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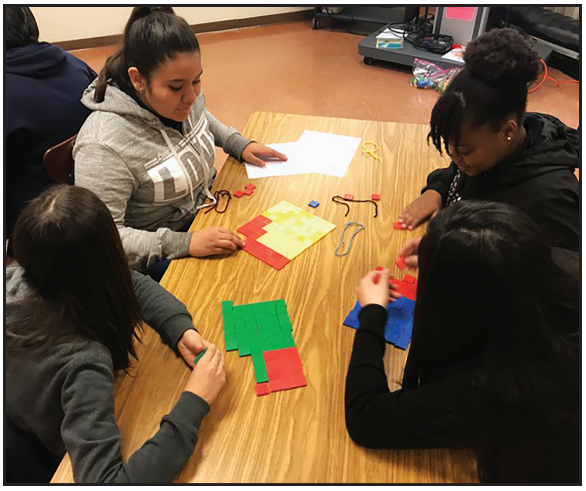
# Developing Reasoning and its Language in Secondary Mathematics Instruction

*by Jeff Zwiers, Researcher—Stanford University*

Language grows the most when it is used to share and build complex ideas—ideas such as the concepts and claims that result from high-quality mathematical reasoning. Most educators would agree that in many classrooms we need to spend more think time and talk time on mathematical reasoning. The problem is, when you get into the real weeds of lesson plans and pacing guides, the first thing to get squeezed out, more often than not, is reasoning. The purpose of this article is to emphasize the need for increasing the instructional time and energy around reasoning and its language, with a particular emphasis on student conversations as vehicles for doing this.



*Student conversations support the development of reasoning in mathematics.*

In my work with teachers and students, I have come to see the need for four overlapping types of reasoning, shown in Figure 1 (pg. 10): deciding what procedures to use, claims, conjectures and generalizations, critiquing the reasoning of others, and representing abstract and complex concepts and ideas. Each type is followed by related prompts that might be used by the teacher or students and sample student responses.

You will likely notice that justification is a major component in all four types of reasoning. Why? Well, justification is really what puts the reason into reasoning. It is the logical support that strengthens and proves one's claim or idea. Justification tends to come from two sources: (1) mathematical rules and concepts, and (2) the wording or visuals of the problem or situation. Take a moment to think about all the rich language that is needed to address these dimensions. Now think about all the rich language that is developed when students work with others to justify a solution. And yes, these can overlap and intertwine in the very same response by a student. A student might start talking about procedures for solving a problem and why, and then come up with a conjecture and a representation for it.

It's exciting to consider these kinds of responses, but how do we operationalize the shift toward more and better reasoning, given the many ingrained and institutionalized ideas about mathematics learning that focus on getting the right answers as quickly and easily as possible? Shifting from finding the right answer at all costs to using problems and right and wrong answers to support reasoning is a major shift.

This shift also challenges students to use more complex and precise language to explain their reasoning-based ideas. But as

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| Type of Reasoning | Sample prompts | Possible student responses |
|---|---|---|
| **Deciding on what procedures to use when solving a problem, with justification.** Depending on the problem, students must use their reasoning to decide if they need to add, subtract, multiply, take the derivative, etc., as they solve a problem. And they need to have a mathematically logical reason, or justification, for it. "Because we did this yesterday," or "because adding is easier than division," are not strong justifications. | What is your plan (next step) for solving this problem? Why? | "I think we should make it *x* because in the problem it changes."<br><br>"I decided to start by dividing the trapezoid into triangles because I think it's made up of triangles with no leftover area." |
| **Claims, conjectures, and generalizations, with justification.** Conjecture tends to mean coming up with a tentative and logical idea about how things work in math, which includes using patterns and explaining how they might be used. Justification of these things means supporting an idea with math principles, patterns, and the information given in a problem. It also means outlining conditions under which the claim is true. | In looking at this tile pattern, can you come up with a claim for how math works? | "I think when you multiply two fractions, the answer will always be smaller because ..."<br><br>"You can never divide by zero because..." |
| **Critiquing the reasoning of others, with justification,** (and critiquing one's own reasoning) in order to sharpen and revise thinking for all. This often includes arguments for or against ideas in math, such as why you choose a certain method for solving a problem. Students need to justify their critiques with logical ideas and specific language. They can't just say, "I liked it," or "It wasn't clear." | After reading (or listening) to this person's reasoning, what is right, wrong, or unclear? Why? | "I don't think you can say it's "always true" based on trying it with just those numbers. It could be another number you didn't try. I think you need to notice in the problem where it says, *except*." |
| **Represent abstract and complex concepts and ideas, with justification.** This includes the use of representations such as symbols, graphs, diagrams—and why you are using them. Representations are often used to show relationships that are impossible to communicate with just words in sentences. | How would you clearly show others your thinking for solving this problem? Come up with at least two different representations. | "I put them into a graph in order to show how the answer is where the two lines cross. "<br><br>"I drew the pencils in different groups to help me times them up". |

*Figure 1. Types of reasoning*

you well know, language can vary a lot, even out of the mouth of the same student on different days. For this reason, it is very difficult to isolate the exact words and sentences that you want students to produce and understand. You can do this with modeling and sentence frames, but the much bigger and more important practice is creating engaging activities and tasks that challenge students to use reasoning and its language with one another to accomplish them—even if you can't frame up all the language used. We don't want to put the cart of language before the horse of understanding.

### *Using Student Conversations to Foster Reasoning and its Language*

Even though we must improve the reasoning in activities that highlight all modes of language (e.g., listening, speaking, reading, writing), here I will focus on conversations. In the space remaining, we will begin to answer the question, "How can we shape and support student conversations so that they build reasoning skills and the language needed for them?" Here are several suggestions:

#### 1. Foster a "building up ideas" mindset.

Students can and often do have the basic "I talk with a partner to get the answer" mindset. In addition, we must strive to cultivate a "conversations help us to build up big mathematical ideas" mindset. This includes dispositions such as:

- I work with my partner to understand and think more deeply about this topic.
- I work with, not against, my partner even if we disagree at times.
- I am open to having my ideas change.

#### 2. Have students clarify and justify often.

To build up ideas, the two main skills of clarifying and justifying are needed. Clarifying can include defining terms, paraphrasing, synthesizing, and asking questions. Justifying, as we saw already,

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includes referring to mathematical principles and/or the information given in a problem or set of problems. Anchor charts with phrases and questions that are typical of these language functions can be made available to students, revisited, and referred to often during the course of any lesson.

### 3. Model and scaffold conversation skills.

We can't just put two students together, tell them to solve a problem and share their reasoning, and expect powerful reasoning-filled conversations. We need to model how to have productive conversations. You can do this with sample transcripts that you analyze, or live modeling with you and a student or two students. Then you can debrief the conversation, highlighting phrases and responses that deepen the thinking and further the conversation, as well as behaviors that allow for respectful and fruitful collaboration.

### 4. Improve conversation purposes and prompts.

Here are four types of prompts that your students' conversations might emphasize. Two or more of these foci often overlap in one conversation.

• Collaborating—Solving a problem through collaboration is the most common purpose for a conversation. It is important for students to learn how to effectively and respectfully talk with each other to solve problems in math—and in other disciplines, for that matter. The sharing, critiquing, and justifying of ideas during conversations can accelerate students' mathematical language development.

> A sample prompt could be, "Collaborate with your partner to come up with at least two methods for solving this problem and be able to describe how the two methods relate to one another."

• Arguing and critiquing—Have students collaborate to make choices and justify them by using mathematical principles and/or the information in the problem(s).

> Here's a sample prompt for an argument-based conversation: "Work with your partner to decide whether the following statement is sometimes, always, or never true: x2 > x. You must cobuild a strong argument for your choice, supporting it with mathematical rules and principles, as well as strategic examples. Argue why it is what you chose and why it can't be the other two. Use examples to justify your ideas."

• Clarifying concepts and big ideas—These conversations focus on how math works, and students use problems as examples to show their understandings, rather than as ends in and of themselves. These conversations can be foundational for many students.

> For example, you might have two students work together to come up with a presentation for other students on the topic of completing the square for quadratic equations. Students try to clearly explain, using problems and drawings, the idea of completing the square, why it's done, when, and how.

• Collaborating to create new math problems—Many students want to add their creativity and interests to learning, which means they tend to put more energy and investment into the process. When they work with another student to create a problem, they need to negotiate both language and math at the same time. To create problems, students must apply the math they have learned to real-world settings, "reverse engineering" the problems, and thereby building a better understanding of how different kinds of problems work.

> For example, if we tell you right now to create a problem that requires using algebra, you immediately need to think of and describe a setting that uses variables and equations. Here's a sample prompt: Work with your partner to create a word problem that requires that your classmates solve it using two linear equations. Both of you contribute ideas and then decide which would make for the most interesting problem for your peers to solve. Make sure to set up what is happening and use consistent units.

## *Conclusion*

Developing reasoning in math lessons can provide rich opportunities to develop complex uses of language. But we must set up learning activities that support and challenge students to communicate their reasoning. Fortunately, conversations between students foster this communication and the practice that students need in building mathematical concepts, ideas, and arguments with others.