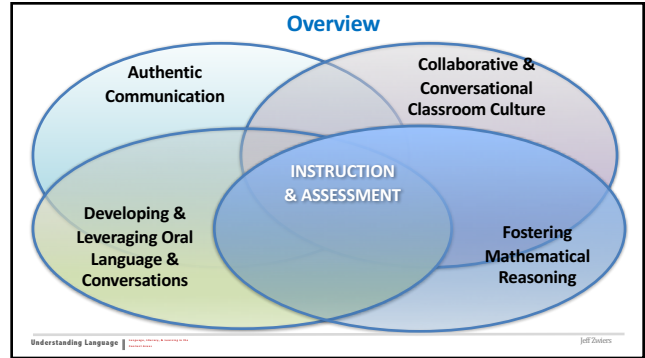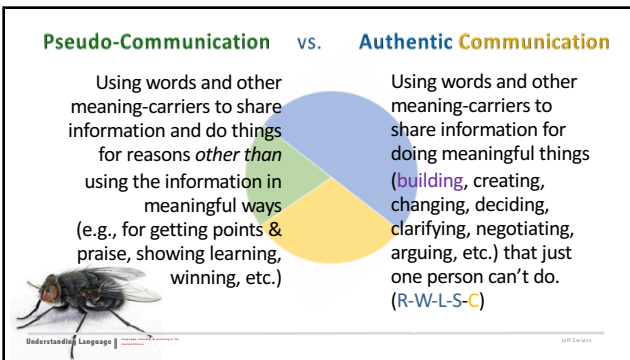**Fortifying Students' Use of Language in Math:**
Focus on Reasoning and Building Ideas

Jeff Zwiers
October 10, 2018
East Side Alliance Mathematics Symposium XIX
Jeffzwiers.org/october10

Understanding Language

Overview

- Authentic Communication
- Collaborative & Conversational Classroom Culture
- INSTRUCTION & ASSESSMENT
- Developing & Leveraging Oral Language & Conversations
- Fostering Mathematical Reasoning

Understanding Language

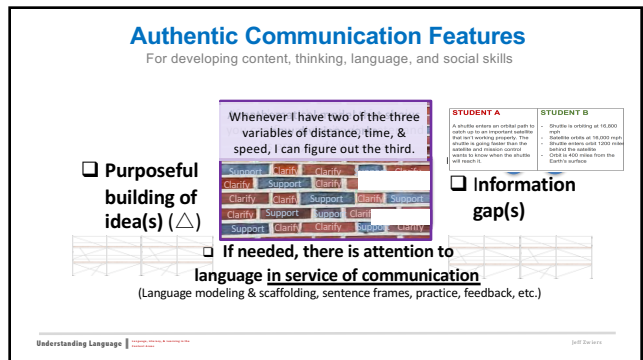## Pseudo-Communication vs. Authentic Communication

Using words and other meaning-carriers to share information and do things for reasons *other than* using the information in meaningful ways (e.g., for getting points & praise, showing learning, winning, etc.)

Using words and other meaning-carriers to share information for doing meaningful things (building, creating, changing, deciding, clarifying, negotiating, arguing, etc.) that just one person can't do. (R-W-L-S-C)

Understanding Language

Authentic Communication Features

For developing content, thinking, language, and social skills

Whenever I have two of the three variables of distance, time, & speed, I can figure out the third.

☐ Purposeful building of idea(s) (△)

☐ Information gap(s)

☐ If needed, there is attention to language in service of communication
(Language modeling & scaffolding, sentence frames, practice, feedback, etc.)

Understanding Language

What are Mathematical Ideas
that students would-could-should build?

Claim
Conclusion
Conjecture
Argument
Solution
Generalization
Plan
Procedure

Interpretation
Concept
Theory
Hypothesis
Approach
Explanation
Relationship
Reason

From just two points on a line I

Pi never changes and I can use it

To figure out the area of crazy

I can transform ratios, decimals, fractions, & percentages into one

If the x has an exponent, its graph

For this type of problem, we come

To figure out volume of

To solve for a single variable (like x) in an equation, I need to isolate it by

By looking at the quadratic, I can decide whether to factor, complete

I can solve systems of equations different ways such as substitution or "subtraction"

Understanding Language

Key Dimensions of Reasoning

DIMENSION OF REASONING	DESCRIPTION
Justify Procedures	Explain the underlying concepts and reasons that support the use of a procedure or method
Justify Claims, Conjectures, & Generalizations	Support an idea about how math works using mathematical principles, problems, and connections
Evaluate Reasoning	Evaluate how strong, accurate, logical, and relevant the reasoning is

Understanding Language Language, Literacy, and Learning in the Content Areas

Stanford | GRADUATE SCHOOL OF EDUCATION

Ideas & Reasoning Should be a Major Focus

In addition to solving problems, students need to be able to explain the mathematics (i.e., reasoning, major concepts, big ideas) that the problems are meant to teach. And a key aspect of the explanation is *using problems as examples and evidence* of the mathematical ideas and reasoning that students are explaining.

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Big Idea Blueprint

Big Idea Blueprint

Big idea in own words...with why it's big, connections to other ideas, and real world applications

Visual ways to clarify, represent & remember this idea

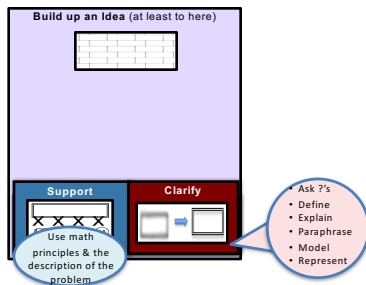Sample problems given to students, found by them, and created by them individually or in groups

Big idea (Key Concept; Standard; Answer to Essential Question) in my own words:
You can predict what a graph will look like by analyzing the function
Why it's big (important): Graphs show relationships
How this connects to or depends on other big ideas: Domain and range
Real world applications (if any): Bacteria growth

Visual Ways to Represent & Remember this Idea (symbols, drawings, graphs, etc.)

Sample Problem	Sample Problem
Barrel over waterfall	Dolphin jumping
Negative quadratic parabola just down	Negative quadratic parabola up & down
Bacteria Growth	Mt. Everest growth
Positive quadratic parabola	Same ratio over time makes a line

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Constructive Conversation Skills: Building an Idea

Build up an Idea (at least to here)

Support — Use math principles & the description of the problem

Clarify

Ask ?'s
• Define
• Explain
• Paraphrase
• Model
• Represent

Constructive Conversation Skills: Building an Idea

Build up an Idea (at least to here)

Support   Clarify   Support
Support   Clarify   Support
Support   Clarify
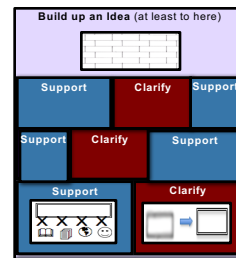
Hand motions

Developing Mathematical Conversation Skills with Structured Interactions:

Stronger-Clearer Each Time Activity

"Stronger & Clearer Each Time" Activities

1. Prompt for an original response that can build
2. Successive partners borrow and use the language, ideas, and justifications from previous partners. Turns get:
   • **Stronger** (often longer) with better support and examples, and
   • **Clearer** with more precise terms and linked, organized, complete sentences.
3. Listeners push for clarification & support, and offer ideas

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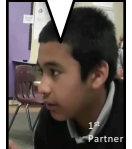
'Stronger & Clearer Each Time' Grid

I think to draw it. Then cut up into ounces of each thing.

Take one or two-word notes switch partners! Remember, stronger & clearer!

I think we gotta find like how much ounces for a dollar it is. Like one dollar you get, I don't know.

Pre-write:

Cut into ounces each bottle. Then add or times.
_ _ _ _ _ _ _ _ _ _ _

Post-write:

David
1st Partner

Darla decides to buy a sports drink. Her choices are a 20-ounce bottle for $1.49 or a 32-ounce bottle for $2.49. Which is the better value? Explain what you did to get your answer and why.

'Stronger & Clearer Each Time' Grid
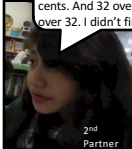
I think to draw it. Then cut up into ounces of each thing.

Take notes & switch partners! Remember to say "because" to justify your steps

I think we gotta find like how much ounces for a dollar it is. Like one dollar you get, I don't

I wanna find how much a dollar can get, like a ounce. So 1 dollar is like 1 over 1.50, two thirds. So I take 2/3 of it?

I kinda did that, but I did for one ounce, its cost. I did 1.49 over 20. I think it's like 70 cents. And 32 over, no, 2.49 over 32. I didn't finish it.

David
2nd Partner

Darla decides to buy a sports drink. Her choices are a 20-ounce bottle for $1.49 or a 32-ounce bottle for $2.49. Which is the better value? Explain what you did to get your answer and why.
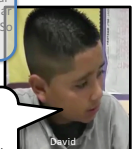

'Stronger & Clearer Each Time' Grid

Pre-write:

Cut into ounces each bottle. Then add or times it.
_ _ _ _ _ _ _ _ _ _ _

Post-write:

For this type of problem, you need to find out how much each ounce is. So I did cost over number of ounces. I got 7 for the 20 bottle.

it. Then cut of each thing.

OK, now write down what you learned. Start with "For this type of problem, you…"

I think we gotta find like how much ounces for a dollar it is. Like one dollar you get, I don't kn

I kinda did that, but I did for one ounce, its cost. I did 1.49 over 20. I think it's like 70 ver, no, 2.49 didn't finish it.

much a dollar nce. So 1 dollar), two thirds. So

First I think how much ounces for a dollar. But Alan said to find out how much costs each ounce. I agree. So I just divided it. So 1.49, divide 20 in it; Alan said 70, but I think it's 7.
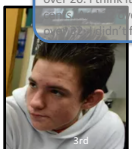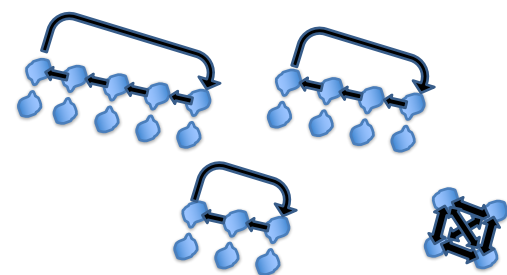
David
3rd Partner

decides to buy a sports drink. Her choices are a 20-ounce for $1.49 or a 32-ounce bottle for $2.49. Which is the better value? Explain what you did to get your answer and why.

Interaction Mini-Lines

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"Stronger & Clearer Each Time" Grid

Name	The school decided to put a rubber walkway of uniform width around the small swimming pool. The pool has a rectangular shape that measures 12 meters by 20 meters. The area of the walkway cannot be more than 528 m² because of the cost of the material. Find the maximum possible width of the walkway.	
Me	(just two or three key words, if any)	
1.		
2.		
3.		
Me		

20
12

I first thought I needed to figure out…because…
I know that some of the lengths are…, so…
I think that we set it equal to….

The answer is 6 and the expression is $4x^2 + 64x = 528$

Listeners can & should:
- Prompt for clarification
- Prompt for support
- Help with "What about…?"

Constructive Conversations

Assess These Conversations

A: I think we can add the volume of the cylinder to the square.
B: I respectfully disagree. I think we subtract them.
A: Why?
B: Because we just want the water that came out.

C: I think there are different ways to solve it.
D: So? Just do what the book example did.
C: But why does the formula work?
D: Who cares? Just do it; it'll get the answer.
C: OK.

Conversation Model or Non-Model?

Ana: What do we need to find?
Ben: How far the boat goes down the river.
Ana: So, how?
Ben: Maybe figure out the time to cross it, like straight, like this (a).
Ana: I think we should just add the speeds together.
Ben: OK, I guess. So that's 5 plus 3 equals 8. Then what?
Ana: We need to use the other number, 30. So divide?
Ben: Why not? OK, so 30 divided by 8 is 3.75.
Ana: 3.75 what?
Ben: Meters, I think, but that doesn't look right.
Ana: No, so what do we do?
Ben: I don't know. Maybe go back to my idea?

> A boat steers straight across a river at 3 meters per second. The river is flowing at 5 m/s. If the river is 30 meters wide, how far downstream will the boat end up on the other side?
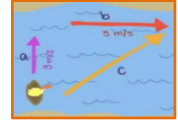
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What does building an idea look like?

(And how do we decide if it's built up enough?)
Prompt: Work with your partner to come up with an explanation of how we can use two or more lines on a graph.

| IDEA |
| CLARIFY |
| SUPPORT |

Linear equations with different slopes intersect on a graph, and that can be a decision point.

What does building an idea look like?

(And how do we decide if it's built up enough?)
Prompt: Work with your partner to come up with an explanation of how we can use two or more lines on a graph.

Linear equations with different slopes intersect on a graph, and that can be a decision point.

It's how the two variables relate; it's the m in y = mx + b

What is a slope and how do we find it? | What's an example of a line?

What does building an idea look like?

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Prompt: Work with your partner to come up with an explanation of how we can use two or more lines on a graph.

Linear equations with different slopes intersect on a graph, and that can be a decision point.

| It's how the two variables relate; it's the m in y = mx + b | Or when you go the same speed and distance | What about when there are two lines? |
| What is a slope and how do we find it? | What's an example of a line? | When you buy more things that are the same price |

What does building an idea look like?

(And how do we decide if it's built up enough?)
Prompt: Work with your partner to come up with an explanation of how we can use two or more lines on a graph.

Linear equations with different slopes intersect on a graph, and that can be a decision point.

Because if not, they never meet	Why?	But the slopes need to be different	Two lines intersect and that's the answer
It's how the two variables relate; it's the m in y = mx + b		Or when you go the same speed and distance	What about when there are two lines?
What is a slope and how do we find it?		What's an example of a line?	When you buy more things that are the same price

What does building an idea look like?
(And how do we decide if it's built up enough?)
Prompt: Work with your partner to come up with an explanation
of how we can use two or more lines on a graph.

Linear equations with different slopes intersect on a graph, and that can be a decision point.

		That means the apples cost less til they cross
Like this apple problem. They have different slopes	And different b's, 2 and 7. If they had the b, that would be the answer	The place starting at 2 costs more per box
Because if not, they never meet	Why? / But the slopes need to be different	Two lines intersect and that's the answer
It's how the two variables relate; it's the m in y = mx + b	Or when you go the same speed and distance	What about when there are two lines?
What is a slope and how do we find it?	What's an example of a line?	When you buy more things that are the same price

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What does building an idea look like?
(And how do we decide if it's built up enough?)
Prompt: Work with your partner to come up with an explanation
of how we can use two or more lines on a graph.

Linear equations with different slopes intersect on a graph, and that can be a decision point.

So for > 8 boxes, we switch	But we can't eyeball it; we gotta set them equal to each other, like the trains, to get x	That means the apples cost less til they cross
Like this apple problem. They have different slopes	And different b's, 2 and 7. If they had the b, that would be the answer	The place starting at 2 costs more per box
Because if not, they never meet	Why? / But the slopes need to be different	Two lines intersect and that's the answer
It's how the two variables relate; it's the m in y = mx + b	Or when you go the same speed and distance	What about when there are two lines?
What is a slope and how do we find it?	What's an example of a line?	When you buy more things that are the same price

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What does building an idea look like?
(And how do we decide if it's built up enough?)
Prompt: Work with your partner to come up with an explanation
of how we can use two or more lines on a graph.

IDEA / CLARIFY / SUPPORT

Linear equations with different slopes intersect on a graph, and that can be a decision point.

So for > 8 boxes, we switch	But we can't eyeball it; we gotta set them equal to each other, like the trains, to get x	That means the apples cost less til they cross
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It's how the two variables relate; it's the m in y = mx + b	Or when you go the same speed and distance	What about when there are two lines?
What is a slope and how do we find it?	What's an example of a line?	When you buy more things that are the same price

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Big Idea Blueprint Conversations

Helpful prompts:

1. Why is this idea important?

2. How does that visual represent the idea?

3. What problems help to show or teach this idea?

4. How does solving that problem support this idea?

Converse to Co-Create a Math Problem

1. Tell pairs that they will be teachers, in a sense, and create math problems similar to the problems they just worked on.
2. Tell them to create problems that are challenging and that show the big math idea that they are building. Encourage real world situations, visuals, and actions.
3. Each pair then solves its own problem & keeps solutions secret.
4. Each pair trades its problem for that of another pair, and solves it together.
5. Now as a group of four, they come to an agreement on how the two problems show the big idea. (e.g., Big Idea Blueprint)
6. Each group of four can then meet with another group of four to present the two problems and how they support the big idea.

Paired Conversation Protocol

To support and scaffold productive student conversation of students who are working in pairs and jointly solving a problem with more than one solution method.

Math Paired Conversation Protocol

PROBLEM:

Paraphrase and clarify problem for one another (in pairs)
(Talk about what is asked; what is given; what happens; what the units are, etc.)
☐ *TALK*

Estimate the answer
(Each partner generate and justify your own estimate; then compare them)
☐ *TALK*

METHOD A *(name it)*	Justify method ☐ *TALK*	METHOD B *(name it)*	Justify method ☐ *TALK*
Visuals, Drawings, Charts, Symbols, Calculations, Solution	Justify what you do	Visuals, Drawings, Charts, Symbols, Calculations, Solution	Justify what you do
	☐ *TALK*		☐ *TALK*

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Math Paired Conversation Protocol, cont.

| Check answer and compare to estimated ones ☐ *TALK* | Check answer and compare to estimated ones ☐ *TALK* |

Discuss (argue) which method you would recommend for problems like this. Why?
☐ *TALK*

Discuss connections between the two methods. How do they relate?
☐ *TALK*

Generate a final explanation for how to solve problems like this; use this problem as an example.
☐ *TALK*

*Co-create a similar problem, write it on the back of this sheet, and solve it
(then share the problem with others)*
☐ *TALK & WRITE*

Math Paired Conversation Protocol

Suppose it takes the Almond River 6 months to fill a reservoir, by itself, and it takes Belfair River 12 months to fill it, on its own. If both are flowing into the reservoir, how long will it take to fill it?

Paraphrase and clarify problem for one another (in pairs)
(Talk about what is asked; what is given; what happens; what the units are, etc.)
☐ *TALK*

Estimate the answer
(Each partner generate and justify your own estimate; then compare them)
☐ *TALK*

METHOD A *(name it)*	Justify method ☐ *TALK*	METHOD B *(name it)*	Justify method ☐ *TALK*
Visuals, Drawings, Charts, Symbols, Calculations, Solution	Justify what you do	Visuals, Drawings, Charts, Symbols, Calculations, Solution	Justify what you do
	☐ *TALK*		☐ *TALK*

Sample Based on the Paired Protocol

A: What do we gotta find?
B: How long they take to fill the reserve.
A: I say less than 6.
B: Why?
A: The Almond takes 6 months itself.
   So with extra water from this other one, less time, right?
B: Maybe. So, I think we draw it for one way to solve.
A: So like two rivers into a tank?
B: Yeah, and it fills up. After 3 months it's half full from Almond, right? But Belfair only fills up like, what?
A: 3 out of 12 is, a... quarter of it full.
B: So, a quarter's not full. So let's just guess it. Like I say/
A: /We can't do that. I think there's a right answer.
B: OK, let's try the other way, like a graph or a table.

Math Paired Conversation Protocol

PROBLEM:

Paraphrase and clarify problem for one another (in pairs)
(Talk about what is asked; what is given; what happens; what the units are, etc.)
☐ *TALK*

Estimate the answer
(Each partner generate and justify your own estimate; then compare them)
☐ *TALK*

METHOD A *(name it)*	Justify method ☐ *TALK*	METHOD B *(name it)*	Justify method ☐ *TALK*
Visuals, Drawings, Charts, Symbols, Calculations, Solution	Justify what you do	Visuals, Drawings, Charts, Symbols, Calculations, Solution	Justify what you do
	☐ *TALK*		☐ *TALK*

Squeezing the Most out of Each Problem

A dolphin jumps straight up out of the water with an initial velocity of 20 ft/s (sea level = starting height is 0). Using the vertical motion model, $h = -16t^2 + vt + s$ where v is initial velocity, s is starting height, t is time in seconds, and h is height.

a) How much time is the dolphin in the air before it hits the water again?
b) How high is the dolphin at its highest point?

How do we squeeze as much reasoning & language out of this problem as possible?

CONTACT INFO

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