W_C^L + \lambda^2 (W_C^L)^2 + \lambda(1-\lambda) W_C^L \hat{W}_C^L$$
$$- (1-\lambda) W_C^U \hat{W}_C^L + \lambda(1-\lambda) W_C^L \hat{W}_C^L + (1-\lambda)^2 (\hat{W}_C^L)^2 \overset{?}{\leq} \lambda (W_C^U)^2 - 2\lambda W_C^U W_C^L$$
$$+ \lambda (W_C^L)^2 + (1-\lambda)(W_C^U)^2 - 2(1-\lambda) W_C^U \hat{W}_C^L + (1-\lambda)(\hat{W}_C^L)^2$$

$$\Leftrightarrow 0 \overset{?}{\leq} (W_C^L - \hat{W}_C^L)^2$$

which always holds. Therefore, the PC constraint $g(W) = \frac{q^2(W_C^U - W_C^L)^2}{4\theta} - W_C^U - W_C^L$ is convex. The remaining limited liability constraint (LL) is linear and therefore convex.

Step 1-c:

Thirdly, since the constraints of centralized problem are convex, we also need to ensure the existence of a set of parameters such that

$$W_C^U + W_C^L > \frac{q^2}{4\theta}(W_C^U - W_C^L)^2$$

and

$$W_C^L > 0$$

hold in order to use the Kuhn-Tucker conditions. Since I have not impose any restrictions on the parameters such that the above inequalities cannot hold, the third conditions is satisfied.

Step 2: I can now justify using the Kuhn-Tucker conditions to solve for the centralized problem with the limited liability constraint.

The Lagrangian of this problem is

$$\mathcal{L} = \frac{2q^2\Delta u}{\theta}[W_C^U - W_C^L - \frac{q}{4\theta}(W_C^U - W_C^L)^2] + 2\hat{u} - W_C^U - W_C^L$$
$$- \lambda[\frac{q^2(W_C^U - W_C^L)^2}{4\theta} - W_C^U - W_C^L] + \mu W_C^L.$$

The Kuhn-Tucker conditions are

$$\frac{\partial \mathcal{L}}{\partial W_C^U} = 0$$

$$\Leftrightarrow \frac{2q^2\Delta u}{\theta}[1 - \frac{q}{2\theta}(W_C^U - W_C^L)] - 1 - \lambda[\frac{q^2}{2\theta}(W_C^U - W_C^L) - 1] = 0 \quad (6)$$

$$\frac{\partial \mathcal{L}}{\partial W_C^L} = 0$$

$$\Leftrightarrow \frac{2q^2\Delta u}{\theta}[-1 + \frac{q}{2\theta}(W_C^U - W_C^L)] - 1 - \lambda[-\frac{q^2}{2\theta}(W_C^U - W_C^L) - 1] + \mu = 0 \quad (7)$$

$$\lambda[\frac{q^2(W_C^U - W_C^L)^2}{4\theta} - W_C^U - W_C^L] = 0 \quad (8)$$

$$\mu W_C^L = 0 \quad (9)$$

as well as $W_C^U + W_C^L \geq \frac{q^2(W_C^U - W_C^L)^2}{4\theta}$, $W_C^L \geq 0$, $\lambda \geq 0$ and $\mu \geq 0$.

Case $\lambda = 0$: By 6, this yields

$$\frac{2q^2\Delta u}{\theta}[1 - \frac{q}{2\theta}(W_C^U - W_C^L)] = 1$$

$$\Leftrightarrow \frac{\theta(2q^2\Delta u - \theta)}{q^3\Delta u} = \Delta W^C. \quad (10)$$

I then put 10 into 7 to get

$$\frac{2q^2\Delta u}{\theta}[\frac{q}{2\theta}\frac{\theta(2q^2\Delta u - \theta)}{q^3\Delta u} - 1] - 1 + \mu = 0$$

which simplifies into

$$\Leftrightarrow \mu = 2.$$

The fact that $\mu > 0$ implies $W_C^L = 0$ by 9 which then implies by 10

$$W_C^U = \frac{\theta(2q^2\Delta u - \theta)}{q^3\Delta u}.$$

So one possible **solution** is $W_C^U = \frac{\theta(2q^2\Delta u-\theta)}{q^3\Delta u}$, $W_C^L = 0$, $\lambda = 0$ and $\mu = 2$. Obviously, this solution only holds when $2q^2\Delta u > \theta$ since $W_C^U \geq 0$ is a necessary condition for PC to hold when $W_C^L = 0$.

By looking at the Kuhn-Tucker conditions, it can be observed that $W_C^U = W_C^L = 0$ is a possible solution. Indeed, if $W_C^U = W_C^L = 0$, then 6 implies

$$\frac{2q^2\Delta u}{\theta} - 1 + \lambda = 0$$

$$\lambda = \frac{\theta - 2q^2\Delta u}{\theta}.$$

The Kuhn-Tucker condition $\lambda \geq 0$ requires $\theta \geq 2q^2\Delta u$, which complements the previous solution. Finally, 7 implies

$$\frac{-2q^2\Delta u}{\theta} - 1 + (\frac{\theta - 2q^2\Delta u}{\theta}) + \mu = 0$$

$$\mu = \frac{4q^2\Delta u}{\theta}.$$

So another **solution** is $W_C^L = 0$, $W_C^U = 0$, $\lambda = \frac{\theta-2q^2\Delta u}{\theta}$ and $\mu = \frac{4q^2\Delta u}{\theta}$.

<u>Case $\lambda > 0$</u>: By condition 8, this implies

$$W_C^U + W_C^L = \frac{q^2}{4\theta}(W_C^U)^2 - \frac{q^2}{2\theta}W_C^U W_C^L + \frac{q^2}{4\theta}(W_C^L)^2$$

$$\Leftrightarrow 0 = (W_C^U)^2 - [\frac{2(2\theta + q^2 W_C^L)}{q^2}]W_C^U + (W_C^L)^2 - \frac{4\theta}{q^2}W_C^L.$$

This yields

$$W_C^U = \frac{2q^2 W_C^L + 4\theta}{2q^2} \pm \frac{1}{2}\sqrt{\frac{4q^4(W_C^L)^2 + 16q^2\theta W_C^L + 16\theta^2}{q^4} - 4(W_C^L)^2 + \frac{16\theta W_C^L}{q^2}}$$

$$\Leftrightarrow W_C^U = \frac{2q^2W_C^L + 4\theta}{2q^2} \pm \frac{1}{2}\sqrt{\frac{32q^2\theta W_C^L + 16\theta^2}{q^4}}$$

$$\Leftrightarrow W_C^U = W_C^L + \frac{2\theta}{q^2} \pm \frac{2}{q^2}\sqrt{\theta(2q^2W_C^L + \theta)}. \tag{11}$$

Subcase $\mu = 0$:

Condition 6 implies

$$\frac{2q^2\Delta u\theta - q^3\Delta W^C\Delta u - \theta^2}{\theta^2} = \lambda(\frac{q^2\Delta W^C - 2\theta}{2\theta})$$

$$\Leftrightarrow \lambda = \frac{2(2q^2\Delta u\theta - q^3\Delta W^C\Delta u - \theta^2)}{\theta(q^2\Delta W^C - 2\theta)}$$

and condition 7 implies

$$\lambda(\frac{q^2\Delta W^C + 2\theta}{2\theta}) = \frac{2q^2\theta\Delta u - q^3\Delta u\Delta W^C + \theta^2}{\theta^2}$$

$$\Leftrightarrow \lambda = \frac{2(2q^2\Delta u\theta - q^3\Delta W^C\Delta u + \theta^2)}{\theta(q^2\Delta W^C + 2\theta)}.$$

Combining both modified conditions, I get

$$\frac{2(2q^2\Delta u\theta - q^3\Delta W^C\Delta u - \theta^2)}{\theta(q^2\Delta W^C - 2\theta)} = \frac{2(2q^2\Delta u\theta - q^3\Delta W^C\Delta u + \theta^2)}{\theta(q^2\Delta W^C + 2\theta)}$$

$$\Leftrightarrow 4\theta(2q^2\Delta u\theta - q^3\Delta u\Delta W^C) = 2q^2\theta^2\Delta W^C$$

$$\Leftrightarrow W_C^U = W_C^L + \frac{4\Delta u\theta}{\theta + 2q\Delta u}. \tag{12}$$

When I combine 11(+) and 12, I get

$$\frac{4\Delta u\theta}{\theta + 2q\Delta u} = \frac{2\theta}{q^2} + \frac{2}{q^2}\sqrt{\theta(2q^2W_C^L + \theta)}$$

$$\Leftrightarrow \frac{2q^2\Delta u\theta}{\theta + 2q\Delta u} - \theta = \sqrt{\theta(2q^2W_C^L + \theta)}$$

$$\Leftrightarrow \frac{4q^4\Delta u^2\theta^2}{(\theta + 2q\Delta u)^2} - \frac{4q^2\Delta u\theta^2}{\theta + 2q\Delta u} + \theta^2 = \theta(2q^2W_C^L + \theta)$$

$$\Leftrightarrow W_C^L = \frac{2q^2\Delta u^2\theta}{(\theta + 2q\Delta u)^2} - \frac{2\Delta u\theta}{\theta + 2q\Delta u}.$$

However, I need $W_C^L \geq 0$, which is equivalent to

$$2q^2\Delta u^2\theta \geq 2\Delta u\theta(\theta + 2q\Delta u)$$

$$\Leftrightarrow q\Delta u(q-2) \geq \theta$$

which does not hold. Repeating the procedure with 11 (-) yields the same outcome. Therefore, a solution with $\lambda > 0$ and $\mu = 0$ is not feasible.

Subcase $\mu > 0$

This implies $W_C^L = 0$. By 8, it also implies $W_C^U = \frac{4\theta}{q^2}$ or $W_C^U = 0$. The case for $W_C^U = W_C^L = 0$ has already been study so it will be ignored.

From 6, this implies

$$\frac{2q^2\Delta u}{\theta} - \frac{q^3\Delta u}{\theta^2}\frac{4\theta}{q^2} - 1 - \lambda = 0$$

$$\Leftrightarrow \lambda = \frac{2q\Delta u(q-2)}{\theta} - 1$$

which is a contradiction with $\lambda \geq 0$. Therefore, a solution with $\lambda \geq 0$ and $\mu \geq 0$ is not feasible.

6.6 Decentralized Lagrangian with a Limited Liability Constraint

Step 1: Once again, to use the Kuhn-Tucker conditions, I must show that three conditions hold. First, the objective function has to be concave. Since $\frac{\Delta u^2 \Delta W^D}{L\theta} + 2\hat{u} - W_D^U - W_D^L$ is linear, it is also concave. Second, the constraint $\frac{\Delta u(\Delta W^D)^2}{4L^2\theta} - W_D^U - W_D^L$ is convex in W_D^U if and only if

$$\frac{\Delta u^2}{4L^2\theta}[\lambda W_D^U + (1-\lambda)\hat{W}_D^U - W_D^L]^2 - \lambda W_D^U - (1-\lambda)\hat{W}_D^U - W_D^L$$
$$\overset{?}{\leq} \frac{\lambda\Delta u^2(W_D^U - W_D^L)^2}{4L^2\theta} - \lambda W_D^U - \lambda W_D^L + \frac{(1-\lambda)\Delta u^2(\hat{W}_D^U - W_D^L)^2}{4L^2\theta} - (1-\lambda)\hat{W}_D^U - (1-\lambda)W_D^L \qquad \lambda \in [0,1]$$

$$\Leftrightarrow 0 \overset{?}{\leq} \lambda(1-\lambda)(W_D^U)^2 + \lambda(1-\lambda)(\hat{W}_D^U)^2 - 2\lambda(1-\lambda)W_D^U\hat{W}_D^U$$

$$\Leftrightarrow 0 \overset{?}{\leq} (W_D^U - \hat{W}_D^U)^2$$

which always hold. It also needs to be convex in W_D^L:

$$\frac{\Delta u^2}{4L^2\theta}[W_D^U - \lambda W_D^L - (1-\lambda)\hat{W}_D^U]^2 - W_D^U - \lambda W_D^L - (1-\lambda)\hat{W}_D^U$$
$$\overset{?}{\leq} \frac{\lambda\Delta u^2(W_D^U - W_D^L)^2}{4L^2\theta} - \lambda W_D^U - \lambda W_D^L + \frac{(1-\lambda)\Delta u^2(W_D^U - \hat{W}_D^U)^2}{4L^2\theta} - (1-\lambda)W_D^U$$
$$- (1-\lambda)\hat{W}_D^U \qquad \lambda \in [0,1]$$

$$\Leftrightarrow 0 \overset{?}{\leq} \lambda(1-\lambda)(W_D^L)^2 + \lambda(1-\lambda)(\hat{W}_D^U)^2 - 2\lambda(1-\lambda)W_D^L\hat{W}_D^U$$

$$\Leftrightarrow 0 \overset{?}{\leq} (W_D^L - \hat{W}_D^U)^2$$

which always hold. Finally, I need to ensure the existence of a set of parameters such that

$$0 > \frac{\Delta u^2(\Delta W^D)^2}{4L^2\theta} - W_D^U - W_D^L$$

and $W_D^L > 0$ hold. Since I have not imposed any restrictions on these parameters, then the above inequalities can hold.

Step 2: The Lagrangian of this problem is

$$\mathcal{L} = \frac{\Delta u^2 \Delta W^D}{L\theta} + 2\hat{u} - W_D^U - W_D^L - \lambda(\frac{\Delta u^2(\Delta W^D)^2}{4L^2\theta} - W_D^U - W_D^L) + \mu W_D^L.$$

I can then use the Kuhn-Tucker conditions

$$\frac{\partial \mathcal{L}}{\partial W_D^U} = 0$$

$$\Leftrightarrow \frac{\Delta u^2}{L\theta} - 1 - \lambda(\frac{\Delta u^2 \Delta W^D}{2L^2\theta} - 1) = 0 \tag{13}$$

$$\frac{\partial \mathcal{L}}{\partial W_D^L} = 0$$

$$\Leftrightarrow -\frac{\Delta u^2}{L\theta} - 1 - \lambda(-\frac{\Delta u^2 \Delta W^D}{2L^2\theta} - 1) + \mu = 0 \quad (14)$$

$$\lambda[\frac{\Delta u^2(\Delta W^D)^2}{4L^2\theta} - W_D^U - W_D^L] = 0 \quad (15)$$

$$\mu W_D^L = 0 \quad (16)$$

as well as $\lambda \geq 0$, $\mu \geq 0$, $W_D^U + W_D^L \geq \frac{\Delta u^2(\Delta W^D)^2}{4L^2\theta}$ and $W_D^L \geq 0$ to find a solution to the decentralized problem.

Case $\lambda = 0$: It can be seen that 13 implies $\frac{\Delta u^2}{L\theta} - 1 = 0$. This clearly indicates the presence of corner solutions. If $\Delta u \geq \sqrt{L\theta}$, then W_D^U should be as high as possible. If $\Delta u \leq \sqrt{L\theta}$, then W_D^U should be zero.

Subcase $\mu = 0$: By 14, this implies $-\frac{\Delta u^2}{L\theta} - 1$ which suggests W_D^L should be as low as possible.

Since W_D^U needs to be at a maximum and W_D^L at a minimum, I will set ΔW^D such that $e_D^* = 1$. This yields $\Delta W^D = \frac{2L\theta}{\Delta u}$. The minimum value W_D^L can have is provided by the participation constraint:

$$W_D^L + W_D^U \geq \frac{\Delta u^2}{4L^2\theta}(\Delta W^D)^2$$

$$\Leftrightarrow 2W_D^L + \Delta W^D \geq \frac{\Delta u^2}{4L^2\theta}(\Delta W^D)^2$$

$$\Leftrightarrow 2W_D^L + \frac{2L\theta}{\Delta u} \geq \frac{\Delta u^2}{4L^2\theta}(\frac{2L\theta}{\Delta u})^2$$

$$\Leftrightarrow W_D^L \geq \frac{\theta(\Delta u - 2L)}{2\Delta u}.$$

Given the corner solution of 14 when $\lambda = \mu = 0$, it can be concluded that $W_D^L = \frac{\theta(\Delta u - 2L)}{2\Delta u}$. Along with $\Delta W^D = \frac{2L\theta}{\Delta u}$, this implies $W_D^U = \frac{\theta(\Delta u + 2L)}{2\Delta u}$. Since the constraint $W_D^L \geq 0$ always has to hold, this solution is only possible if $\Delta u \geq 2L$ and $\Delta u \geq \sqrt{L\theta}$.

A possible **solution** is therefore $\lambda = \mu = 0$, $W_D^L = \frac{\theta(\Delta u - 2L)}{2\Delta u}$ and $W_D^U = \frac{\theta(\Delta u + 2L)}{2\Delta u}$ when $\Delta u \geq 2L$. But since I have assumed $\Delta u \leq L$ as a parameter restriction, this solution is not feasible.

Subcase $\mu > 0$: By 14, this implies that $\mu = 1 + \frac{\Delta u^2}{L\theta}$. Since $\mu > 0$, 16 requires $W_D^L = 0$. This implies $W_D^U = \Delta W^D$. If $\Delta u \geq \sqrt{L\theta}$, then $W_D^U = \Delta W^D$ needs to be as high as possible. Since the benefits of ΔW^D comes from motivating the agent, the upper boundary for ΔW^D is set by $e_D^* = 1$ which results in $W_D^U = \frac{2L\theta}{\Delta u}$.

Since the participation constraint always has to hold,

$$\frac{2L\theta}{\Delta u} \geq \frac{\Delta u^2}{4L^2\theta}(\frac{4L^2\theta^2}{\Delta u^2})$$

$$\Leftrightarrow 2L \geq \Delta u$$

has to hold. Therefore, a another possible **solution** is $\lambda = 0$, $\mu = 1 + \frac{\Delta u^2}{L\theta}$, $W_D^L = 0$ and $W_D^U = \frac{2L\theta}{\Delta u}$ if $\Delta u \in [\sqrt{L\theta}, 2L]$.

<u>Case $\lambda > 0$</u>: This case and 15 implies $\frac{\Delta u^2(\Delta W^D)^2}{4L^2\theta} = W_D^U + W_D^L$.

Subcase $\mu > 0$: By 16, this implies $W_D^L = 0$ and, by 15, $W_D^U = \frac{4L^2\theta}{\Delta u^2}$.

By 13, this implies

$$\frac{\Delta u^2 - L\theta}{L\theta} = \lambda.$$

By 14, this implies

$$\mu + \lambda 3 = \frac{\Delta u^2}{L\theta}.$$

This implies

$$\mu = \frac{2(2L\theta - \Delta u^2)}{L\theta}.$$

In order for the conditions $\lambda \geq 0$ and $\mu \geq 0$ to be satisfied simultaneously, $\Delta \in [\sqrt{L\theta}, \sqrt{2L\theta}]$ is required. Therefore, it can be seen that, if $\Delta u^2 \in [L\theta, 2L\theta]$, a **solution** is $W_D^U = \frac{4L^2\theta}{\Delta u^2}$, $W_D^L = 0$, $\lambda = \frac{\Delta u^2 - L\theta}{L\theta}$ and $\mu = \frac{2(2L\theta - \Delta u^2)}{L\theta}$.

Subcase $\mu = 0$: This implies $W_D^L \geq 0$.

If $\mu = 0$ and $W_D^L = 0$, then $W_D^U = \frac{4L^2\theta}{\Delta u^2}$. By 13 and 14, $\Delta u = \sqrt{2L\theta}$. But this is simply part of the previous solution.

If $\mu = 0$ and $W_D^L > 0$, then 15 implies

$$\frac{\Delta u^2[(W_D^U)^2 - 2W_D^U W_D^L + (W_D^L)^2]}{4L^2\theta} = W_D^U + W_D^L$$

$$\Leftrightarrow \Delta u^2(W_D^U)^2 - 2\Delta u^2 W_D^U W_D^L + \Delta u^2(W_D^L)^2 = 4L^2\theta W_D^U + 4L^2\theta W_D^L$$

$$\Leftrightarrow (W_D^U)^2 - W_D^U(\frac{2\Delta u^2 W_D^L + 4L^2\theta}{\Delta u^2}) + (W_D^L)^2 - \frac{4L^2\theta}{\Delta u^2}W_D^L = 0$$

This yields

$$W_D^U = \frac{2\Delta u^2 W_D^L + 4L^2\theta}{2\Delta u^2} \pm \frac{1}{2}\sqrt{\frac{4\Delta u^4(W_D^L)^2 + 16\Delta u^2 L^2\theta W_D^L + 16L^4\theta^2}{\Delta u^4} - 4(W_D^L)^2 + \frac{16L^2\theta W_D^L}{\Delta u^2}}$$

$$\Leftrightarrow W_D^U = \frac{2\Delta u^2 W_D^L + 4L^2\theta}{2\Delta u^2} \pm \frac{\sqrt{32\Delta u^2 L^2\theta W_D^L + 16L^4\theta^2}}{2\Delta u^2}$$

$$\Leftrightarrow W_D^U = \frac{2\Delta u^2 W_D^L + 4L^2\theta}{2\Delta u^2} \pm 4L\frac{\sqrt{2\Delta u^2\theta W_D^L + L^2\theta^2}}{2\Delta u^2}. \tag{17}$$

Also, 13 implies

$$\frac{\Delta u^2}{L\theta} - 1 - \lambda(\frac{\Delta u^2\Delta W^D - 2L^2\theta}{2L^2\theta}) = 0$$

$$\Leftrightarrow \lambda = \frac{2L(\Delta u^2 - L\theta)}{\Delta u^2\Delta W^D - 2L^2\theta} \tag{18}$$

and 14 implies

$$\lambda(\frac{\Delta u^2\Delta W^D + 2L^2\theta}{2L^2\theta}) = \frac{\Delta u^2}{L\theta} + 1$$

$$\Leftrightarrow \lambda = \frac{2L(\Delta u^2 + L\theta)}{\Delta u^2\Delta W^D + 2L^2\theta}. \tag{19}$$

Equations 18 and 19 imply

$$\frac{2L(\Delta u^2 + L\theta)}{\Delta u^2\Delta W^D + 2L^2\theta} = \frac{2L(\Delta u^2 - L\theta)}{\Delta u^2\Delta W^D - 2L^2\theta}$$

$$\Leftrightarrow \Delta u^4 \Delta W^D - \Delta u^2 2L^2\theta + L\theta \Delta u^2 \Delta W^D - 2L^3\theta^2$$
$$= \Delta u^4 \Delta W^D + \Delta u^2 2L^2\theta - L\theta \Delta u^2 \Delta W^D - 2L^3\theta^2$$

$$\Leftrightarrow 2L\theta \Delta u^2 \Delta W^D = 4L^2\theta \Delta u^2$$

$$\Leftrightarrow \Delta W^D = 2L$$

$$W_D^U = W_D^L + 2L. \tag{20}$$

I can then combine equation 17 (with +) and 20 to get

$$W_D^L = W_D^L + \frac{2L^2\theta}{\Delta u^2} + \frac{2L\sqrt{2\Delta u^2 \theta W_D^L + L^2\theta^2}}{\Delta u^2} - 2L$$

$$\Leftrightarrow (\Delta u^2 - L\theta)^2 = 2\Delta u^2 \theta W_D^L + L^2\theta^2$$

$$\Leftrightarrow W_D^L = \frac{\Delta u^2 - 2L\theta}{2\theta}$$

which implies

$$W_D^U = \frac{\Delta u^2 + 2L\theta}{2\theta}.$$

These prizes lead to

$$\lambda = \frac{(\Delta u^2 + L\theta)2L}{\Delta u^2 2L + 2L^2\theta} \Leftrightarrow \lambda = 1.$$

It can then be verified that a combination of 17 (with -) and 20 yield the same solution. If $\Delta u \geq \sqrt{2L\theta}$, then the **solution** is $W_D^L = \frac{\Delta u^2 - 2L\theta}{2\theta}$, $W_D^U = \frac{\Delta u^2 + 2L\theta}{2\theta}$, $\mu = 0$ and $\lambda = 1$.

It can also be observed that $W_D^L = W_D^U = 0$ satisfies both 15 and 16. $W_D^U = W_D^L = 0$ and 13 implies

$$\frac{\Delta u^2}{L\theta} - 1 + \lambda = 0 \Leftrightarrow \lambda = \frac{L\theta - \Delta u^2}{L\theta}.$$

By 14, it can also be seen that

$$-\frac{\Delta u^2}{L\theta} - 1 + (\frac{L\theta - \Delta u^2}{L\theta}) + \mu = 0$$

$$\Leftrightarrow \mu = \frac{2\Delta u^2}{L\theta}.$$

Therefore, if $\Delta u \leq \sqrt{L\theta}$, then $W_D^L = W_D^U = 0$, $\mu = \frac{2\Delta u^2}{L\theta}$ and $\lambda = \frac{L\theta - \Delta u^2}{L\theta}$ is a **solution**.

The following Lemmas are necessary in order to decipher which solution dominates for a specific set of parameters.

Lemma 6: The set of prizes $W_D^L = 0$ and $W_D^U = \frac{2L\theta}{\Delta u}$ is more profitable than the set of prizes $W_D^L = 0$ and $W_D^U = \frac{4L^2\theta}{\Delta u^2}$ if $\Delta u \geq \sqrt{L\theta}$.

First, it must be notice that the set of prizes $W_D^L = 0$ and $W_D^U = \frac{4L^2\theta}{\Delta u^2}$ results in $e_D^* = \frac{2L}{\Delta u}$. However, since $\Delta u \leq L$ holds by assumption, then the set of prizes $W_D^L = 0$ and $W_D^U = \frac{4L^2\theta}{\Delta u^2}$ results in $e_D^* = 1$. By comparing the profits from both set of prizes,

$$2\Delta u - \frac{2L\theta}{\Delta u} \overset{?}{\geq} 2\Delta u - \frac{4L^2\theta}{\Delta u^2} \Leftrightarrow 2L \overset{?}{\geq} \Delta u$$

Lemma 6 can be seen to clearly holds.

Lemma 7: The set of prizes $W_D^L = \frac{\Delta u^2 - 2L\theta}{2\theta}$ and $W_D^U = \frac{\Delta u^2 + 2L\theta}{2\theta}$ is less profitable than $W_D^L = 0$ and $W_D^U = \frac{2L\theta}{\Delta u}$ if $\Delta u \in [\sqrt{2L\theta}, L]$.

First, it must be observed that the set of prizes $W_D^L = \frac{\Delta u^2 - 2L\theta}{2\theta}$ and $W_D^U = \frac{\Delta u^2 + 2L\theta}{2\theta}$ yield an effort level of $e_D^* = \frac{\Delta u}{\theta}$. Second, it must also be observed that for the interval $[\sqrt{2L\theta}, L]$ to be non-empty, $L \geq 2\theta$ must hold. Third, $\Delta u \geq \sqrt{2L\theta}$ is equivalent to $\frac{\Delta u^2}{2\theta} \geq L$. Combining the second and third points, it can be deduced that $\Delta u \geq 2\theta$, which implies that the set of prizes $W_D^L = \frac{\Delta u^2 - 2L\theta}{2\theta}$ and $W_D^U = \frac{\Delta u^2 + 2L\theta}{2\theta}$ yields $e_D^* = 1$. The profits from both sets of prizes can then be compared:

$$2\Delta u - \frac{2L\Delta u}{\theta} \overset{?}{\geq} 2\Delta u - \frac{\Delta u^2}{\theta}$$

$$\Leftrightarrow \Delta u^2 \Delta u \geq (2L\theta)\theta.$$

Since $\Delta u \geq \sqrt{2L\theta}$ and $\Delta u \geq \theta$ by assumption, Lemma 7 is proven.

By Lemmas 6 and 7, it can be seen that the set of prizes $W_D^L = 0$ and $W_D^U = \frac{2L\theta}{\Delta u}$ is optimal and feasible for $\Delta u \in [\sqrt{L\theta}, L]$. Therefore, the solutions are the following.

If $\theta \geq L$, then the solutions are $W_D^U = W_D^L = 0$ with $\Pi_{LL}^D = 2\hat{u}$.

If $\theta \leq L$, then the solutions are $W_D^L = 0$ and

$$W_D^U = \begin{cases} 0 & \text{if } \Delta u \leq \sqrt{L\theta} \\ \frac{2L\theta}{\Delta u} & \text{if } \Delta u \in [\sqrt{L\theta}, L]. \end{cases}$$

which results in

$$\Pi_{LL}^D = \begin{cases} 2\hat{u} \equiv \Pi_0^D & \text{if } \Delta u \leq \sqrt{L\theta} \\ 2\hat{u} + 2\Delta u - \frac{2L\theta}{\Delta u} \equiv \Pi_1^D & \text{if } \Delta u \in [\sqrt{L\theta}, L]. \end{cases}$$

6.7 Proof of Proposition 3

The threshold in result 3-i is simply based on the centralized profit function. When $q < \sqrt{\frac{\theta}{2\Delta u}}$, then $\Pi^C = 2\hat{u}$, which means $\Pi^D \geq \Pi^C$.

Part ii) can be derived by comparing both profit functions:

$$2\Delta u - \frac{2L\theta}{\Delta u} \leq 2q\Delta u - \frac{2\theta}{q} + \frac{\theta^2}{2q^3\Delta u}$$

$$\Leftrightarrow \frac{\Delta u^2}{\theta}(1-q) + \frac{\Delta u}{q} - \frac{\theta}{4q^3} \leq L.$$

Furthermore, a sufficient condition for centralization to weakly dominate decentralization is for $\Delta u \leq \sqrt{L\theta} \Leftrightarrow \frac{\Delta u^2}{\theta} \leq L$. Therefore, if $L \geq L^* \equiv \min\{\frac{\Delta u^2}{\theta}, \frac{\Delta u^2}{\theta}(1-q) + \frac{\Delta u}{q} - \frac{\theta}{4q^3}\}$, then centralization dominates decentralization.

Part iii) can be derived by once again comparing both profit functions:

$$2\Delta u - \frac{2L\theta}{\Delta u} \geq 2q\Delta u - \frac{2\theta}{q} + \frac{\theta^2}{2q^3\Delta u}$$

$$\Leftrightarrow \Delta u^2 + [\frac{\theta}{q(1-q)}]\Delta u - [\frac{4L\theta q^3 + \theta^2}{4q^3(1-q)}] \geq 0.$$

The next step is using the formula quadratic:

$$[\frac{-\theta}{q(1-q)} \pm \sqrt{\frac{\theta^2}{q^2(1-q)^2} + \frac{4L\theta q^3 + \theta^2}{q^3(1-q)}}]\frac{1}{2}.$$

Subtracting the square root results in a negative value. This in turn implies that any $\Delta u \geq \sqrt{L\theta}$ would result decentralization being more profitable than centralization, which is incorrect. Therefore, I use the expression that adds the square root and get:

$$= \frac{-\theta}{2q(1-q)} + \sqrt{\frac{\theta^2 + 4L\theta q^2(1-q)}{4q^3(1-q^2)}}.$$

Since it is also necessary for $\Delta u \geq \sqrt{L\theta}$ to hold, the threshold is $\Delta u^* \equiv \max\{2L, \frac{-\theta}{2q(1-q)} + \sqrt{\frac{\theta^2+4L\theta q^2(1-q)}{4q^3(1-q^2)}}\}$

QED

6.8 Proof of Proposition 4

i) An increase in production noise L has no impact on centralized profits. In contrast, it will depress decentralized profits when $\Delta u \geq \sqrt{L\theta}$:

$$\frac{\partial(\Pi^D_{LL} - \Pi^C_{LL})}{\partial L} = \frac{-2\theta}{\Delta u}.$$

ii) A reduction in communication noise (an increase in q) has no impact on decentralized profits but can only increase centralized profits:

$$\frac{\partial \Pi^C_{LL}}{\partial q} \geq 0 \tag{21}$$

$$\Leftrightarrow 2\Delta u + \frac{2\theta}{q^2} - \frac{3\theta^2}{2q^4\Delta u} \geq 0.$$

In terms of the above inequality, the most restrictive value Δu can take is $\frac{\theta}{2q^2}$:

$$4q^2(\frac{\theta^2}{4q^4}) + 4q^2\theta(\frac{\theta}{2q^2}) \geq 3\theta^2.$$

Since the above inequality holds, inequality 21 also holds, which proves result 4-ii.

iii) It can be seen that

$$\frac{\partial(\Pi^D_{LL} - \Pi^C_{LL})}{\partial \Delta u} \overset{?}{\geq} 0$$

$$\Leftrightarrow 2(1-q) + \frac{2L\theta}{\Delta u^2} + \frac{\theta^2}{2q^3 \Delta u^2} \overset{?}{\geq} 0$$

which holds. Furthermore, the participation constraint of the agents is slack in both a centralized setting

$$\frac{1}{2}[\frac{\theta(2q^2\Delta u - \theta)}{q^3 \Delta u}] \geq \frac{\theta}{2}[\frac{\theta}{2\theta}\frac{\theta(2q^2\Delta u - \theta)}{q^3\Delta u}]^2$$

$$\Leftrightarrow \frac{2q^2\Delta u - \theta}{q^3\Delta u} \geq \frac{(2q^2\Delta u - \theta)^2}{4q^4\Delta u^2}$$

$$\Leftrightarrow q\Delta u(4-2q) \geq -\theta$$

as well as the decentralized setting

$$\frac{1}{2}(\frac{2L\theta}{\Delta u}) \overset{?}{\geq} \frac{\theta}{2}[\frac{\Delta u}{2L\theta}(\frac{2L\theta}{\Delta u})]^2$$

$$\Leftrightarrow 2L \overset{?}{\geq} \Delta u$$

which holds by assumption ($\Delta u \leq L$). Therefore, an increase in the reservation utility R starting from $R = 0$ has no impact on either the centralized or decentralized profits. This proves result 4-iii.

6.9 Proof of Proposition 5

i) It must be observed that the spread of prizes in a centralized setting with no limited liability constraint is $\Delta W^C_{noLL} = \frac{4\Delta u\theta}{2q\Delta u + \theta}$. It can then be compared to the spread of prizes in a centralized setting with limited liability constraint:

$$\Delta W^C_{noLL} \overset{?}{\geq} \Delta W^C_{LL}$$

$$\Leftrightarrow \frac{4\Delta u\theta}{2q\Delta u + \theta} \overset{?}{\geq} \frac{\theta(2q^2\Delta u - \theta)}{q^3\Delta u}$$

$$\Leftrightarrow 2q\Delta u\theta(1-q) \overset{?}{\geq} -\theta^2$$

which holds.

ii) The spread of the prizes in a decentralized setting without a limited liability constraint is

$$\Delta W^D_{noLL} = \begin{cases} 2L & \text{if } \Delta u \leq \theta \\ \frac{2L\theta}{\Delta u} & \text{if} \Delta u \geq \theta \end{cases}$$

while the spread of the prizes in a decentralized setting with a limited liability constraint is

$$\Delta W^D_{LL} = \begin{cases} 0 & \text{if } \Delta u \leq \min\{L, \sqrt{L\theta}\} \\ \frac{2L\theta}{\Delta u} & \text{if } \Delta u \in [\sqrt{L\theta}, L]. \end{cases}$$

Clearly, the introduction of a limited liability constraint decreases the spread of prizes if Δu is too low ($\Delta u \leq \min\{L, \sqrt{L\theta}\}$) or maintains it if Δu is sufficiently high $\Delta u \in [\sqrt{L\theta}, L]$.

iii) The decentralized profits with no limited liability constraint is made up of two functions, depending on the value of Δu relative to θ. However, when $\Delta u \in [\sqrt{L\theta}, L]$, it implies that $\Delta u \geq \theta$ ($\Delta u \leq L$, $\sqrt{L\theta} \leq L \Leftrightarrow \theta \leq L$ and $\Delta u \geq \sqrt{L\theta}$ imply $\Delta u \geq \theta$) so that the profit function $2\Delta u - \frac{2L\theta}{\theta} + 2\hat{u}$ can only be compared to $2\Delta u - \theta + 2\hat{u}$.

For $\Delta u \geq \theta$, I must show that

$$\frac{\partial(\Pi^D_{LL} - \Pi^C_{LL})}{\partial \Delta u} \overset{?}{\geq} \frac{\partial(\Pi^D_{noLL} - \Pi^C_{noLL})}{\partial \Delta u}.$$

$$\partial(2\Delta u - \frac{2L\theta}{\Delta u} - 2q\Delta u + \frac{2\theta}{q} - \frac{\theta^2}{2q^3\Delta u})/\partial \Delta u \overset{?}{\geq} \partial(2\Delta u - \theta - \frac{4q^2\Delta u^2}{2q\Delta u + \theta})/\partial \Delta u$$

$$\Leftrightarrow \frac{2L\theta}{\Delta u^2} - 2q + \frac{\theta^2}{2q^3\Delta u^2} \overset{?}{\geq} -[\frac{8q^2\Delta u(2q\Delta u + \theta) - 4q^2\Delta u^2(2q)}{(2q\Delta u + \theta)^2}].$$

$$\Leftrightarrow \frac{2L\theta}{\Delta u^2} + \frac{\theta^2}{2q^3\Delta u^2} + \frac{8q^2\Delta u(q\Delta u + \theta)}{(2q\Delta u + \theta)^2} \overset{?}{\geq} 2q. \tag{22}$$

To do so, I first have to show that the right hand side of inequality 22 increases with θ:

$$\partial[\frac{2L\theta}{\Delta u^2} + \frac{\theta^2}{2q^3\Delta u^2} + \frac{8q^3\Delta u(q\Delta u + \theta)}{(2q\Delta u + \theta)^2}]/\partial\theta \overset{?}{\geq} 0$$

$$\Leftrightarrow \frac{2L}{\Delta u^2} + \frac{\theta}{q^3 \Delta u^2} - \frac{16q^4 \Delta u^2}{(2q\Delta u + \theta)^3} + [\frac{8q^3\Delta u(2q\Delta u+\theta)^2 - 8q^3\Delta u\theta 2(2q\Delta u+\theta)}{(2q\Delta u+\theta)^4}] \overset{?}{\geq} 0$$

$$\Leftrightarrow \frac{2Lq^3+\theta}{q^3\Delta u^2} \overset{?}{\geq} \frac{8q^3\Delta u\theta}{(2q\Delta u+\theta)^3}$$

$$(2Lq^3+\theta)(8q^3\Delta u^2 + 8q^2\Delta u^2\theta + 2q\Delta u\theta^2 + 4q^2\Delta u^2 + 4q\Delta u\theta^2 + \theta^3)\ 8q^6 \overset{?}{\Delta} u^3\theta\ . \tag{23}$$

A sufficient condition for 23 to hold is $8q^3\Delta u^3\theta \geq 8q^6\Delta u^3\theta \Leftrightarrow 1 \geq q^3$ which clearly holds.

Therefore, in order to check if 22 holds, I let θ be as close to zero as possible and I check if 22 still holds. This results in

$$\frac{8q^2\Delta u(q\Delta u)}{(2q\Delta u)^2} \overset{?}{\geq} 2q \Leftrightarrow 2q \overset{?}{\geq} 2q$$

which holds, thereby proving result 5-iii.

6.10 Proof of Proposition 6

Proof of proposition 6: Result i) is straightforward and can be obtained from solving the principal's problem. The boundaries of the function can be derived from comparing $q^* = \frac{8}{3}\frac{\theta}{\Delta W} - (\frac{\theta}{\Delta W})^2 \frac{2\theta^P}{3\Delta u}$ to both 0 and 1.

For result ii), first observe that the conditions for $-\frac{\partial q^*}{\partial \theta} > 0$ are the same as those for $\frac{\partial q^*}{\partial \Delta W} > 0$. It can be seen that

$$-\frac{\partial q^*}{\partial \theta} > 0$$

$$\Leftrightarrow \frac{8}{3\Delta W} - \frac{4\theta^P}{3\Delta u}(\frac{\theta}{\Delta W^2}) > 0$$

$$\Leftrightarrow \frac{\theta^P}{\Delta u} > \frac{2\Delta W}{\theta}$$

whereas

$$\frac{\partial q^*}{\partial \Delta W} > 0$$

$$\Leftrightarrow -\frac{\theta}{\Delta W^2}(\frac{8}{3}) + 2(\frac{\theta^2}{\Delta W^3})\frac{2\theta^P}{3\Delta u} > 0$$

$$\Leftrightarrow \frac{\theta^P}{\Delta u} > \frac{2\Delta W}{\theta}. \tag{24}$$

Then, by looking at equation 24, it can be seen that there exist a certain threshold $\frac{\Delta W}{\theta}$ that determines the direction of $\frac{\partial q^*}{\partial \Delta W}$ and $-\frac{\partial q^*}{\partial \theta}$.

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