

**Communicating to Cultivate Academic Languages & Rich Content Learning Across Disciplines: Conversing in Math**

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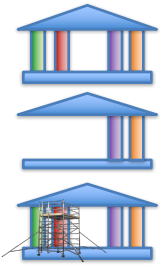
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**REVIEW: Strengthen Language Development Features**

\_\_\_ There is a **useful & engaging purpose**. Students use language to do/build/change something beyond just answering questions for praise or points. Clarity matters.

\_\_\_ There are **information gaps**. Students get or give information that they want, need, or don't have. (interdependence)

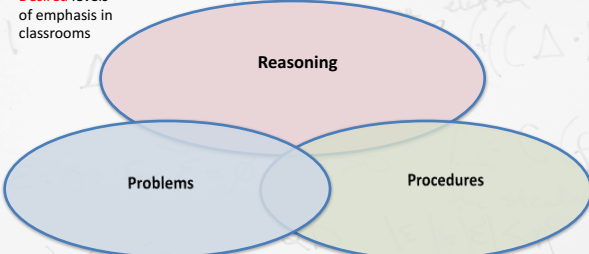
\_\_\_ There is attention to language **in service of communication?** There is extra work (modeling, practicing, giving feedback, scaffolds) on language used.



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**REVIEW: Math Lesson Emphases**

Desired levels of emphasis in classrooms



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**Assessing Reasoning in Conversations**

External Authority	Informal Reasoning	Formal Reasoning
<ul style="list-style-type: none"> <li>Teacher</li> <li>Peer</li> <li>Text</li> <li>Role procedure</li> </ul>	<ul style="list-style-type: none"> <li><b>Non-strategic examples</b> (e.g., says that 1 or 5 or 10 examples is "enough")</li> <li><b>Symbols, objects, movements, and visual examples</b> (tables, graphs, diagrams, drawings) without formal explanation</li> </ul>	<ul style="list-style-type: none"> <li><b>Principles, properties, definitions, axioms, theorems, and/or previously established results</b></li> <li><b>Counter-examples</b></li> <li><b>Strategic examples</b></li> <li><b>Constraints</b></li> <li><b>Verification of results</b></li> <li><b>Structure, regularity, patterns</b></li> <li><b>Symbols, objects, movements, and visuals</b> (tables, graphs, diagrams, drawings) with solid explanation</li> </ul>

Come up with an example of each.

**REVIEW: Key Dimensions of Reasoning**

DIMENSION	SAMPLE PROMPTS	SAMPLE STUDENT RESPONSES
<b>Procedures &amp; Plans with Justification</b>	What is your plan (next step) for solving this problem, and why?	<i>I think we should... because... I decided to start by... Because...</i>
<b>Claims, Conjectures, &amp; Generalizations with Justification</b>	In looking at this pattern, can you come up with a claim for how math works?	<i>I think when you multiply two fractions, the answer will always be smaller because... You can never divide by zero because...</i>
<b>Critiques with Justification</b>	After reading (or listening) to this person's reasoning, what is right, wrong, or unclear, and why?	<i>I don't think you can say it's 'always true' based on trying it with just those numbers. It could be another number you didn't try.</i>
<b>Representations with Justification</b>	How would you clearly show others your thinking for solving this problem. Come up with at least two different representations.	<i>I put them into a graph in order to show how the answer is where the two lines cross. I drew the pencils in different groups to help me times them up.</i>

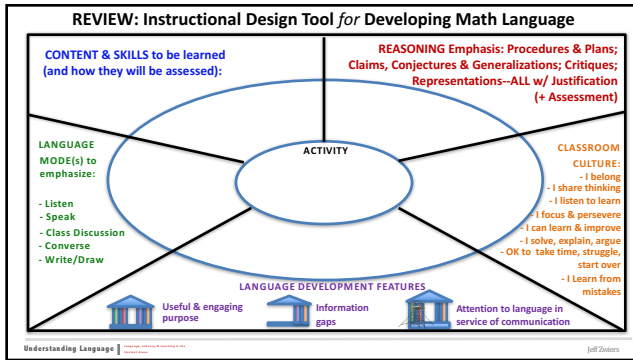
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**REVIEW: Choose an Activity(s)**

Consider the first three dimensions, your students' needs, and curriculum resources...

PURPOSE	MODE	Activities & Routines that Foster REASONING & Its Language	
		Covered in the course	Other
Solve a Problem	Listening & Speaking	<ul style="list-style-type: none"> <li>Stronger &amp; Clearer Each Time Grid</li> <li>Information Gap A &amp; B</li> <li>Farms &amp; Gardens</li> <li>Listening Graphic Organizer</li> </ul>	<ul style="list-style-type: none"> <li>Gallery Walk</li> <li>Think-Pair-Share</li> <li>Compare and Connect Ideas</li> </ul>
	Class Discussion	<ul style="list-style-type: none"> <li>Talk Moves</li> </ul>	<ul style="list-style-type: none"> <li>Central Focus Semantic Map</li> <li>Note-Taking Supports</li> </ul>
Co-create a problem or question	Conversing	<ul style="list-style-type: none"> <li>Paired Conversation Protocol</li> </ul>	<ul style="list-style-type: none"> <li>Conversation Skills</li> </ul>
	Reading/Viewing/Writing/Drawing	<ul style="list-style-type: none"> <li>Three Reads</li> <li>Write down your Thinking</li> </ul>	<ul style="list-style-type: none"> <li>Sentence Stems</li> <li>Note-taking organizers</li> <li>Gallery Walk</li> </ul>

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**WHAT HAVE WE DONE?**

- Communication,
- Info Gaps,
- Stronger Clearer,
- Critique-Correct-Clarify...

**Constructive Conversation Skills, Icons, & Motions**

**Conversations Develop Language & Content**

**INPUT:** Every other turn, a learner is pushed to understand the partner's talk. Language is reinforced as it is used multiple times. Content is clarified.

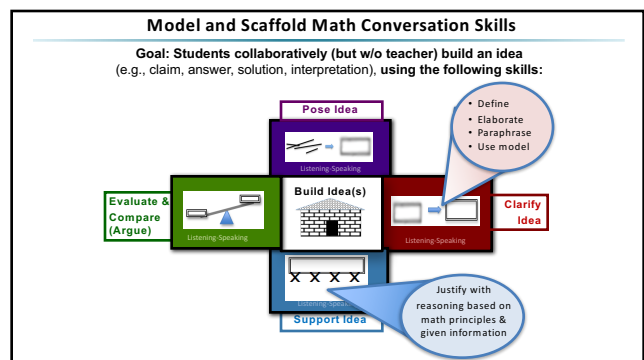
**OUTPUT:** Every other turn, output pushes a learner to try new ways of constructing and clarifying messages, putting ideas into words and sentences that others can understand (Swain, ).

**MINI-CHALLENGES:** Each turn presents a "mini-challenge" for both partners to overcome, usually at each person's ZPD. (Zwiers, et al., 2016)

**Foster Conversational Mindsets**

*I will try to:*

- Help my partner to understand & think more deeply about this concept or problem.
- Be as clear as possible every time I talk.
- Allow my partner to help me understand and think more deeply about this concept or problem.
- Work with my partner, not against, even if we disagree at times.
- Be open to learning new ideas and having my ideas change.



### Conversation Observation & Analysis Tool - Math

**Quantity**

# of turns  
 Length of turns  
 Equity of voice

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**Quality**

Use conversation skills to co-construct & argue ideas

- Turns build on previous turns
- Students pose or choose a relevant initial idea(s) that is focused on learning objective(s)
- Students clarify idea(s) by paraphrasing, defining, elaborating, asking questions, etc.)
- Students justify ideas (using reasoning\*, examples, explanations, wording from the problem, etc.)
- *If there are two or more competing ideas (argue/decide):*
  - students build up both ideas and
  - (a) evaluate the strength/weight of the justification for each idea
  - (b) compare the strengths/weights and choose the

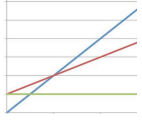
### Conversation Samples

**Laura:** On this graph I think he shouldn't spend more than 50 dollars.

**Eli:** I think it should be 100.

**Laura:** Why?

**Eli:** I don't know, but just wait and he'll tell us.




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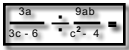
**Mansur:** I think there are different ways to solve it.

**Lynn:** So? Just do what the teacher did.

**Mansur:** But why did she use the formula?

**Lynn:** Who cares? Just use it and it'll work.

**Mansur:** OK.




### Improving Conversations

**Alex:** You can cut it into two triangles.

**Carlos:** Why?

**Alex:** Cuz it's easier to get the area, I think.

**Carlos:** I don't think so.




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**Manny:** I think we should use a table, like yesterday.

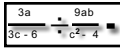
**Sara:** I hate those things. They're so boring.

**Manny:** So, what should we do?

**Sara:** Maybe just times it.

**Manny:** Why?

**Sara:** Cuz that's what we did yesterday.



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### Assess These Conversations

**Laura:** I think it's like a cone.

**Eli:** I disagree cuz it's got branches.

**Laura:** I respectfully disagree with you.

**Eli:** Then we just agree to disagree, right?




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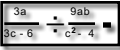
**Asha:** I think there are different ways to solve it.

**Juan:** So? Just do what the book example did.

**Asha:** But why do you divide by this?

**Juan:** Who cares? Just do it; it'll work.

**Lisa:** OK.



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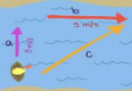
### Conversation Sample - Math

**A:** What do we need to find?

**B:** How far the boat goes down the river.

**A:** So, how?

**B:** Maybe figure out the time to cross it, like straight, like this (a).



**A:** I think we should just add the speeds together.

**B:** OK, I guess. So that's 5 plus 3 equals 8. Then what?

**A:** We need to use the other number, 30. So divide?

**B:** Why not. OK, so 30 divided by 8 is 3.75.

**A:** 3.75 what?


**B:** Meters, I think, but that doesn't look right.

**A:** No, so what do we do?

**B:** I don't know.

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### Types of Conversations in MATH



- Collaborating to solve a problem
- Collaborating to create new math problems
- Modeling math concepts and experimenting with numbers, symbols, and shapes

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### Math Conversation Prompts

- There is an engaging purpose for conversing that (a) connects to lesson objectives, and (b) requires thinking & doing something with ideas (e.g., create, clarify, argue => consensus, decide, solve, evaluate, compare, choose...)(+ Agency)
- There is a need to talk (info gaps; bring unique ideas)
- There are clear directions for how to converse (language use, thinking, content concepts...)

1) Work with your partner to come up with two or more different ways to

2) Work with your partner to create a word problem that requires the solver to

3) In your conversation, come up with a clear explanation for when and how to divide fractions, and what is happening when you divide fractions. Create a poster that can help you, along with example problems that you can use to show the concept. Both of you need to be prepared to do the presentation on your own. Use terms such as inverse, in order to, whenever you need to...,

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## Fostering Conversations in Math with

- Math Paired Conversation Protocol
- Observation Cards
- Co-create a Problem

Suppose it takes the Almond River 4 months to fill a reservoir, by itself, and it takes Belfair River 6 months to fill it on its own. If both are flowing into the reservoir, how long will it take to fill it?

**PROBLEM:**

Paraphrase and clarify problem for one another (in pairs)  
*(Talk about what is asked; what is given; what happens; what the units are, etc.)*

TALK

Estimate the answer  
*(Each partner generate and justify your own estimate; then compare them)*

TALK

METHOD_A (name it)	METHOD_B (name it)
Justify method <input type="checkbox"/> TALK	Justify method <input type="checkbox"/> TALK
Visuals, Drawings, Charts, Symbols, Calculations, Solution	Visuals, Drawings, Charts, Symbols, Calculations, Solution
Justify what you do <input type="checkbox"/> TALK	Justify what you do <input type="checkbox"/> TALK

### Math Paired Conversation Protocol

Check answer and compare to estimated ones <input type="checkbox"/> TALK	Check answer and compare to estimated ones <input type="checkbox"/> TALK
Discuss (argue) which method you would recommend for problems like this. Why? <input type="checkbox"/> TALK	
Discuss connections between the two methods. How do they relate? <input type="checkbox"/> TALK	
Generate a final explanation for how to solve problems like this; use this problem as an example. <input type="checkbox"/> TALK	
Co-create a similar problem, write it on the back of this sheet, and solve it (then share the problem with others) <input type="checkbox"/> TALK & WRITE	

### Sample Conversation Using the Paired Protocol

A: What do we gotta find?  
 B: How long they take to fill the reserve.  
 A: I say less than 4.  
 B: Why?  
 A: The Almond takes 4 months itself. So with extra water from this other one, less time, right?  
 B: Maybe. So we can't average 'em. So, maybe we draw it for one way to solve.  
 A: So like two rivers into a tank, like a box?  
 B: Yeah, and it fills up. After 2 months it's half full from Almond, right? But Belfair only fills up like, what?  
 A: 2 out of 6 is, a third of it full on that side.  
 B: So, not full. So let's just guess it. Like I say/  
 A: /We can't do that. I think there's a right answer.  
 B: OK, let's try another way, like a graph or a table.

Suppose it takes the Almond River 4 months to fill a reservoir, by itself, and it takes Belfair River 6 months to fill it on its own. If both are flowing into the reservoir, how long will it take to fill it?

### Math Paired Conversation Protocol

The grade 8 class is going to the planetarium. There is a budget of \$100 for the outing and the planetarium can accommodate 100 people. The admission price is as follows: Adults = \$10, Chaperones = \$2.50, Students = \$0.50. What is the optimal number of teachers, chaperones and students that will be going if all 100 seats are to be filled and all the \$100 is to be spent? OR

A plane left the airport on a bearing of 45° traveling at 400 mph. The wind was blowing at a bearing of 135° at a speed of 40 mph. What is the actual velocity of the plane? Solve and explain.

**PROBLEM:**

Paraphrase and clarify problem for one another (in pairs)  
*(Talk about what is asked; what is given; what happens; what the units are, etc.)*

TALK

Estimate the answer  
*(Each partner generate and justify your own estimate; then compare them)*

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Justify method <input type="checkbox"/> TALK	Justify method <input type="checkbox"/> TALK
Visuals, Drawings, Charts, Symbols, Calculations, Solution	Visuals, Drawings, Charts, Symbols, Calculations, Solution
Justify what you do <input type="checkbox"/> TALK	Justify what you do <input type="checkbox"/> TALK

### Math Conversation Observation Cards

Ask for a mathematical reason to support the idea	Paraphrase what your partner said	Ask your partner to clarify (Why...How...)
Show that you are listening with your eyes, nods, posture	Pose an alternative idea and start to build it up	Stay focused on the conversation prompt/purpose
Encourage your partner to talk more	Summarize the conversation up until now	

### Using the Cards – 7<sup>th</sup> Gr. Math

1 Karla: So, MNBC needs to be 40% of the area of it all

2 Maya: So if x is bigger, it's a bigger area.

3 Karla: Yeah.

4 Maya: Yeah. (pause)

5 Karla: Yeah...  
OK, why does x make the area bigger?

6 Maya: It makes it less triangle and more square. Square has more area than triangles.

7 Karla: OK, so it looks like the area is a triangle plus a rectangle.

8 Maya: I don't see it...  
How is it triangle plus rectangle?

9 Karla: Look. The triangle is 10 times (20 mm  $\times$  5) times one half. See? And the rectangle area is x times 10. ...but I don't know what to do with the 40.

10 Maya: It's 40%. So those equal 40%.

11 Karla: OK.  
Why do you do that?

12 Maya: It says the area is 40% of the big rectangle, so  $(10 \times 15)/2 = 10(20x)$

Ask your partner to clarify why x makes the area bigger

Ask your partner to clarify

Ask for justification (Why?)

### Live Use of Cards in Triads

1. One person is the observer, C, with the cards, and the other two are the conversers, A and B.
2. All three take some individual time to understand and start solving the problem.
3. As the two talk, the observer listens and notices where A, B, or both might benefit from a support card.

### Co-Create a Math Problem

1. Tell the class that they will be teachers, in a sense, and create math problems similar to the problems they just worked on.
2. Encourage them to create problems that are slightly more challenging than the problems they just worked on.
3. In pairs, students collaborate to create one or more problems that are clear to readers and helps solvers to learn a certain concept or skill.
4. Each pair then solves its own problem & keeps solutions secret.
5. Each pair trades its problem for the problem of another pair, and solves it together.
6. Now as a group of four, they argue/agree on the solution method(s) that they will present for both problems.
7. Each group of four meets with another group of four to present the two problems.

### Fortifying Typical Math Activities

### Instructional Design Tool for Developing Math Language

**CONTENT & SKILLS to be learned**  
(and how they will be assessed):

**REASONING Emphasis:** Procedures & Plans; Claims, Conjectures & Generalizations; Critiques; Representations--ALL w/ Justification (+ Assessment)

LANGUAGE MODE(s) to emphasize:

- Listen
- Speak
- Class Discussion
- Converse
- Write/Draw

CLASSROOM CULTURE:

- I belong
- I share thinking
- I listen to learn
- I focus & persevere
- I can learn & improve
- I solve, explain, argue
- OK to take time, struggle, start over
- I Learn from mistakes

LANGUAGE DEVELOPMENT FEATURES

Useful & engaging purpose

Information gaps

Attention to language in service of communication

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**Next Steps**

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