INFERENCE FOR A POPULATION MEAN

- t-Distribution
- Critical Values using t
- Testing a Population Mean When σ is Unknown
- Effect of Knowing σ
- Inference for a Population Mean Using R

University of Pittsburgh

REVIEW: SAMPLING DISTRIBUTIONS

- If σ is known and $\bar{X}$ is approximately normal, then $\bar{X}$ is standardized:

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

- Because it is a parameter, σ is usually _________. Instead, we estimate σ using the ______________________________. However:

$$Z \neq \frac{\bar{X} - \mu}{s / \sqrt{n}}$$

- Instead…

$$t = \frac{\bar{X} - \mu}{s / \sqrt{n}}$$

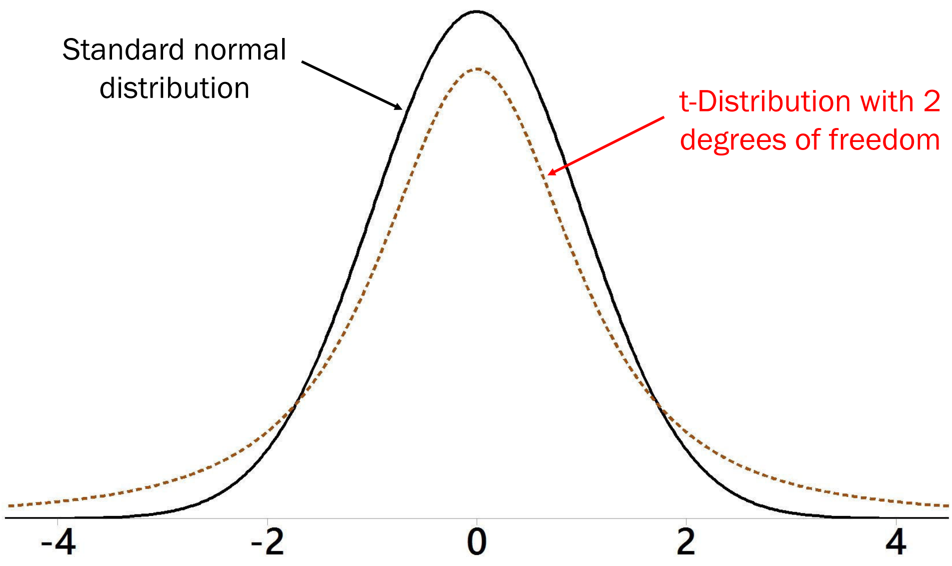
where t stands for the ___________________.

T-DISTRIBUTION

- **t-Distribution:** continuous probability distribution similar to the standard normal in that it is:
  - Symmetric and bell-shaped
  - Centered at 0

but differs from the standard normal because it:
  - Is a family of distributions whose shape changes depending on the **degrees of freedom**
  - Has fatter tails and is shorter in the middle

Standard Normal vs. T-Distribution



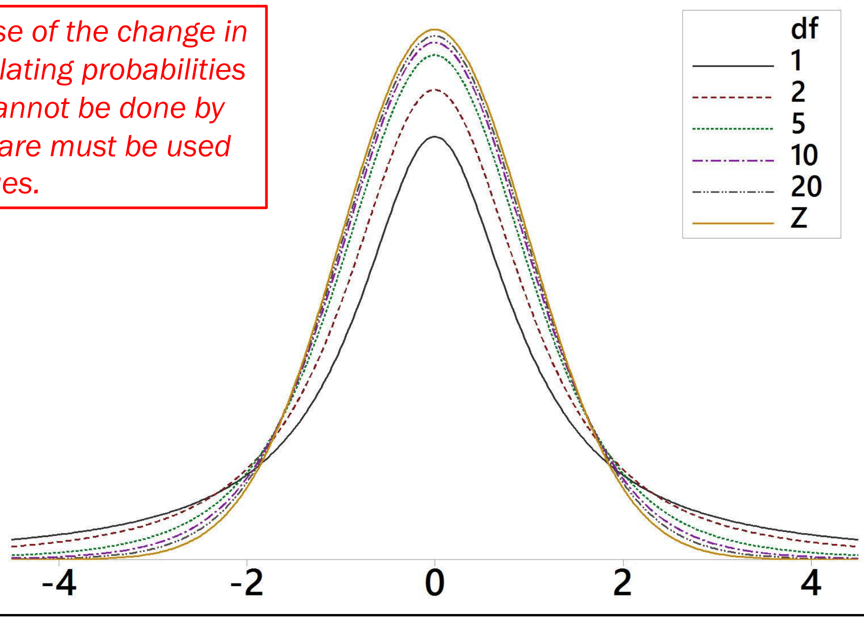
Degrees of Freedom

- **Degrees of freedom:** measure of how much information is contained in a sample that determines the shape of the distribution that is appropriate for the situation
  - Occurs in the t-distribution, chi-squared distribution, and F-distribution
  - Range from 1 to infinity and cause the distributions to change shape
  - Often denoted by the letter v and is placed in the subscript of the statistic (i.e. t_v, χ^2_v, F_{v_1, v_2})
    - More on the chi-square distribution and F-distribution later in the semester…

- In the **t-distribution**, degrees of freedom are dependent upon the sample size.
  - ______________ → ______________ → ________________________

Examples of T-Distributions

Note: Because of the change in shape, calculating probabilities in the tails cannot be done by hand. Software must be used to find p-values.



T-Distribution Table

Critical Values of the t-Distribution

Values give upper tail critical values for area in the upper tail of the t-distribution

Degrees of Freedom	0.25	0.20	0.15	0.10	0.05	0.025	0.02	0.01	0.005	0.0025	0.001	0.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.610	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745

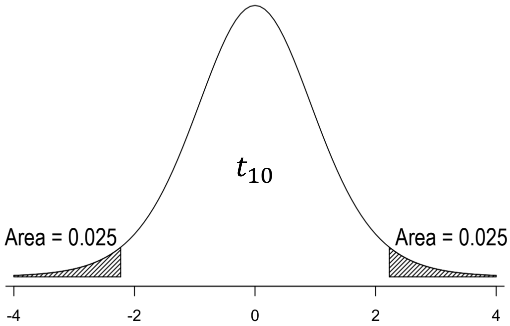
↓ Table continues ↓

- **Top row:** area in upper tail of t-distribution
  - Multiply chosen area by 2 and subtract from 1 to get confidence level
- **Left column:** degrees of freedom
- **Intersection:** t-statistic corresponding to chosen degrees of freedom and area

<u>Note</u>: Complete t-distribution table posted on Canvas.

Example: Critical Values

- **Question:** What critical value should be used for a 95% confidence interval with 10 degrees of freedom?
- **Answer:** ________________
  - Leaves out ___ total in tails – ______ in each
  - Recall: ________________



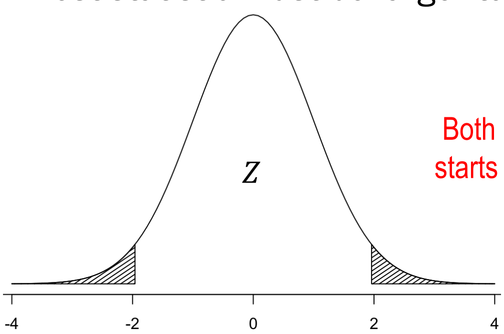
Critical Values of the t-Distribution

Values give upper tail critical values for area in the upper tail of the t-distribution

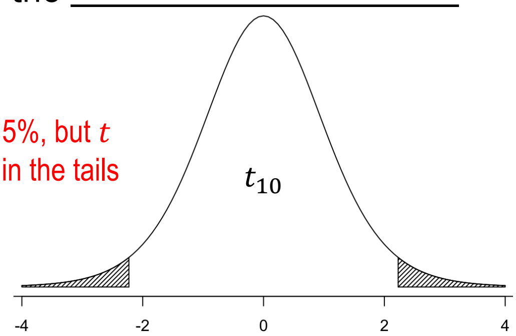
Degrees of Freedom	0.25	0.20	0.15	0.10	0.05	0.025	0.02	0.01	0.005	0.0025	0.001	0.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.610	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745

Comparing Standard Normal and T-Distribution

- **Question:** What do you notice about the critical values when using the standard normal distribution compared to the t-distribution?
- **Answer:** The critical values for the t-distribution are all _______ than the corresponding values from the standard normal distribution
  - Requires _____________ to reject H_0 as degrees of freedom get ________
  - Test statistic must be larger to get into the ______________________



Both areas total 5%, but t starts farther out in the tails



MOTIVATION: ONE-SAMPLE T-TEST

- **Scenario:** Recent study found that college students are more likely to drop out if they work at a part-time job more than 20 hours per week. A professor surveys her students who have jobs to find out how many hours they worked at their job last week.
- **Question:** Is there sufficient evidence to conclude that college students are working more than 20 hours per week on average?
- **Observations:**
  - Doing inference on a ________________ (average hours worked per week)
  - Population standard deviation is ____________
  - Because ___ is unknown, the ________ for a population mean is _________
  - Can calculate the ________ standard deviation __ and use the ___________

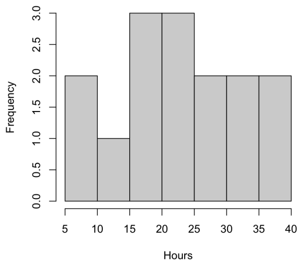
ONE-SAMPLE T-TEST

Step	Description
Used for	Performing inference on a single unknown population mean when the population standard deviation (σ) is unknown
Conditions	Shape of sampling distribution of sample mean must be normal Population standard deviation unknown
Test Statistic	$t = \frac{\bar{X}-\mu_0}{s/\sqrt{n}}$ with $n-1$ degrees of freedom
Confidence Interval	$\bar{X} \pm t_{n-1}\frac{s}{\sqrt{n}}$ with $n-1$ degrees of freedom

***Note:** Everything about this test is identical to the one-sample Z-test except the distribution and knowledge of the standard deviation. If you know the population standard deviation, use the one-sample Z-test.*

EXAMPLE: ONE-SAMPLE T-TEST

- **Question:** Is there sufficient evidence to conclude that college students are working more than 20 hours per week on average using a 5% level of significance?
- **Summary:**




```
> mean(hours$hours)
[1] 24.2
>
> sd(hours$hours)
[1] 9.480808
>
> nrow(hours)
[1] 15
```

- **Hypotheses:** __________________ vs. __________________
- **Normality:** ______________
  - Rule of Thumb ___: Histogram close to ________, but sample size is ___

EXAMPLE: ONE-SAMPLE T-TEST (CONT.)

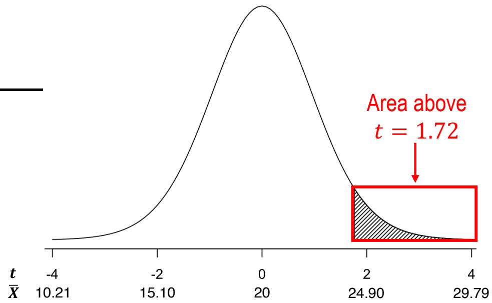
- **Summary:**

```
> mean(hours$hours)
[1] 24.2
```

```
> sd(hours$hours)
[1] 9.480808
```

```
> nrow(hours)
[1] 15
```

- **Test Statistic:** $t =$ ____________ $=$ ________
  - **Interpretation:** If college students actually work __________________ on average, then ______ is ______ standard errors ________ the mean.
- **Degrees of Freedom:** $df =$ ________ $=$ ___
- **P-Value:** $p =$ ________
  - Area above _________ in a t-distribution with ____ df



EXAMPLE: ONE-SAMPLE T-TEST (CONT.)

- **Summary:**

```
> mean(hours$hours)
[1] 24.2
```

```
> sd(hours$hours)
[1] 9.480808
```

```
> nrow(hours)
[1] 15
```

- **95% Confidence Interval:** ___________________ $=$ ________________
  $=$ ________________
- **Conclusions:**
  - ______________ and conclude that college students are ____________ ________________________________ hours per week at jobs.
    - $p =$ __________
  - We are _____________ that the average number of hours that all college students work at jobs is _________________________. This is consistent with the hypothesis test because ______________________________.

EXAMPLE: EFFECT OF KNOWING σ

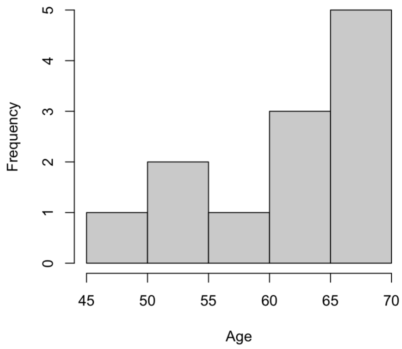
- **Scenario:** Suppose we had known that $\sigma = 9.48$.
- **Question:** What would have been different about the test?
- **Answer:** Would have used __________________
  - **Test Statistic:** $Z =$ _________ $=$ _____
    - Value does ___________, but distribution ___________
  - **P-Value:** $p =$ ______
    - _____ area in the tail from using ___
  - **Confidence Interval:** ___________________
    - Critical value is _____ (from __) instead of 2.145 (from t)
  - **Decision and Conclusion:** __________ and conclude that college students ___________________________ than 20 hours per week working.

Example: Effect of Small Sample Size

- **Scenario:** Failed to reject $H_0: \mu = 20$ and concluded that 20 was a plausible value for the mean number of hours worked per week.
- **Question:** Why is a small sample size damaging to rejecting the null hypothesis in a one-sample t-test?
- **Answer:**
  - Makes the _______________ larger, which keeps the test statistic ______
    - Knew this from one-sample Z-test
  - Keeps the ____________________ smaller
    - Makes the critical value _______ in confidence intervals
    - ____________ to get a small p-value

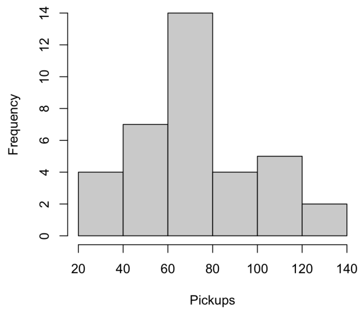
Example: Small Skewed Dataset

- **Scenario:** Random sample of ages of retirement for 12 people in a company who retired this year
- **Question:** What is a 95% confidence interval for the mean retirement age?
- **Answer:** _________________ with the calculation
  - Histogram ____________________ and _______
    - Don't know if the shape of the sampling distribution is _________
    - No _______________ apply
  - Nonparametric methods could be used, but our job is done as they go beyond the scope of this course



Example: One-Sample T-Test Using R

- **Scenario:** The average adult picks up their cell phone 58 times per day. We want to study the habits of college students, so we take a random sample of 36 and ask each student to report the number of phone pickups yesterday.
- **Question:** Is 58 a plausible value for the average number of times college students pick up their phone each day using $\alpha = 0.01$?



- **Hypotheses:** _____________ vs. _____________
- **Normality:** ______________
  - Rule of Thumb ___: Histogram is ____________, but sample size is ____________

One-Sample T-Test Using R

- Because the shape of the t-distribution changes depending on the degrees of freedom, there is no easy way to get p-values by hand.
- R can perform a one-sample t-test and provide an exact p-value so critical values do not need to be used.

- **Function:**

X (data frame$variable) — Alternative hypothesis form

```
t.test(phone$pickups, alternative = "two.sided",
       mu = 58, conf.level = 0.99)
```

Hypothesized mean — Confidence level as a decimal

- **Note:** The one-sided tests are "`less`" and "`greater`".

Example: One-Sample T-Test Using R (Cont.)

- **Output:**

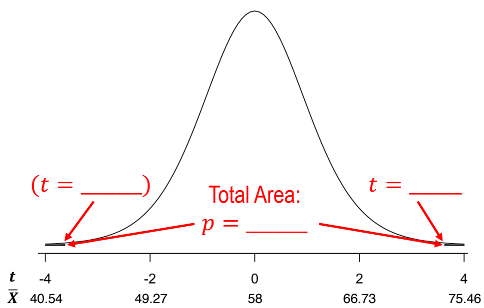
```
> t.test(phone$pickups, alternative = "two.sided", mu = 58, conf.level = 0.99)

        One Sample t-test

data:  phone$pickups
t = 3.6339, df = 35, p-value = 0.0008871
alternative hypothesis: true mean is not equal to 58
99 percent confidence interval:
 61.97238 85.74984
sample estimates:
mean of x
 73.86111
```

```
> mean(phone$pickups)
[1] 73.86111
>
> sd(phone$pickups)
[1] 26.1885
>
> nrow(phone)
[1] 36
```

- **Test Statistic:** $t =$ __________ $=$ ______
- **P-Value:** $p =$ __________
- **99% Confidence Interval:** ____________



Example: One-Sample T-Test Using R (Cont.)

- **Conclusions:**
  - __________ and conclude that the average number of times that college students pick up their cell phone each day _________________________.
    - $p =$ ______ $<$ ______
  - We are ____________ that the average number of cell phone pick ups by college students is ______________________. This is consistent with the hypothesis test because _______________________ in the interval.
  - It appears as if college students pick up their cell phones _____________ than the general population.

EXAMPLE: CHOICE OF ALTERNATIVE

- **Scenario:** Previous test yielded a sample mean of 73.86 pickups and found that $\mu = 58$ was not plausible.
- **Question:** Based on what we know about college students, was a two-sided test reasonable?
- **Answer:** ______________
  - College students spend ________________________
  - Probably should have suspected that the true mean was _______
  - ________________ alternative was probably more appropriate
- **Takeaway:** __________ tests should be used if you are __________ with the population, but use a __________ alternative if you have a _______ about the direction of the alternative.

EXAMPLE: ADJUSTING ALTERNATIVE HYPOTHESIS IN R

- **Question:** What would change about the R function if we wanted to use an upper one-sided test?
- **Answer:** Only the _____________ argument

```
t.test(phone$pickups, alternative = "__________",
       mu = 58, conf.level = 0.99)
```

- **Question:** What would have been different about the test results?
- **Answer:** P-value would be ____________
  - Still __________ but conclude that _______

EXAMPLE: COMPARING ONE-SIDED AND TWO-SIDED OUTPUTS

$H_0: \mu = 58$ vs. $H_A: \mu > 58$

```
> t.test(phone$pickups, alternative = "greater", mu = 58, conf.level = 0.99)

        One Sample t-test

data:  phone$pickups
t = 3.6339, df = 35, p-value = 0.0004435
alternative hypothesis: true mean is greater than 58
99 percent confidence interval:
 63.22106      Inf
sample estimates:
mean of x
 73.86111
```

Do not use the confidence interval from the one-sided test

$H_0: \mu = 58$ vs. $H_A: \mu \neq 58$

```
> t.test(phone$pickups, alternative = "two.sided", mu = 58, conf.level = 0.99)

        One Sample t-test

data:  phone$pickups
t = 3.6339, df = 35, p-value = 0.0008871
alternative hypothesis: true mean is not equal to 58
99 percent confidence interval:
 61.97238 85.74984
sample estimates:
mean of x
 73.86111
```

If you need a confidence interval to go along with a one-sided test, you must run a second line of code with a two-sided alternative and combine the results.