Student Collaboration that Develops Mathematical Reasoning and Language
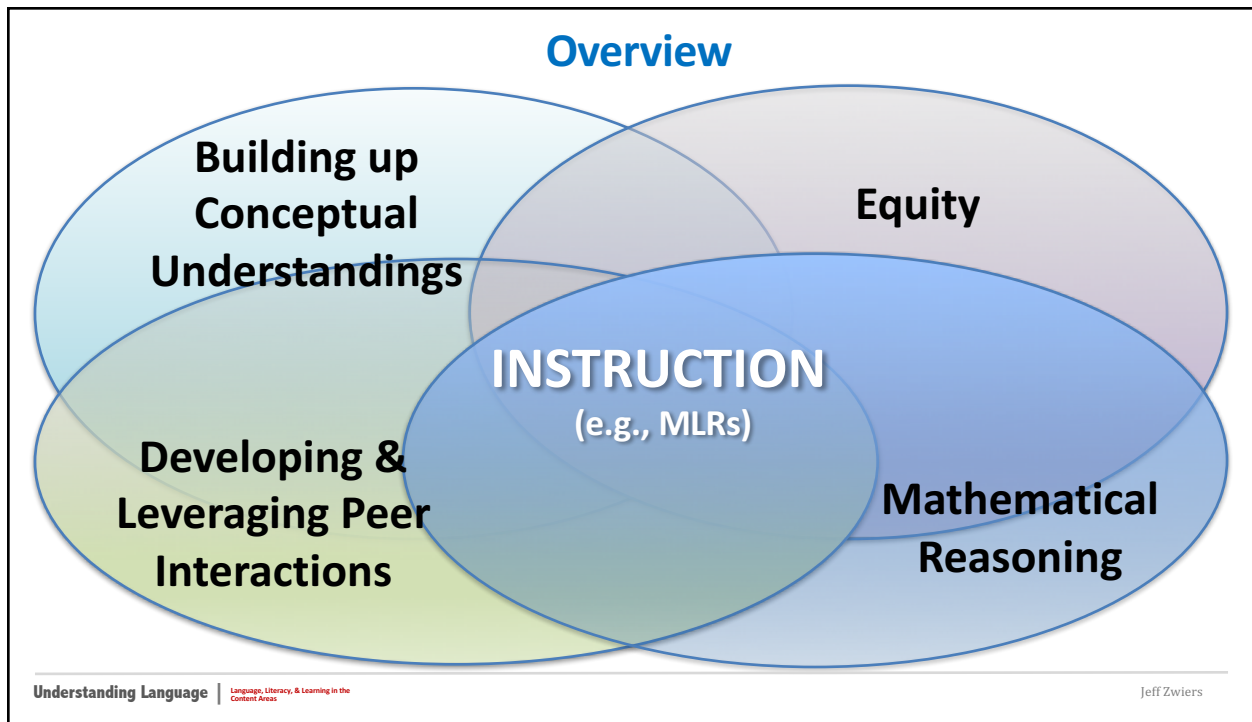
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Understanding Language | Language, Literacy, & Learning in the Content Areas

Jeff Zwiers

Overview

Building up Conceptual Understandings

Equity

INSTRUCTION
(e.g., MLRs)

Developing & Leveraging Peer Interactions

Mathematical Reasoning

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Language Development in Math

Mathematical language grows more—paradoxically—when its development is not the focus. It thrives the most:

❑ when the focus is on **doing engaging things that require the use of rich language** (e.g., building ideas, creating problems, reasoning), &

❑ when teachers understand **how to best support** this language use.

Equity

Creating, supporting, and scaffolding opportunities for collaboration that:

• encourages all **voices** to be heard.
• views all student knowledge and language as **assets**.
• **amplifies academic English** while also making the classroom a **safe space** for students to use variants of English and languages other than English.

How can academic interactions between students foster equity?

Building up Conceptual Understandings

My big idea is that we can **model situations using graphs and equations with variables in them**. For linear ones, the input is the x on the graph and the y is the output, and all the points end up on the line. For example, if you save the same amount of money each week, you can graph it and predict what you'll have after any number of weeks in the future. And you can also predict when something will catch something else. Like the problems where a faster car or a train tries to catch a slower one that left earlier. We used the formula for distance, speed times time. Then we set the two equations equal to each other and got the time they meet.

CCSS.MATH.CONTENT.8.EE.B.5
CCSS.MATH.PRACTICE.MP4

How do we get students to "think above" the problems that they solve?

Fostering Mathematical Reasoning

Standards for Mathematical Practice

- Reason abstractly & quantitatively
- Construct viable arguments and critique the reasoning of others
- Model with mathematics
- Look for and make use of structure
- Look for and express regularity in repeated reasoning
- Make sense of problems and persevere in solving them
- Use appropriate tools strategically
- Attend to precision

Logical justification of choices, ideas & conceptual understandings (e.g., claims, patterns, conjectures, procedures, generalizations, etc.)

3 Important Dimensions of Reasoning

III. Evaluate Reasoning	
I. Justify Procedures	**II. Make & Justify Claims, Conjectures, Generalizations, etc.**

Using principles, properties, definitions, theorems, counter-examples, patterns, representations, verifications, problems as examples, and/or wording and constraints of problems

Mathematical Language Routines

What makes something an MLR?

❑ Can be used all year
❑ Creates time, space, support, and motivation to authentically use language
❑ Language and content mutually reinforce each other
❑ Promotes equity
❑ Supports standards-aligned instruction

MLR Examples

I. **Information Gap Cards**

II. **Big Idea Blueprint Jigsaw**

III. **Stronger and Clearer Each Time**

IV. **Third Observer Conversation Coaching**
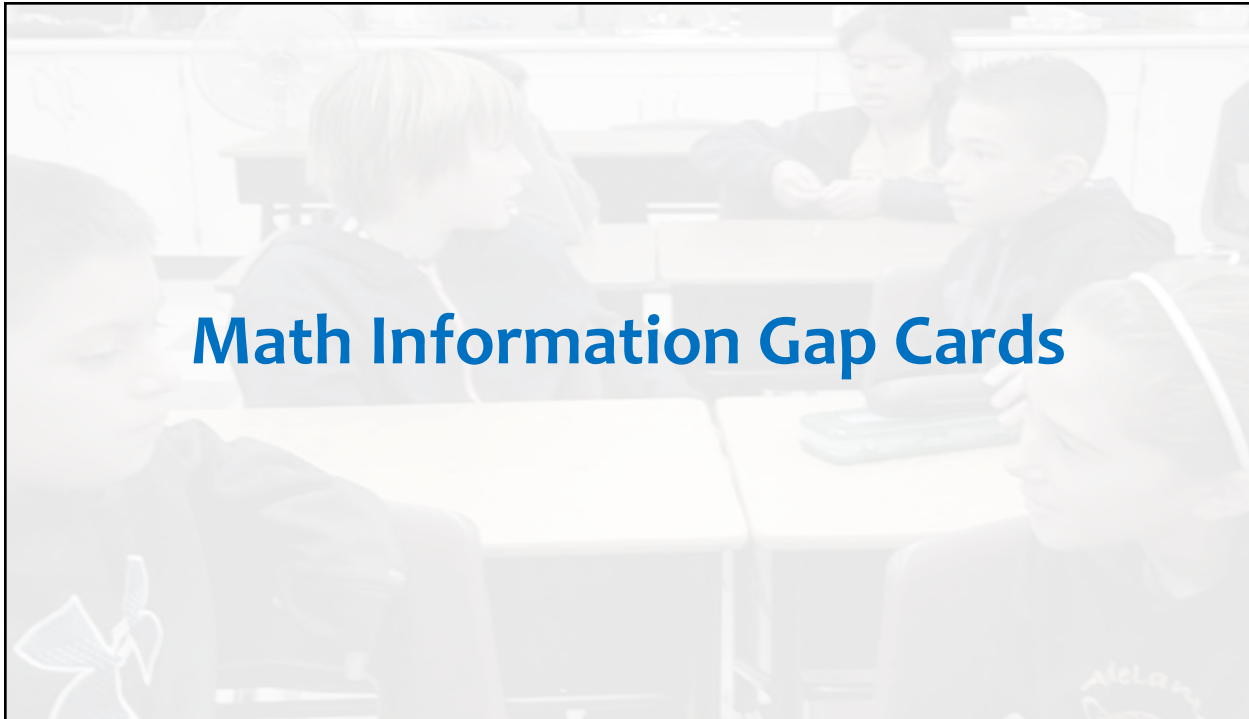
Math Information Gap Cards

Info Gap Cards: Procedure

1. **READ:** A summarizes situation to B. B paraphrases back to A, for agreement.

2. **QUESTION:** A asks B for specific information. Before answering, B prompts A to tell why the information is needed.

3. **JUSTIFICATIONS:** A explains how he or she will use the information.

4. **SOLVE:** A solves problem aloud, explaining, while B asks "Why?" and helps, if needed.

5. **BUILD up an Idea** for how math works and how to solve problems like this.

A: Leo has a party coming up. His friend decided to give him a party with the theme being his lucky number. So the friend bought a wide range of things at the store. He bought the same number of each thing, which was Leo's lucky number. He bought cupcakes, special balloons, streamers, games, and T-shirts. He also had to pay tax on the final total. What is Leo's lucky number?

B:
- cupcakes = 1 dollar each,
- special balloons = 3 dollars each
- streamers = 5 dollars each
- games = 12 dollars each,
- T-shirts = 10 dollars each.
- tax = 23 dollars.
- Total amount paid = 240 dollars.

Info Gap Cards: Sample Interaction

A: Do you know how fast the shuttle is orbiting?
B: Yes, but why do you want to know that?
A: Cuz I need to know it to figure out how long it takes to catch the satellite.
B: How will knowing the speed help you do that?
A: I'll use it and the satellite speed and the distance.
B: OK, it's going 16,800 miles per hour
A: Thanks. And how fast is the satellite going?
B: Why do you need to know that?
A: To know how long it'll take. If it's just a little slower, it'll take longer. I'll make an equation and put them equal to each other cuz that's where they meet.
B: That makes sense. The satellite is going 16,000 mph
A: Thanks. And how far are they apart when the shuttle starts its orbit?
B: Why?

A: Situation	B: Data
A shuttle enters an orbital path to catch up to an important satellite that isn't working properly. The shuttle is going faster than the satellite and mission control wants to know when the shuttle will reach it.	• Shuttle is orbiting at 16,800 mph • Satellite orbits at 16,000 mph • Shuttle enters orbit 1200 miles behind the satellite • Orbit is 400 miles from the Earth's surface

CLASSROOM VIDEO

Context

- 4th math
- Review of volume
- Info gap cards

A: Because of an algae bloom, you need to treat the sea water in the jellyfish tank at the Monterey Bay Aquarium. Your partner measured the tank this morning. You need to know to know how many drops of treatment liquid to use.

B:
- The bottom of the tank is 10 feet by 20 feet.
- The tank is 14 feet high.
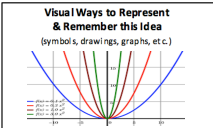- The treatment solution is one drop per cubic foot of water.

Idea Building Blueprint

Jigsaw

Idea Building Blueprint Jigsaw
New Example of MLR7 (Compare & Connect)

1. **Give** groups of 4 students 4 different problems that have some similarities in how they foster and support a possible claim or key concept they are supposed to learn.

2. **Solve.**

3. **Share** problems and solution methods.

4. **Pair up** and answer the question, "What big mathematical concept, idea, or claim can you make using all four problems?"

5. **Meet** in group to come up with a final "top" claim or two.

Idea Building Blueprint - Math

IDEA (Key Concept; Standard; Answer to Essential Question) **in my own words:**

You can predict what a graph will look like by analyzing the function

Why it's important:
Graphs show relationships

How this connects to or depends on other ideas:
Domain and range

Real world applications (if any):
Bacteria growth

Visual Ways to Represent & Remember this Idea
(symbols, drawings, graphs, etc.)

Principle, Property, Theorem, Definition	Principle, Property, Theorem, Definition
Lowest or highest limit of parabola is vertex	
Sample Problem (☐ Given to me ☐ Found by me ☒ Created by me/us)	**Sample Problem** (☒ Given to me ☐ Found by me ☐ Created by me/us)
Barrel over waterfall	Dolphin jumping
How this problem supports/shows the big idea Negative quadratic parabola – just down	*How this problem supports/shows the big idea* Negative quadratic parabola, up & down
Sample Problem (☐ Given to me ☒ Found by me ☐ Created by me/us)	**Sample Problem** (☒ Given to me ☐ Found by me ☐ Created by me/us)
Bacteria Growth	Mt. Everest growth
How this problem supports/shows the big idea Positive quadratic parabola	*How this problem supports/shows the big idea* Same ratio over time makes a line

Stronger-Clearer
Each Time Activities
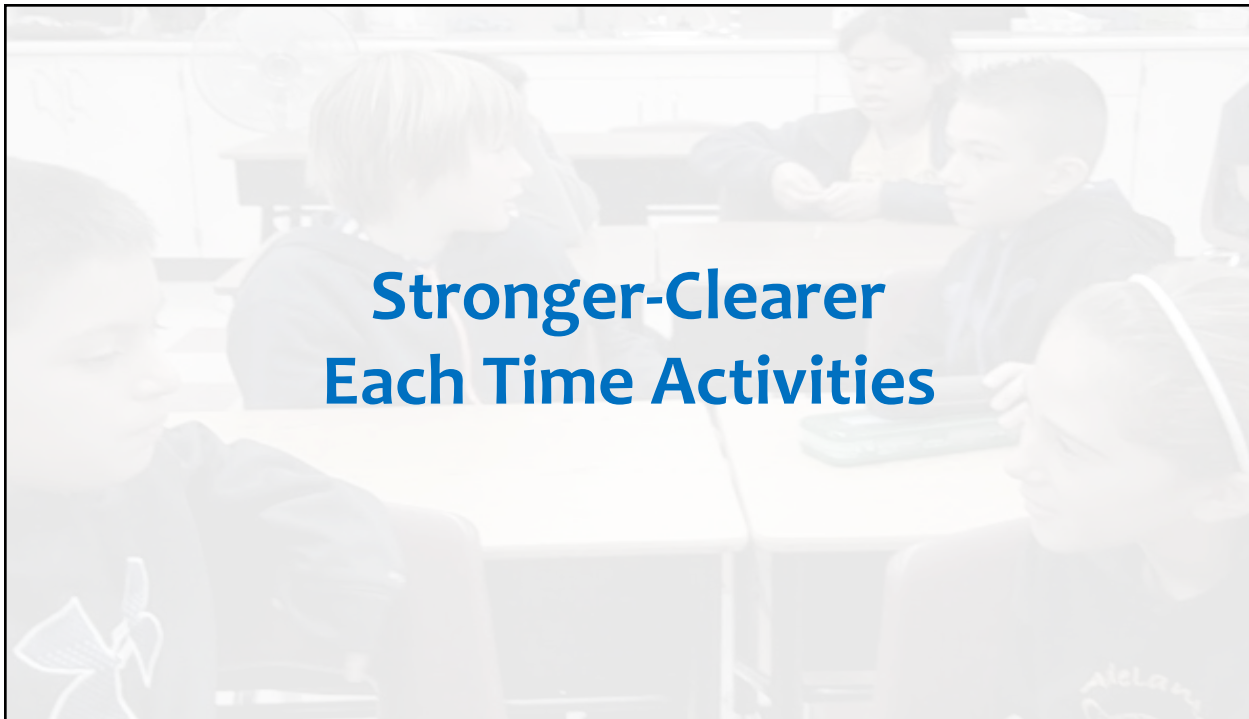
"Stronger & Clearer Each Time" Activities

1. Prompt for an original response that can build
2. Successive partners borrow and use the language, ideas, and justifications from previous partners. Turns get:
   - **Stronger** (often longer) with better justification, details, and examples, and
   - **Clearer** with more precise terms, better representations, and linked, organized, complete sentences.
3. Listeners push for clarification & support, and offer ideas

Understanding Language | Language, Literacy, & Learning in the Content Areas

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'Stronger & Clearer Each Time' Grid

I think to draw it. Then cut up into ounces of each thing.

Take one or two-word notes switch partners! Remember, stronger & clearer!

I think we gotta find like how much ounces for a dollar it is. Like one dollar you get, I don't know.
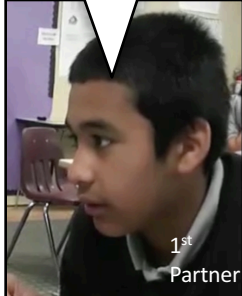
Pre-write:

Cut into ounces each bottle. Then add or times.

_ _ _ _ _ _ _ _ _ _ _

Post-write:

David

1st Partner

Darla decides to buy a sports drink. Her choices are a 20-ounce bottle for $1.49 or a 32-ounce bottle for $2.49. Which is the better value? Explain what you did to get your answer and why.

'Stronger & Clearer Each Time' Grid

I think to draw it. Then cut up into ounces of each thing.

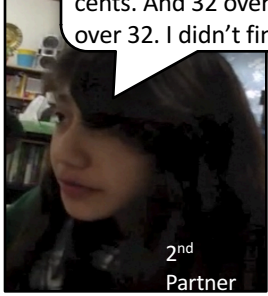Take notes & switch partners! Remember to say "because" to justify your steps

I think we gotta find like how much ounces for a dollar it is. Like one dollar you get, I don't

I wanna find how much a dollar can get, like a ounce. So 1 dollar is like 1 over 1.50, two thirds. So I take 2/3 of it?

I kinda did that, but I did for one ounce, its cost. I did 1.49 over 20. I think it's like 70 cents. And 32 over, no, 2.49 over 32. I didn't finish it.

David

2nd Partner

Darla decides to buy a sports drink. Her choices are a 20-ounce bottle for $1.49 or a 32-ounce bottle for $2.49. Which is the better value? Explain what you did to get your answer and why.

'Stronger & Clearer Each Time' Grid

Pre-write:

Cut into ounces each bottle. Then add or times it.

_ _ _ _ _ _ _ _ _ _ _ _ _ _

Post-write:

For this type of problem, you need to find out how much each ounce is. So I did cost over number of ounces. I got 7 for the 20 bottle.

it. Then cut of each thing.

OK, now write down what you learned. Start with "For this type of problem, you…"

y much a dollar nce. So 1 dollar , two thirds. So

I think we gotta find like how much ounces for a dollar it is. Like one dollar you get, I don't know

I kinda did that, but I did for one ounce, its cost. I did 1.49 over 20. I think it's like 70 cents over, no, 2.49 over 32. I didn't finish it.

First I think how much ounces for a dollar. But Alan said to find out how much costs each ounce. I agree. So I just divided it. So 1.49, divide 20 in it; Alan said 70, but I think it's 7.
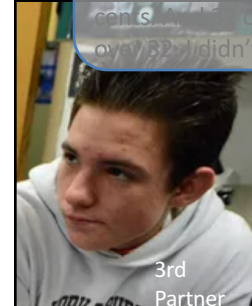
David

3rd Partner

decides to buy a sports drink. Her choices are a 20-ounce e for $1.49 or a 32-ounce bottle for $2.49. Which is the better value? Explain what you did to get your answer and why.

CLASSROOM VIDEO
for Stronger & Clearer Each Time Pairs

Context

- HS Algebra
- The prompt was, "What is a function? Use the Dinner Game task to support your idea.

- [Dinner Game Task: Roll a die and if odd, you get that amount in dollars; if even, you spin for a possible 0, 50 cents, or 1 dollar.] (from Margaret Sullivan)
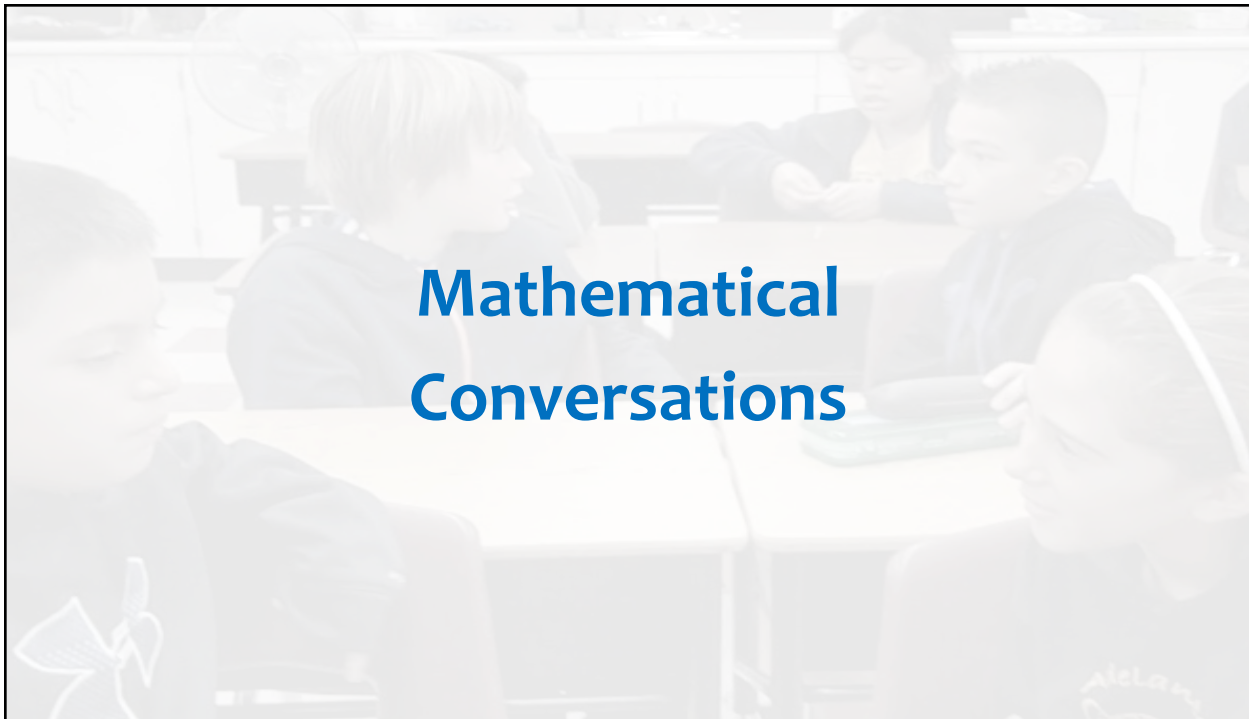
"Stronger & Clearer Each Time" **Grid**

Name	If a tire rotates at 400 revolutions per minute when the car is traveling 72km/h, what is the circumference of the tire? Estimate the answer, come up with two solution methods, and be able to explain them both to others to help them solve any future problems like this.
Me	(just two or three key words, if any)
1.	
2.	
3.	
Me	

Listeners can & should:
- *Prompt for clarification*
- *Prompt for support*
- *Help with "What about…?"*

I first thought I needed to figure out…because…
I know that in …, the tire goes…
I wonder ….

Mathematical Conversations

Constructive Conversation Skills: Building an Idea

Build up an Idea (at least to here)

| Support | Clarify |

Use math principles & the description of the problem

- Ask ?'s
- Define
- Explain
- Paraphrase
- Model
- Represent

Constructive Conversation Skills: Building an Idea

Build up an Idea (at least to here)

Support · Clarify · Support

Support · Clarify · Support

Support · Clarify

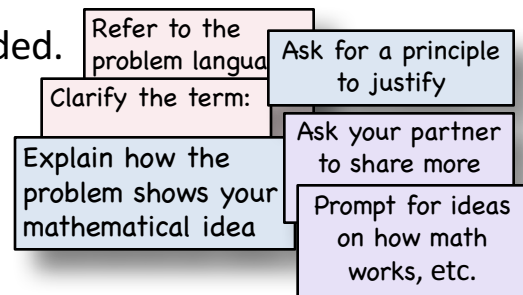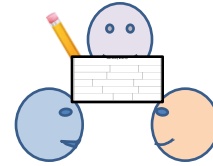Use math principles & the description of the problem
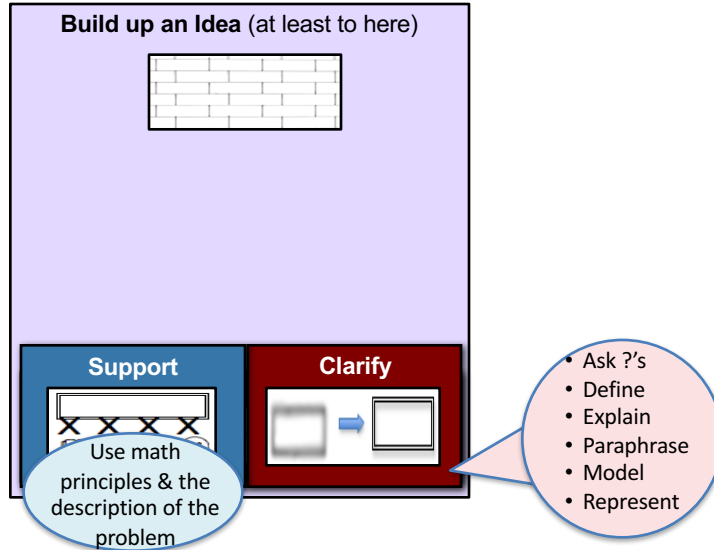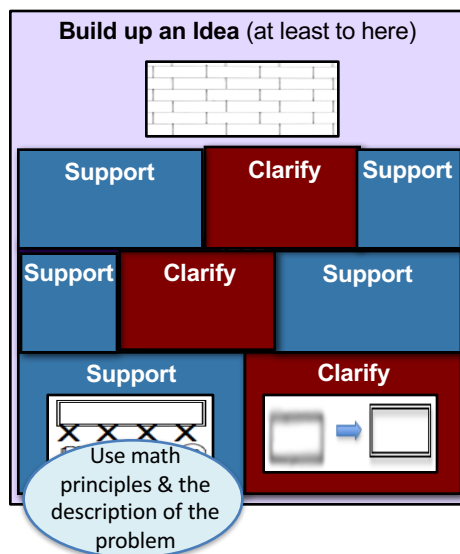
Hand motions

Third Observer Notes & Coaching
(Example of MLR8 - Discussion Supports)

1. As talkers converse, observer takes notes on their justifying of procedures and any other claims or ideas discussed.

2. Makes suggestions verbally (or silently on sticky notes), when needed. (e.g., suggestions that prompt for justification of procedures, conjectures, claims, patterns, & conceptual understandings)

Refer to the problem langua

Clarify the term:

Ask for a principle to justify

Explain how the problem shows your mathematical idea

Ask your partner to share more

Prompt for ideas on how math works, etc.

Conversation Sample

A: So, what's happening?
B: Elvia looks at the two places to decide, like my mom does in the store sometimes. What about you?
A: I remember that problem yesterday of the two cars. They went different speeds. It's like the two different costs of apples.
B: Oh yeah, she did a graph thing and the lines crossed.
A: I think that's what we have to do.
B: OK….
   Why?
A: Like here are two lines. They cross cuz of different prices going up.
B: Oh yeah. Like the car problem. So let's draw the graph.

Elvia wanted to pick apples from an orchard as cheaply as possible. Palomar Orchards sells apples for 8 dollars for every ten pounds, plus a flat entrance fee of 10 dollars. Ted's Orchard sells apples at 1 dollar per pound with no entrance fee. At what amount would she change from which orchard to the other? *Discuss how to solve this in more than on way, and then how to solve future problem like this one.*

Conversation Sample
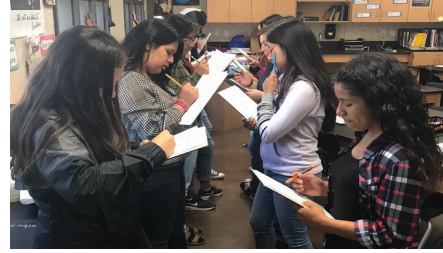
A: Yeah. But we can't just draw the lines without the numbers.

B: Why not?

A: Cuz we have to use numbers, like the points, like x and y. But they aren't there now.

B: Why don't they just give us the points?

A: Yeah, that'd be better. I don't know. May be too easy. (silence)…

B: Maybe we can do a table, like yesterday. It had lots of numbers.

A: Oh yeah. OK, put how much on the left, here. And cost here. You buy more and the price goes up. So pounds is x and cost is y.

B: That's just for one orchard. I think we gotta do two tables.

A: OK. So for Palomar, 10 pounds is 8 dollars.

B: Don't forget the fee, the entrance fee.

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