



Integrating Language Development and Content Learning in Math: Focus on Oral Language & Reasoning

East Side Alliance Mathematics Symposium XVII
Jeffzwiers.org/january17

WARM-UP: Transition Improv (Math Situations)

Topics: Addition-Subtraction, Area-Perimeter, Multiplication-Division,
 Sine-Cosine, Volume-Surface Area, Linear-Quadratic
 Fractions-Decimals, Integral-Derivative, Constant-Variable

Transitions: **However,**
On the other hand,
Then again,

~~but~~

Frames: You need to add when ... because...
 When you ... you need to subtract because...
 You need to calculate volume when... because...
 When you ... you need to use decimals because...

*Director can prompt
 for clarification
 and/or spark ideas,
 if needed.*

Today's Objectives

1. Develop practical ways to increase authentic math communication
2. Improve at developing as much reasoning and language as possible from each problem and activity (includes setting up, processing, and ending phases)
3. Develop teacher practices and activities that foster students' reasoning language in 3 modes: listening, speaking, and conversing



Key Dimensions of Reasoning

Reasoning

DIMENSION	SAMPLE PROMPTS	SAMPLE STUDENT RESPONSES
Procedures with Justification	What procedures do you need to use for solving this problem, and why?	<i>I think we should... because...</i> <i>I decided to start by... Because...</i>

Authentic Communication in School is

the use of words

(and/or other meaning-carriers)

to share information

for doing meaningful things

(creating, changing, deciding, clarifying, etc.)

that just one person can't do.



Why is it so important?

Fostering a **Mindset** for Growing Mathematical Language: Features for Language Development

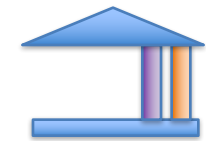
___ **Is there a useful & engaging purpose?** In the activity, do students *use* (and *need to use*) language to do something meaningful and engaging beyond just to answer questions or get points? (e.g., language, content, thinking...)

Does clarity matter?

___ **Is there an information gap?** In the activity, do students get or give information that they want, need, or don't have?

___ **Is there attention to language in service of communication?**

In the activity, is there extra teaching and assessment focused on improving how language is used?



SPEAKING & LISTENING

Information Gap Activities

Purpose: To create a need for students to listen and talk. Students need to orally (and often visually) share their ideas and information in order to bridge the gap accomplish a task.

Info-Gap Cards

I need the floor size to know the wall lengths. I need the wall heights to multiply by their lengths to get the area that needs painting

Can you tell me the size of the room and the height of the walls?

Why do you need to know those measurements?


A: Emma wants to paint the four walls of her room. But she isn't sure if she has enough money to buy the paint. Does she have enough?

B: The floor of Emma's room is 12 feet by 10 feet. The walls are 8 feet tall. A gallon of paint covers 100 square feet. A gallon costs \$24. Emma has \$75.

Info Gap Cards - Procedure

1. **READ:** A (Situation card) and B (Data card) read their cards. A summarizes situation to B. B paraphrases back to A, for agreement.
2. **QUESTION 1:** A asks B for specific information.
3. **QUESTION 2:** Before answering, B must ask for justification: "Why do you need that information?" (even if B knows why, already)
4. **JUSTIFICATIONS:** A explains how he or she will use the information. B decides if the justification is solid enough to give the data to A.
5. **SOLVE:** A solves problem aloud, explaining process, while B asks Why? And helps, if needed.

A: You need to paint the walls and ceiling in a classroom. Your partner went to the school and did some measurements. Ask your partner for the information that you think you need in order to know how many square meters you will paint in total.	B: <ul style="list-style-type: none"> - 4 walls in each classroom - Each wall is 8 meters long and 3 meters high - The ceiling has an area of 64 meters squared
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A: You need to buy carpet to cover the floors of the classrooms in a small school. Your partner went to the school and made some measurements. Ask your partner for the information that you think you need in order to know how many square meters of carpet to buy.	B: 
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Info Gap Activities

Info Gap Cards Sample Conversation

- A: Do you know how fast the shuttle is orbiting?
 B: Yes, but why do you want to know that?
 A: Cuz I need to know it to figure out how long it takes to catch the satellite.
 B: How will knowing the speed help you do that?
 A: I'll use it and the satellite speed and the distance.
 B: OK, it's going 16,800 miles per hour
 A: Thanks. And how fast is the satellite going?
 B: Why do you need to know that?
 A: To know how long it'll take. If it's just a little slower, it'll take longer. I'll make an equation and put them equal to each other cuz that's where they meet.
 B: That makes sense. The satellite is going 16,000 mph
 A: Thanks. And how far are they apart when the shuttle starts its orbit?
 B: Why?



A: Model A shuttle enters an orbital path to catch up to an important satellite that isn't working properly. The shuttle is going faster than the satellite and mission control wants to know when the shuttle will reach it.	B: Model <ul style="list-style-type: none"> - Shuttle is orbiting at 16,800 mph - Satellite orbits at 16,000 mph - Shuttle enters orbit 1200 miles behind the satellite - Orbit is 400 miles from the Earth's surface
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Info Gap Cards: Your Turn

A:

The Silver Star train left the station early της τρισε φαλλσ οφφ Βρενδανεσ βικε, ωηιχη τηεψ λεαπε λοχκεδ υπ αρουνδ α τρεε. Τηεψ δεχιδε τηατ Βρενδαν ωιλλ ωαλκ φορ α ωηιλε ανδ Σηαων ωιλλ ριδε ηισ βικε, λεαπινγ ιτ φυρτηερ υπ τηε ροαδ ανδ ωαλκινγ τηε ρεστ οφ τηε ωαψ. Ωηεν Βρενδαν ρεαχηεσ Σηαων'σ βικε, ηε ωιλλ ριδε ιτ ηομε. Ηοω φαρ σηουλδ Σηαων ριδε τηε βικε φορ βοτη το αρριπε ηομε ατ τηε σαμε τιμε?



B:

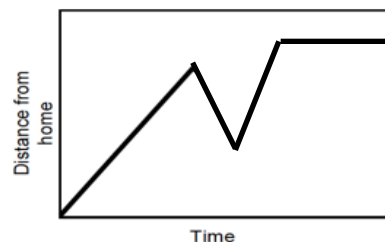
- The Silver Star left at πειντε κμ φρομ ηομε
- The Golden Arrow left ωαλκσ ατ χινθυε κμ/η ανδ ριδεσ ατ δοδιχηι κμ/η.
- Σηαων ωαλκσ ατ χετιρι κμ/η ανδ ριδεσ ατ διεχι κμ/η.

Info Gap Activities: Card Matching

Procedure

1. Give 'story' cards out to Student A in each pair
2. Give graph cards to B
3. Have A read a card silently and picture what is happening.
4. Then A describes the type of graph he/she is looking for and B finds it.
5. B asks A for any information missed
6. A watches to help B, if needed
7. Students can also draw a graph and have the partner make up the story for it.

Elia walked away from her home. Then she realized that she forgot her lunch and ran toward home. Halfway back she decided to buy lunch, so she turned around to run to the bus stop and waited for the bus.



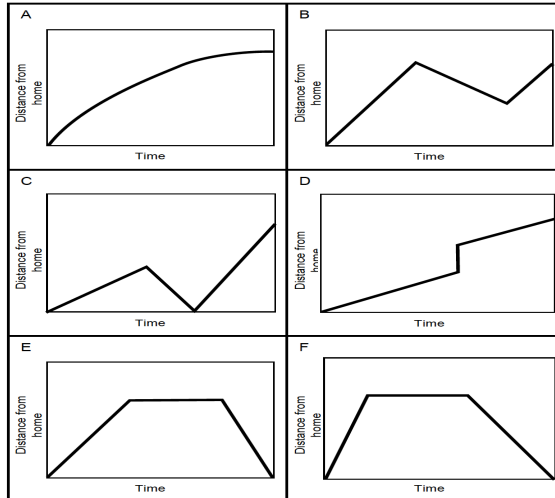
<http://map.mathshell.org/lessons.php>

Let's try it out

1 Tom ran from his home to the bus stop and waited. He realized that he had missed the bus so he walked home.

7 Tom went out for a walk with some friends. He suddenly realized he had left his wallet behind. He ran home to get it and then had to run to catch up with the others.

4 Tom has a mutation that allows him to be two places at once.



<http://map.mathshell.org/lessons.php>

APPLY

Think about how you might use **information gap activities** and their features in your upcoming lessons.





Designing “Stronger & Clearer Each Time” Activities



1. Prompt for an original response (e.g., solve problem)
2. Successive partners: borrow and use the language, ideas, and justifications each time; responses become:
 - Stronger (often longer) with better justifications and examples, and
 - Clearer with more precise terms and linked, organized, complete sentences.
3. Scaffolds are reduced during the activity.

'Stronger & Clearer Each Time' Grid

I think to draw it. Then cut up into ounces of each thing.

Take one or two-word notes switch partners! Remember, stronger & clearer!

I think we gotta find like how much ounces for a dollar it is. Like one dollar you get, I don't know.

Pre-write:

Cut into ounces each bottle. Then add or times.

Post-write:



Darla decides to buy a sports drink. Her choices are a 20-ounce bottle for \$1.49 or a 32-ounce bottle for \$2.49. Which is the better value? Explain what you did to get your answer and why.

'Stronger & Clearer Each Time' Grid

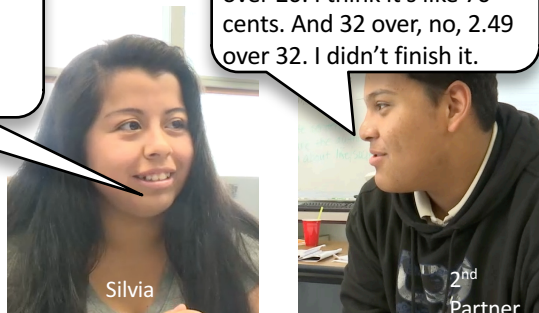
I think to draw it. Then cut up into ounces of each thing.

Take notes & switch partners! Remember to say "because" to justify your steps

I think we gotta find like how much ounces for a dollar it is. Like one dollar you get, I don't

I wanna find how much a dollar can get, like of ounces. So 1 dollar is like 1 over 1.50, two thirds. So I take 2/3 of it?

I kinda did that, but I did for one ounce, its cost. I did 1.49 over 20. I think it's like 70 cents. And 32 over, no, 2.49 over 32. I didn't finish it.



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'Stronger & Clearer Each Time' Grid

Pre-write:
 Cut into ounces each bottle.
 Then add or times it.

Post-write:
 You need to find out how much each ounce is. So I did cost over number of ounces. I got 7 for the 20 bottle.

I think we gotta find like how much ounces for a dollar it is. Like one dollar you get, I don't know. I kinda did that, but I did for one ounce, its cost. I did 1.49 over 20. I think it's like 70 cents. And 32 over, no, 2.49 over 32. I didn't finish it.

First I think how much ounces for a dollar. But Alan dijo que hay que buscar cuánto cuesta each ounce. I agree. So I just divide. So 1.49, divide 20 in it; Alan said 70, but I think is 7.



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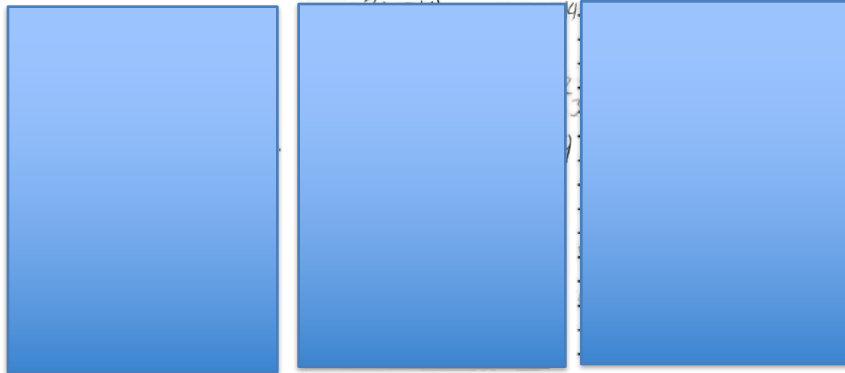
Looking at Student Work (Stronger-Clearer)

Essential Question: How do you find a solution to a system of equations when both equations are in standard form?

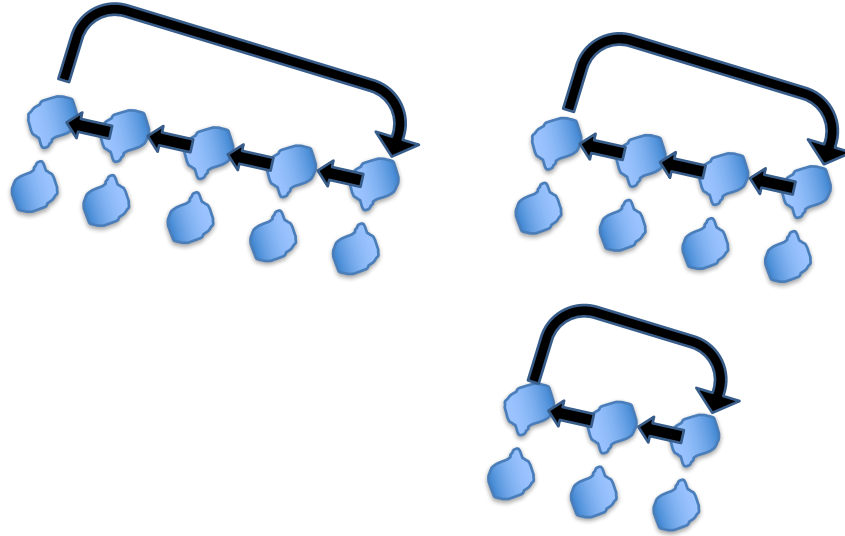
1st

2nd

3rd



Practically Speaking: Interaction Mini-Lines



“Stronger & Clearer Each Time” Grid

Name	<p>A plane takes off at 1:00 p.m. heading northeast with an average airspeed of 300 mph. Right after takeoff, the compass breaks and a wind starts blowing northwest at 60 mph. How far is the plane from the airport at 3 p.m.? Solve and explain, justifying your ideas.</p>	<p><i>The plane has not reported in and you need to know where to look for it.</i></p>
Me	(just two or three key words, if any)	
1.		
2.		
3.		
Me	<p><i>Listeners should ask for clarification & justification (& can offer idea seeds)</i></p>	

- I first thought I needed to figure out...because...
- I know the wind is blowing for/against, so the resulting velocity must be...
- I wonder
- ~~I disagree with you...~~

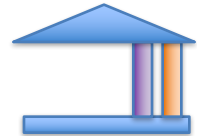
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Does clarity matter?

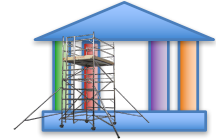


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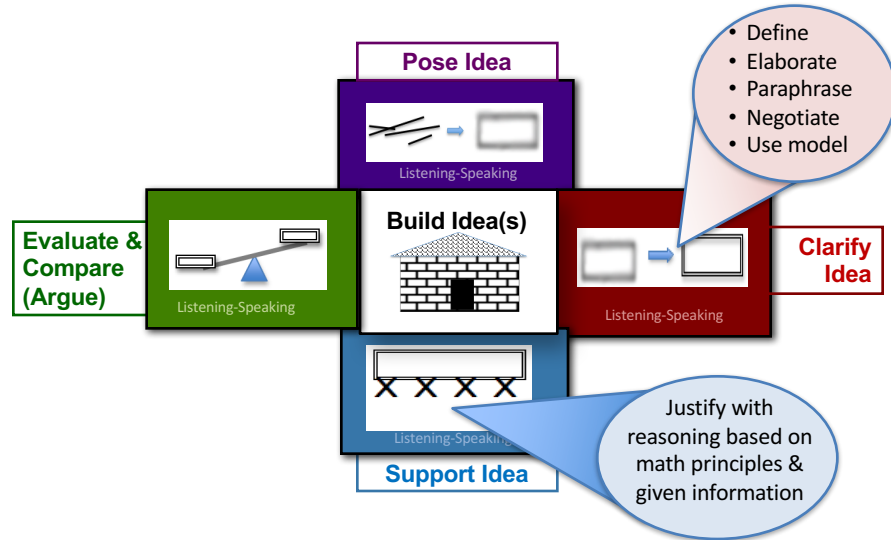
In the activity, is there extra teaching and assessment focused on improving how language is used?



MATH CONVERSATION Activities



Model & Scaffold Math Conversation Skills



Conversation Model

1A: So, what's happening?

2B: Elvia looks at the two places to decide, like my mom does in the store sometimes. What about you?

3A: I remember that problem yesterday of the two cars. They went different speeds. It's like the two different costs of apples.

4B: Oh yeah, she did a graph thing and the lines crossed.

5A: I think that's what we have to do. It's when it switches.

6B: Why?

7A: Like here are two lines. They cross cuz of different prices.

8B: Oh yeah. Like the car problem.

Elvia wanted to pick apples from an orchard as cheaply as possible. Palomar Orchards sells apples for 8 dollars for every ten pounds, plus a flat entrance fee of 10 dollars. Ted's Orchard sells apples at 6 dollars per 10 pounds with no entrance fee. At what amount would she change from which orchard to the other?

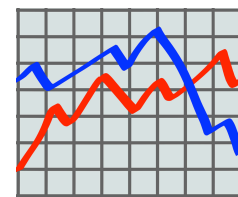
Conversation Model

- 9A: Yeah. But we can't just draw the lines without the numbers.
 10B: Why not?
 11A: Because we have to use the numbers in the problem. But they aren't there now. We have to come up with them.
 12B: Why don't they just give us all the numbers?
 13A: We do a table, like yesterday. It had lots of numbers.
 14B: Oh yeah. OK, put how much on the left, here. And cost here. You buy more and the price goes up. So pounds is x and cost is y .
 15B: That's just for one orchard. I think we have to do two tables.
 16A: OK. So for Palomar, 10 pounds is 8 dollars.
 17B: Don't forget the entrance fee.

Conversation Non-models

- Laura: On this graph I think he shouldn't spend more than 50 dollars.
 Eli: I think it should be 100.
 Laura: Why?
 Eli: I don't know, but just wait and he'll tell us.

Build up the
1st idea 1st!



- Mansur: I think there are different ways to solve it.
 Lynn: Maybe. But we should do what the teacher did so it's right.
 Mansur: But why did she use that formula?
 Lynn: Does it matter? Just use it and it'll work.
 Mansur: OK.

$$\frac{3a}{3c-6} \div \frac{9ab}{c^2-4} =$$

Conversation Non-model

Ana: What do we need to find?

Ben: How far the boat goes down the river.

Ana: So, how?

Ben: Maybe figure out the time to cross it, like straight, like this (a).

Ana: I think we should just add the speeds together.

Ben: OK, I guess. So that's 5 plus 3 equals 8. Then what?

Ana: We need to use the other number, 30. So divide?

Ben: Why not? OK, so 30 divided by 8 is 3.75.

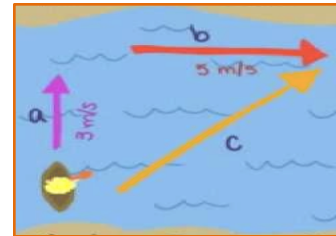
Ana: 3.75 what?

Ben: Meters, I think, but that doesn't look right.

Ana: No, so what do we do?

Ben: I don't know. Maybe go back to my idea?

A boat steers straight across a river at 3 meters per second. The river is flowing at 5 m/s. If the river is 30 meters wide, how far downstream will the boat end up on the other side?



ACTIVITY:

Paired Conversation Protocol

To support and scaffold productive student conversation of students who are working in pairs and jointly solving a problem with more than one solution method.

Math Paired Conversation Protocol

PROBLEM:

Paraphrase and clarify problem for one another (in pairs) <i>(Talk about what is asked; what is given; what happens; what the units are, etc.)</i>			
<input type="checkbox"/> TALK			
Estimate the answer <i>(Each partner generate and justify your own estimate; then compare them)</i>			
<input type="checkbox"/> TALK			
METHOD A <i>(name it)</i>	Justify method <input type="checkbox"/> TALK		METHOD B <i>(name it)</i>
Visuals, Drawings, Charts, Symbols, Calculations, Solution	Justify what you do <input type="checkbox"/> TALK	Visuals, Drawings, Charts, Symbols, Calculations, Solution	Justify what you do <input type="checkbox"/> TALK

Math Paired Conversation Protocol, cont.

Check answer and compare to estimated ones <input type="checkbox"/> TALK	Check answer and compare to estimated ones <input type="checkbox"/> TALK
Discuss (argue) which method you would recommend for problems like this. Why? <input type="checkbox"/> TALK	
Discuss connections between the two methods. How do they relate? <input type="checkbox"/> TALK	
Generate a final explanation for how to solve problems like this; use this problem as an example. <input type="checkbox"/> TALK	
Co-create a similar problem, write it on the back of this sheet, and solve it <i>(then share the problem with others)</i> <input type="checkbox"/> TALK & WRITE	

Math Paired Conversation Protocol

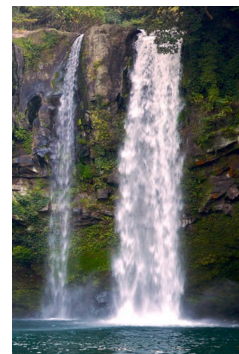
Suppose it takes the Almond River 6 months to fill a reservoir, by itself, and it takes Belfair River 12 months to fill it, on its own. If both are flowing into the reservoir, how long will it take to fill it?

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Visuals, Drawings, Charts, Symbols, Calculations, Solution	Justify what you do <input type="checkbox"/> TALK	Visuals, Drawings, Charts, Symbols, Calculations, Solution	Justify what you do <input type="checkbox"/> TALK

Sample Conversation Using the Paired Protocol

- A: What do we gotta find?
 B: How long they take to fill the reserve.
 A: I say less than 6.
 B: Why?
 A: The Almond takes 6 months itself.
 So with extra water from this other one, less time, right?
 B: Maybe. So, I think we draw it for one way to solve.
 A: So like two rivers into a tank?
 B: Yeah, and it fills up. After 3 months it's half full from Almond, right? But Belfair only fills up like, what?
 A: 3 out of 12 is, a... quarter of it full.
 B: So, a quarter's not full. So let's just guess it. Like I say/
 A: /We can't do that. I think there's a right answer.
 B: OK, let's try the other way, like a graph or a table.

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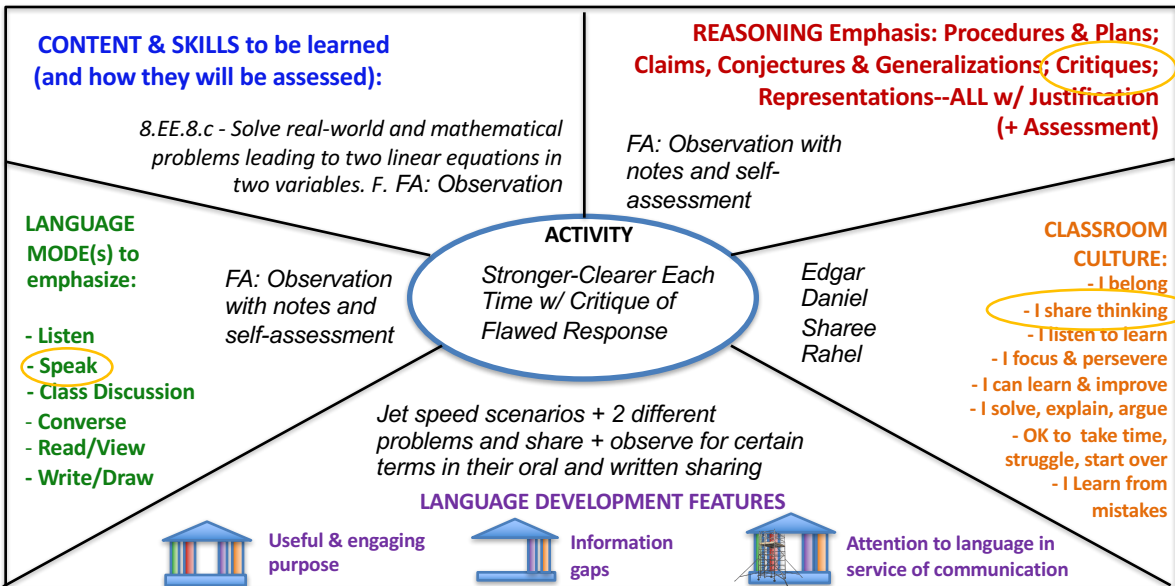


APPLY

Think about how you might use the **Paired Conversation Protocol** in your upcoming lessons.



Instructional Design Tool for Developing Math Language



Next Steps

1. Authentic communication
2. Students push selves and others for clarity
3. Emphasize reasoning
4. Work on conversation skills



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Free Online PD Courses (Language in Math) --> ell.stanford.edu