Developing Academic Discourse in Mathematics Lessons

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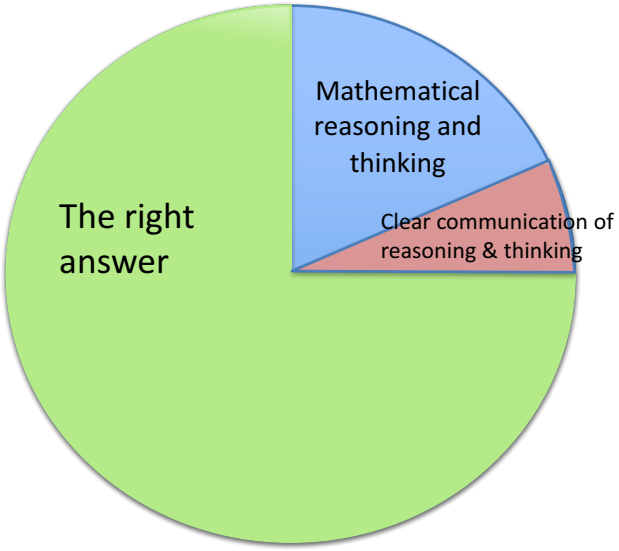
Sanger, CA

Previous Emphases



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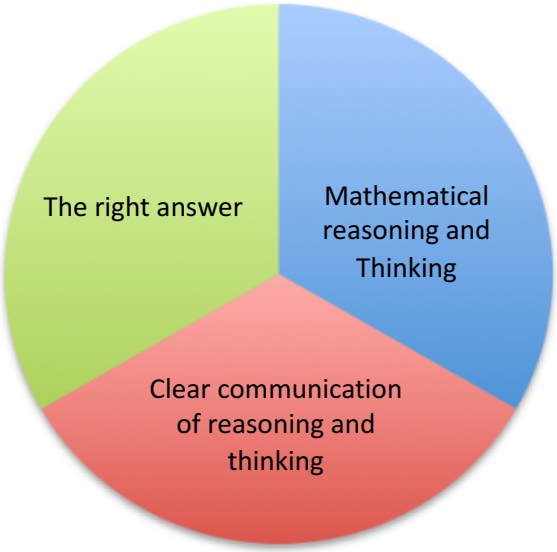


Focus of Today

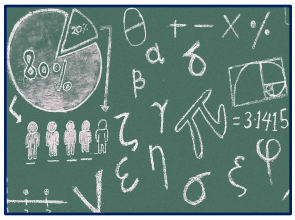
The right answer is just a destination.

We must emphasize the importance of the "journey," its many twists and turns, and how to describe it as best we can.

Therefore, ?

Objectives

1. Better understand the role of language in learning and showing learning of new math standards (& practices)



2. Learn how to better assess students' uses of language for doing math during lessons

3. Develop teacher practices and activities that foster students' math language in 5 dimensions: listening, reading, speaking, conversing, and writing

4. Learn from each other

8 Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

How does language play a role in each of these?

4 Teacher Practices for Developing Math Language

2. Help students **MAKE SENSE** of math language (Listening & Reading)	3. Help students **EXPRESS** clear & strong ideas (Speaking & Writing)	4. Help students **CONVERSE** productively about math ideas
1. Foster a **CULTURE of communication & collaboration**		

How are you already doing these things? Which practice do you want to work on most? Why?

The "Communicativeness Test" for an Activity's Power for Developing Language

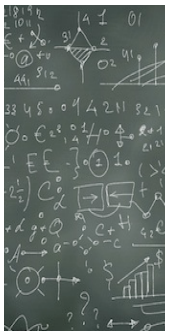
____ **Is there a useful & engaging purpose?** In the activity, do students use language to do something meaningful and engaging (beyond just to answer questions or get points)? Does it prepare students to use language for academic purposes?

____ **Is there an information gap?** In the activity, do students get or give information that they want, need, or don't have?

____ **Is there attention to language <u>in service of communication</u>?** In the activity, is there extra teaching and assessment focused on improving how language is used? This includes modeling, practicing, giving feedback, and/or scaffolds.

Math Language Routines

1. **Stronger and Clearer Each Time**



2. **Critique, Correct, & Clarify**

3. **Information Gap Activities**

4. **Math Paired Conversation Protocol**
   (w/ 5. Connecting & Comparing
   & 6. Co-Crafting Questions & Problems)

How do these support the 8 CC SMPs?

1. Stronger & Clearer Each Time Activity:
Stronger-Clearer Grid

Purpose: To provide a structured and interactive opportunity for students to revise and refine both their ideas and their verbal and written output. Students write individually about a response, use a successive pairing strategy to have multiple opportunities to refine and clarify the response through conversation, and then revise their original written response.

Designing "Stronger & Clearer Each Time" Activities

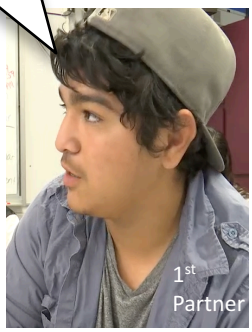
1. Prompt for an **original response**
2. Successive partners: **borrow and use the language, ideas, and evidence** each time.
3. Ideas become **stronger** (often longer) with better supporting reasoning.
4. Ideas become **clearer** with more precise terms and linked, organized, complete sentences.
5. Scaffolds **are reduced during** the activity.
6. Do not say "I disagree with you…."

"Stronger & Clearer Each Time" Grid (Math)

I think to draw it. Then cut up to ounces of each thing.

Take one or two-word notes and switch partners!

I think we gotta find like how much ounces for a dollar it is. Like one dollar you get, I don't know.



Darla decides to buy a sports drink. Her choices are a 20-ounce bottle for $1.49 or a 32-ounce bottle for $2.49. Which is the better value? Explain.

"Stronger & Clearer Each Time" Grid (Math)

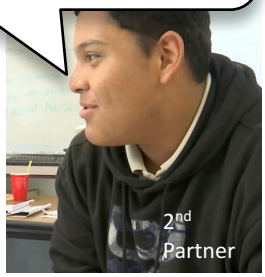
I think to draw it. Then cut up to ounces of each thing.

Take notes & switch partners! Remember to say "because" to justify your steps

I think we gotta find like how much ounces for a dollar it is. Like one dollar you get, I don't

I wanna find how much a dollar can get, like of ounces. So 1 dollar is like 1 over 1.50, two thirds. So I take 2/3 of it?

I kinda did that, but I did for one ounce, its cost. I did 1.49 over 20. I think it's like 70 cents. And 32 over, no, 2.49 over 32. I didn't finish it.



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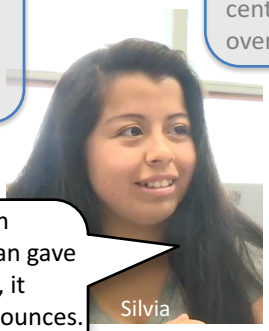
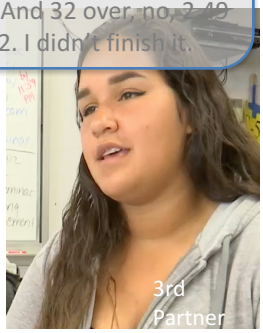
First I thought to find how much ounces for a dollar. But then Alan gave me the idea to find each ounce, it costs. So I just do cost over the ounces. So like 1.49, divide 20 into it; Alan said 70 but I think its like 7.





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$2.49. Which is the better value? Explain.

Looking at Student Work (Before & After Grid Partners)

EssentialQuestion: How do you find a solution to a system of equations when both equations are in standard form?

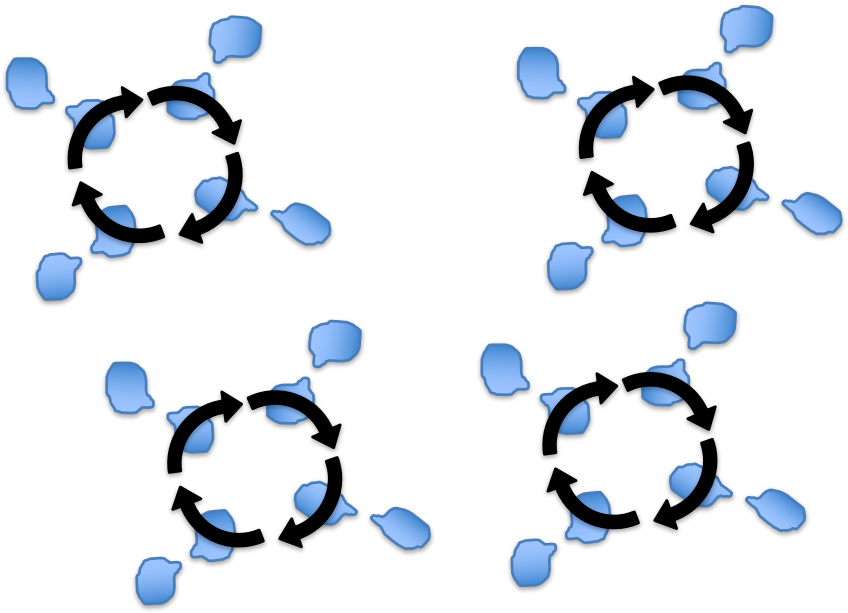
Stronger

Clearer

First Attempt

System is 2 equations
I know that at the
end of the steps
your answer needs to
be in coordinate pairs.

Practically Speaking: Interaction Mini-Circles



II. "Stronger & Clearer Each Time" **Grid**

Name	Two trains were heading toward each other on the same track, but didn't know it, in the fog. Train A was going 70 miles per hour and Train B was going 100 mph. Then when they were a half mile from each other, they saw each other and hit the brakes. Braking distance is 150 feet for every 10 mph of a train's speed. Create a good question and explain how you answer it.
Me	
1.	
2.	
3.	
Me	

One way to draw this is by…
I chose this question because …
I changed a few things based on what my previous partner said. For example..

(Teacher can have listeners ask clarifying and supporting questions)

The "Communicativeness Test" for an Activity's Power for Developing Language

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APPLICATION TIME

Think about how you might use Stronger & Clearer Each Time activities and their features in your upcoming lessons. Try it out with your table group and notice the language being used.

2. Critique, Correct, & Clarify

Purpose: To give students a sample of mathematical thinking/writing to analyze, reflect on, and develop. The intent is to prompt student reflection with an incorrect, incomplete, or ambiguous written argument or explanation, and for students to improve upon the written work by correcting errors and clarifying meaning.

Critique, Correct, & Clarify:

Critique an Unclear, Partial, or Flawed Response

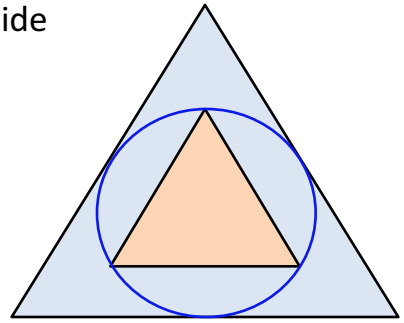
1. **PRESENT:** Present the problem and allow time for the students to understand and solve it. Optionally, provide a solution method and answer.
2. **SHOW** a partial/flawed argument, explanation, or solution method. The response can include a common error and should include an ambiguous term or phrase, or an informal way of expressing a mathematical idea. Teacher can play the role of the person who produced the response, and ask for help in fixing it.
3. **PROMPT:** Prompt students to identify the error(s) or ambiguity, analyze the response in light of their own understanding of the problem, and work both individually and in pairs to help the person who created the response.
4. **SHARE & REFINE:** Pairs share out their draft feedback responses, share model answers, and refine their own responses.

Critique, Correct, & Clarify:

Critique a Flawed Response (Your Turn)

One equilateral triangle is inside another. Find the ratio of the areas of the triangles.

Without the inscribed circle, you can't. Why?



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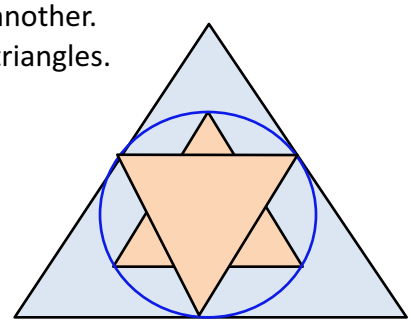


One method is to spin the triangle 180 degrees.

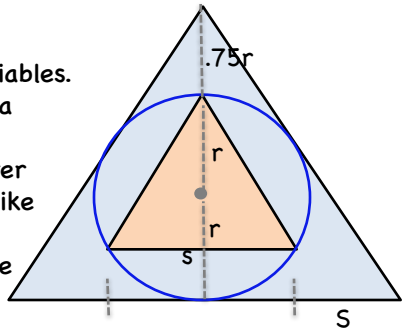
How do you know that all four triangles have equal areas?

The answer is 4:1

Critique, Correct, & Clarify:

Critique a Flawed Response (Your Turn)

One equilateral triangle is inside another. Find the ratio of the areas of the triangles.



- There are no numbers, so I use variables.
- S is a side of the big triangle; s is a side of the small one.
- The center of the circle is the center of the triangle, so its height looks like 1 and ½ times r.
- So 1.5r is the height of the triangle
- So 1.5r x s x ½ = .75rs = the area of the small triangle.
- It looks like 2.75 r's fit in the height of the big triangle and S looks like 2 times the small triangle side, s.
- So A = 2.75r x 2s x ½ = 2.75rs
- Now set equal to each other and cancel out:
  .75rs = 2.75rs → rs:rs → 1:3.7

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APPLICATION TIME

Think about how you might use critique, correct, and clarify activities and their features in your upcoming lessons. Come up with a sample flawed response for an upcoming lesson and try it out on a couple partners. Listen for the language that they use.

OPTION A: You have $50,000 that you would like to invest in two different funds. The Stock Fund yields 14% interest in one year. The Bond Fund yields 6% interest. You want to earn as much as you can. However, because of college financial aid limits how much interest income you can earn, you can't afford to earn more than $4,500 in interest income this year. How much should you invest in each fund, assuming these interest rates remain fixed?

OPTION B: Choose a problem from your current unit of study.

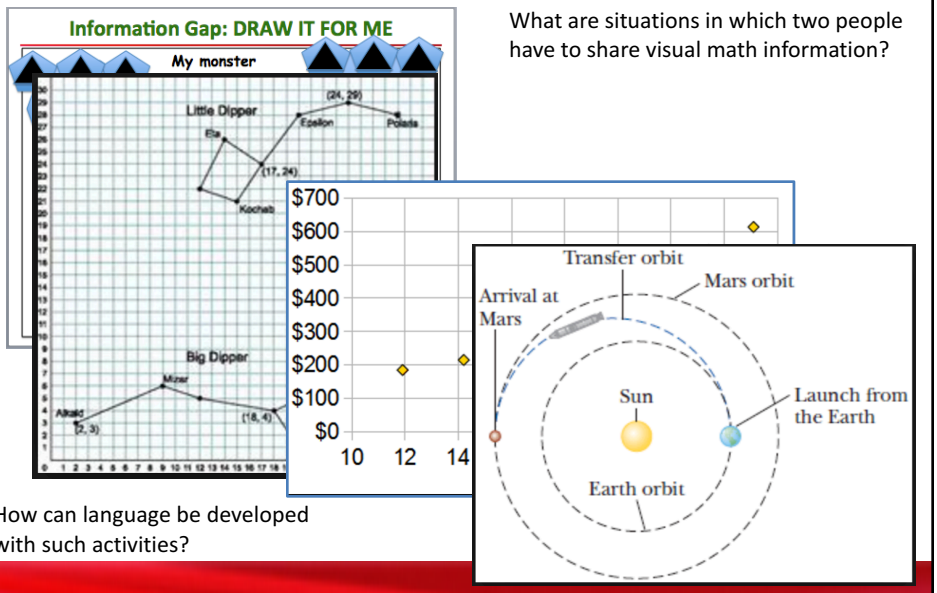
3. Information Gap Activities

Purpose: To create a need for students to talk. This routine allows teachers to facilitate meaningful interactions by giving partners different pieces of necessary information that must be used together to solve a problem, accomplish a task, or play a game. With an information gap, students need to orally (and often visually) share their ideas and information in order to bridge the gap.

Info Gap Activities:

Draw It For Me

Info Gap Activities: *Draw It For Me*



What are situations in which two people have to share visual math information?

How can language be developed with such activities?

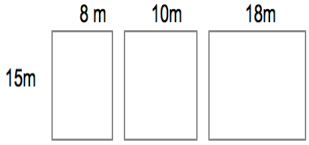
Info Gap Activity: *Info Gap Cards*

A:
You need to paint the walls and ceiling in a classroom. Your partner went to the school and did some measurements. Ask your partner for the information that you think you need in order to know how many square meters you will paint in total.

B:

- 4 walls in each classroom
- Each wall is 8 meters long and 3 meters high
- The ceiling has an area of 64 meters squared

A:

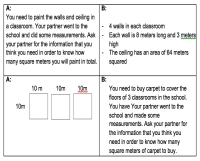


B:
You need to buy carpet to cover the floors of the classrooms in a small school. Your partner went to the school and made some measurements. Ask your partner for the information that you think you need in order to know how many square meters of carpet to buy.

Info Gap Cards - Overview

One partner has the general problem on a card, and the other partner has the information needed to solve it on the "data card." Data cards can also contain diagrams, tables, graphs, etc. The "problem" partner needs to realize what is needed and ask for information that is provided on the partner's data card. The data partner should not share information unless the problem partner specifically asks for it. Neither partner should read their cards to one another nor show their cards to their partners. As they work the problem, they justify their responses using clear and connected language.

Info Gap Cards - Procedure



1. READ: The problem card partner reads his or her card silently and thinks about what information will be needed to solve it. Partner B reads the data card silently and thinks about what the problem might be.
2. QUESTION 1: Partner B asks, "What specific information do you need?" Partner A needs to ask for specific information
3. QUESTION 2: When partner A asks, Partner B should ask for justification: "Why do you need that information?" before telling it to Partner A in complete sentences.
4. EXPLANATIONS: Partner A then explains how he or she will use the information to solve the problem. Partner B helps and asks for explanations--even if he or she understands what Partner A is doing.
5. Have them switch roles with new cards.
6. FOLLOW-UP: As a follow-up step, have both students use blank cards to write their own similar problem card and data card for other pairs to use.

What are situations in which two people have to share math information?

Info Gap Activities:
Info Gap Cards (Model)

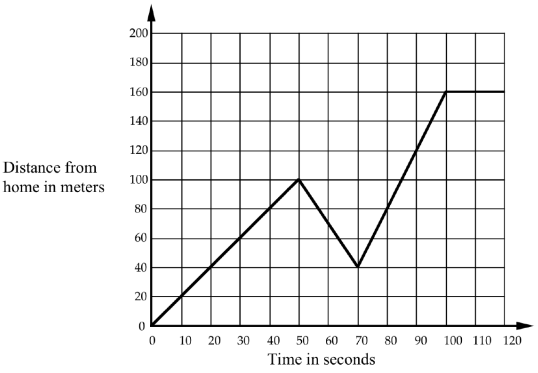
A: Model	B: Model
A shuttle enters an orbital path to catch up to an important satellite that isn't working properly. The shuttle is going faster than the satellite and mission control wants to know when the shuttle will reach it.	- Shuttle is orbiting at 16,800 mph - Satellite orbits at 16,000 mph - Shuttle enters orbit 1200 miles behind the satellite - Orbit is 400 miles from the Earth's surface

Info Gap Activities:
Info Gap Cards (Your Turn)

Info Gap Activities:
Card Matching

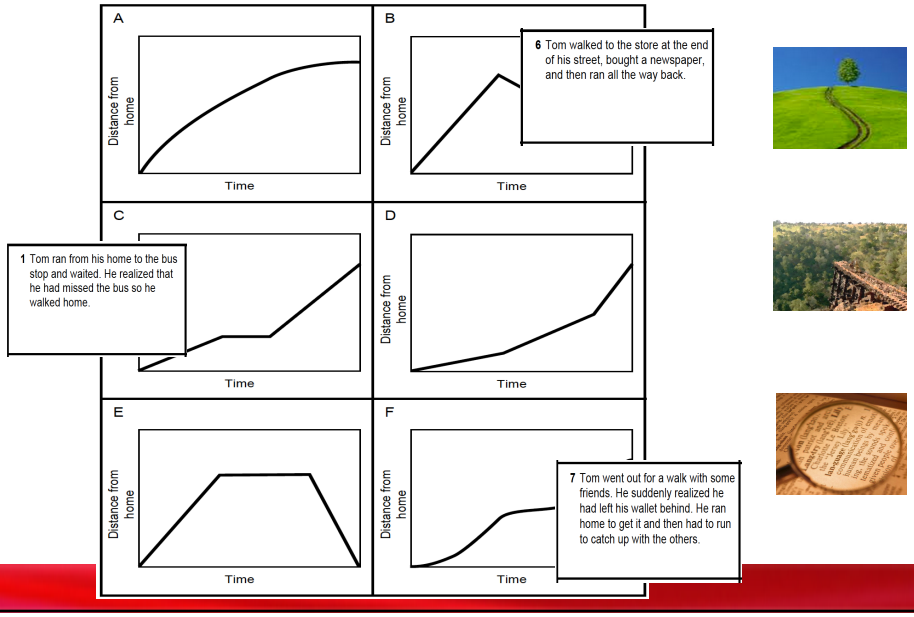
Journey to the Bus Stop

Every morning Tom walks along a straight road from his home to a bus stop, a distance of 160 meters.
The graph shows his journey on one particular day.



1. Describe what may have happened.
   You should include details like how fast he walked.

Info Gap Activities:
Card Matching (You Design)



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APPLICATION TIME

Think about how you might use information gap activities and their features in your upcoming lessons. If there is time, try one out at your table and take notes on the language being used. This is language that you might want to scaffold.

4. Math Paired Conversation Protocol

PURPOSE: To support and scaffold productive student conversation of students who are working in pairs to jointly solve a problem with more than one solution method.

w/

5. Comparing & Connecting

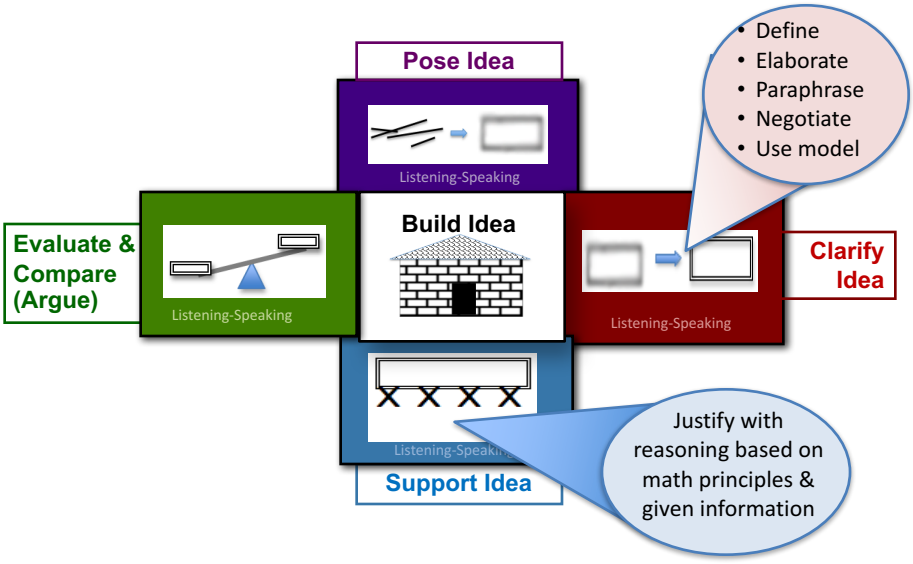
PURPOSE: To foster students' meta-awareness as they identify, compare, and contrast different mathematical approaches, representations, and language.

6. Co-Crafting Questions & Problems

PURPOSE: To allow students to understand contexts before feeling pressure to produce answers, and to create space for students to produce the language of mathematical questions themselves.

FOUNDATION: Math Constructive Conversation Skills

Goal: Students collaboratively (but w/o teacher) build an idea (e.g., claim, answer, solution, interpretation), **using the following skills:**



Good Math Conversations?

Laura: On this graph I think he shouldn't spend more than 50 dollars.
Eli: I think it should be 100.
Laura: Why?
Eli: I don't know, but just wait and he'll tell us.

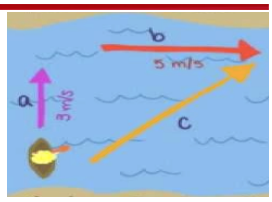
Mansur: I think there are different ways to solve it.
Lynn: So? Just do what the teacher did.
Mansur: But why did she use the formula?
Lynn: Who cares? Just use it and it'll work.
Mansur: OK.

$$\frac{3a}{3c-6} \div \frac{9ab}{c^2-4} =$$

Constructive Conversation - Math

A: What do we need to find?
B: The smallest distance the car has to stay back when they are going 30 miles an hour. And 60 I think.
A: In feet, right?
B: Yeah. It's a two-second rule, so two seconds behind, so we got find how far the car goes in 2 seconds.
A: I think we should get everything into seconds and feet first.
B: Why?
A: Cuz they need to be the same. You can't do feet and miles.
B: Why not?
A: You just can't. It's like a math rule or something.
B: OK, so 30 miles per hour is 30 x 5280 feet in a mile, that's 158,400 feet per hour.
A: Now get it into seconds. Multiply by 60 times 60, 3600.
B: I think that's gonna be too big. Try dividing.
A: OK. 44 feet per second. That seems right. So in two seconds that's 88 feet. Seems right. But my dad drives way closer.

Conversation Sample - Math



A: What do we need to find?
B: How far the boat goes down the river.
A: So, how?
B: Maybe figure out the time to cross it, like straight, like this (a).
A: I think we should just add the speeds together.
B: OK, I guess. So that's 5 plus 3 equals 8. Then what?
A: We need to use the other number, 30. So divide?
B: Why not. OK, so 30 divided by 8 is 3.75.
A: 3.75 what?
B: Meters, I think, but that doesn't look right.
A: No, so what do we do?
B: I don't know.

Math Paired Conversation Protocol

PROBLEM:

Paraphrase and clarify problem for one another (in pairs) *(Talk about what is asked; what is given; what happens; what the units are, etc.)* ☐ *TALK*			
Estimate the answer *(Each partner generate and justify your own estimate; then compare them)* ☐ *TALK*			
METHOD A *(name it)*	Justify method ☐*TALK*	**METHOD B** *(name it)*	Justify method ☐*TALK*
Visuals, Drawings, Charts, Symbols, Calculations, Solution	Justify what you do ☐*TALK*	Visuals, Drawings, Charts, Symbols, Calculations, Solution	Justify what you do ☐*TALK*

Math Paired Conversation Protocol

Check answer and compare to estimated ones ☐ *TALK*	Check answer and compare to estimated ones ☐ *TALK*
Discuss (argue) which method you would recommend for problems like this. Why? ☐ *TALK*	
Discuss connections between the two methods. How do they relate? ☐ *TALK*	
Generate a final explanation for how to solve problems like this; use this problem as an example. ☐ *TALK*	
Co-create a similar problem, write it on the back of this sheet, and solve it (then share the problem with others) ☐ *TALK & WRITE*	

Suppose it takes the Almond River 3 years to fill a reservoir, Belfair Creek 6 years to fill it, and Campbell Creek 10 years to fill it. If all are flowing into the reservoir, how long will it take to fill it?

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Estimate the answer *(Each partner generate and justify your own estimate; then compare them)* ☐ *TALK*			
METHOD A *(name it)*	Justify method ☐ *TALK*	**METHOD B** *(name it)*	Justify method ☐ *TALK*
Visuals, Drawings, Charts, Symbols, Calculations, Solution	Justify what you do ☐ *TALK*	Visuals, Drawings, Charts, Symbols, Calculations, Solution	Justify what you do ☐ *TALK*

Sample Conversation Using the Paired Protocol

Suppose it takes the Almond River 6 months to fill a reservoir, by itself, and it takes Belfair River 12 months to fill it, on its own. If both are flowing into the reservoir, how long will it take to fill it?

A: What do we gotta find?
B: How long they take to fill the reserve.
A: I say less than 6.
B: Why?
A: The Almond takes 6 months itself.
So with extra water from this other one, less time, right?
B: Maybe. So, I think we draw it for one way to solve.
A: So like two rivers into a tank?
B: Yeah, and it fills up. After 3 months it's half full from Almond, right? But Belfair only fills up like, what?
A: 3 out of 12 is, a… quarter of it full.
B: So, a quarter's not full. So let's just guess it. Like I say/
A: /We can't do that. I think there's a right answer.
B: OK, let's try the other way, like a graph or a table.

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APPLICATION TIME

Think about how you might use Paired Conversation Protocol, Comparing & Connecting, and/or Co-Crafting Questions & Problems in your upcoming lessons. If there is time, have partners try out an idea and notice the language needed.

Assessing Mathematical Reasoning

External Authority	Informal Reasoning	Formal Reasoning
• Teacher • Peer • Text • Rote procedure	• **Non-strategic examples** (e.g., says that 1 or 5 or 10 examples is "enough") • **Symbols, objects, movements, and visual examples** (tables, graphs, diagrams, drawings) **without formal explanation**	• **Principles, properties, definitions, axioms, theorems, and/or previously established results** • **Counter-examples** • **Strategic examples** • **Constraints** • **Verification of results** • **Structure, regularity, patterns** • **Symbols, objects, movements, and visuals** (tables, graphs, diagrams, drawings) **with solid explanation**

FA & Feedback: Math Conversation Observation Cards

- Ask for a mathematical reason to support the idea
- Paraphrase what your partner said
- Ask your partner to clarify (Why…How…)
- Show that you are listening with your eyes, nods, posture
- Pose an alternative idea and start to build it up
- Stay focused on the conversation prompt/purpose
- Encourage your partner to talk more
- Summarize the conversation up until now
-

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CONTENT TO BE LEARNED

DML Practices & Notes

Foster a **CULTURE** of communication and collaboration

Help students **MAKE SENSE** of complex math language (Listening & Reading)

Help students **EXPRESS** clear and strong ideas (Speaking & Writing)

Help students have productive **CONVERSATIONS**

Choose an Activity:
(Stronger-Clearer, Critique-Correct, Info Gap, Co-Craft Questions, Connect-Compare, Paired Protocol)
or another ACTIVITY:

_____

and take notes on how the activity fosters (or can be strengthened for) language development with practices and features:

Do the activity in class, notice language strengths and needs (understanding problems, explaining, arguing), and model and scaffold for stronger language

Features of Communicativeness

Engaging purpose

Information gaps

Attention to language in service of communication

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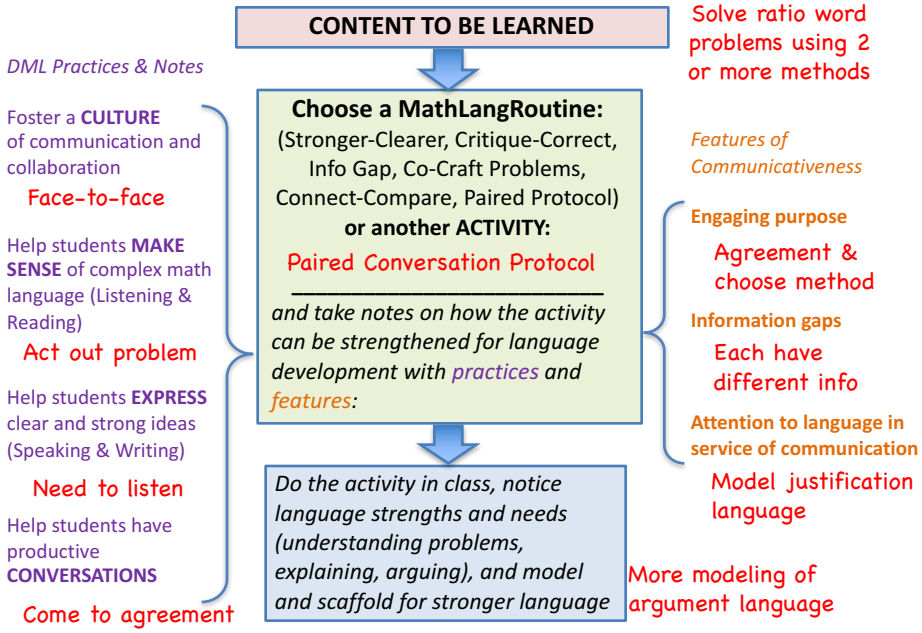

APPLICATION TIME

Think about how you might use anything from today in your upcoming lessons. If there is time, have partners try out an idea and notice the language needed.

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References

Mercer, N. (2000). *The Guided Construction of Knowledge: Talk amongst teachers and learners.* Clevedon, UK: Multilingual Matters.

Zwiers, J. & Soto, I. (2016) *Academic Language Mastery: Conversational Discourse in Context.* Corwin Press.

Zwiers, O'Hara, & Pritchard (2014) *Common Core Standards in diverse classrooms: Essential practices for developing academic language & disciplinary literacy*. Stenhouse.

A boat travels for three hours with a current of 3 mph and then returns the same distance against the current in four hours. What would be the boat's speed in calm water? How far did the boat travel one way?

Pump A can fill a tank of water in 4 hours. Pump B can fill the same tank in 6 hours. Both pumps are started at 8:00 a.m. to fill the same empty tank. An hour later, pump B breaks down and took one hour to repair and was restarted again. When will the tank be full? (round your answer to the nearest minute).

A riverboat heads east on a river that flows north. The riverboat is moving at 5.1 m/s with respect to the water. The water moves north with respect to the shore at a speed of 3.6 m/s. a. Determine the resultant velocity of the riverboat (velocity with respect to the shore). b. If the river is 71.0 m wide, then determine the time required for the boat to cross the river. c. Determine the distance that the boat will travel downstream